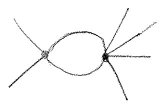
Renormalizability

Ⓐ power corrections

Once we assume Wilson's point of view ($\Rightarrow$ cut-off Λ on the effective action) there is no problem with introducing non-renorm. irrelevant Ops $\Rightarrow$ they remain irrelevant but renormalize relevant & marginal operators.

Eg.

$$\frac{c_1}{\Lambda^2}\phi^6 \Rightarrow \delta\lambda \sim \frac{c_1}{\cancel{\Lambda^2}}\frac{\cancel{\Lambda^2}}{16\pi^2}$$



$$\frac{c_1}{\Lambda^2}\phi^6 \otimes \lambda\phi^4 \Rightarrow \delta c_1 \sim \frac{c_1\lambda}{16\pi^2}\log(\Lambda^2)$$

$\Rightarrow$ All operators allowed by the symmetries of the system are generated (even if $\emptyset$ at the tree-level)

$\Rightarrow$ If the couplings are natural $\Rightarrow$ remain natural
" there is at least one natural couplings $\Rightarrow$ all other coupling tend to become natural [otherwise fine tuning]

$\Downarrow$

- The real problem are the RELEVANT operators since they receive power corrections

- The fact that the irrelevant operators renormalize the relevant/marginal ones seems to imply we will never be able to compute these couplings

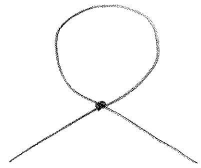
  $\hookrightarrow$ problem solved with "mass-independent" renormalization procedures

Power-corrections & fine tuning

The problem with relevant ops. are the power corrections

E.g.:

$$\delta m^2 = \frac{\lambda^2}{16\pi^2}\Lambda^2$$



Within a NATURAL EFT, either there are no relevant operators (e.g. only mass-less states protected by some symmetry principles) or their natural scale is set by the EFT cut-off

E.g.: (SM Higgs mass term)

$$\delta \mu_H^2 \sim \frac{1}{16\pi^2}\Lambda^2 \sim (100\ \text{GeV})^2_{exp}$$

Higgs mass term

$$\Downarrow$$

$$\Lambda \lesssim (4\pi) \times 100\ \text{GeV}$$

↳ why we hope to see physics beyond SM @ LHC

Counter-example: the cosmological constant

$$\Omega_{obs} \approx (10^{-3}\ \text{eV})^4$$

$$\Downarrow$$

2 possible "solutions"

Anthropic principle

$\Omega \ll \Lambda^4$ "selected" in our Universe

Modification of gravity at small scales

$$d \lesssim \frac{1}{10^{-3}\,\text{eV}} \sim 10^{-4}\ \text{m}$$

B) Logarithmic corrections → scaling violations, dim. regularization, matching

The logarithmic dependence from the cut-off introduced by renormalization is qualitatively different and much more interesting than power corrections:

(1) ⇒ Cannot be completely reabsorbed into coupling-redefinition and leads to observable effects

$$\Rightarrow \quad \delta\lambda \sim \frac{\lambda^2}{16\pi^2} \log\left(\frac{\Lambda^2}{s}\right)$$

I.R. sensitivity

(see also example of QED)

(2) ⇒ Modify the naive scaling of marginal operators

As you now from QFT courses

$$\lambda\phi^4 \rightarrow \underbrace{\lambda\left[1 + \frac{c\lambda^2}{16\pi^2}\log\left(\frac{\Lambda^2}{\mu^2}\right)\right]\phi^4}_{\lambda(\mu)\,\phi^4} + \text{generic non local terms}$$

$$\parallel$$

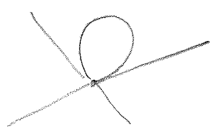
$$\lambda\phi^4\left(\frac{\mu}{\Lambda}\right)^{+\frac{c}{16\pi^2}\lambda^2}$$

anomalous dimension

All marginal operators either become relevant or irrelevant

$\overline{MS}$ and matching

The renormalization of a EQFT becomes particularly simple if one adopts a mass-independent regularization procedure such as the dimensional regularization + MS



$C_1 \dfrac{\phi^6}{\Lambda^2} \Rightarrow$ no $\delta\lambda$ $\Rightarrow$

the (power-correction) shift in the coupling was not a physical effect and can be re-absorbed in the def. of the coupling

Logarithmic corrections remains unchanged.

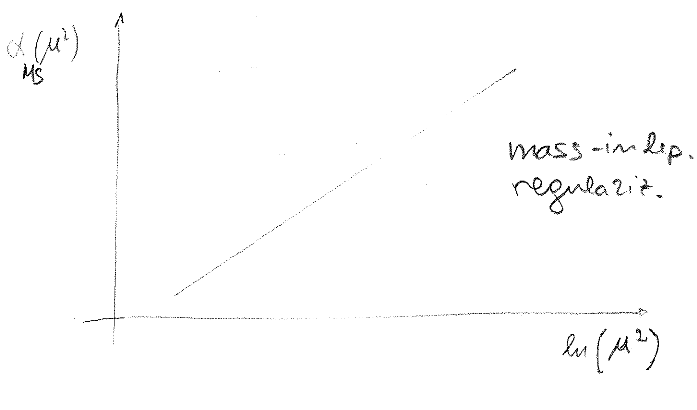
$\Rightarrow$ The only problem with $\overline{MS}$ is that heavy fields do not decouple explicitly

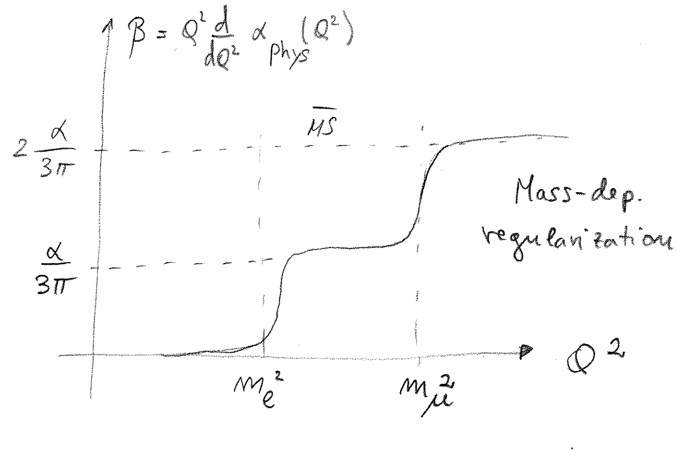
E.g.: QED with 2 non-degenerate fermions (e, μ)

$$\Pi(q^2)\Big|_{\substack{\text{dim}\\ \text{reg.}}} = -\frac{2\alpha_0}{\pi}\int_0^1 dx\, x(1-x)\left[\ln\left(\frac{m_e^2 - q^2 x(1-x)}{\mu^2}\right) + \ln\left(\frac{m_\mu^2 - q^2 x(1-x)}{\mu^2}\right) + 2\,C(\mu^2)\right]$$

part subtracted to define $\alpha(\mu)$ in $\overline{MS}$ scheme

$$\alpha_{\overline{MS}}(\mu^2) = \frac{\alpha_0}{\left[1 - \frac{\alpha_0}{3\pi}\, 2\,C(\mu^2)\right]}$$





In the $\overline{MS}$ we have to "impose by hand" the decoupling by an appropriate matching procedure.

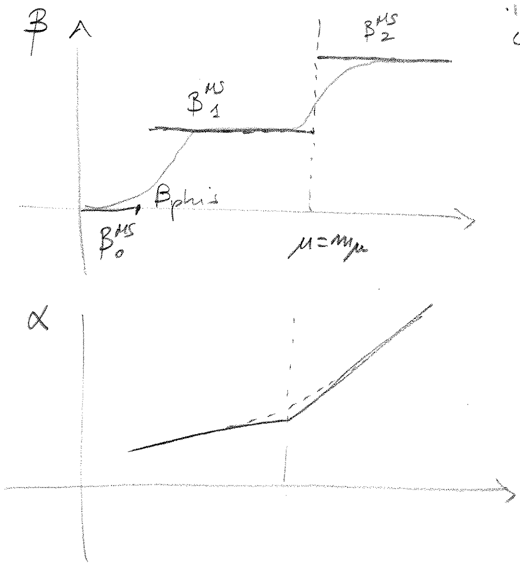
$\underline{\underline{\text{N.B}}}$: If we don't impose by hand the decoupling we have a baddly behaved pert. theory

$\underline{\text{E.g.}}$: $\quad q^2 = -Q^2 \qquad m_e^2 \ll Q^2 \ll m_\mu^2 \qquad + \quad$ choose MS renormalization condition $\mu^2 = Q^2$

$$\Delta\Pi(Q^2, \mu^2) = -\frac{e^2}{2\pi}\int_0^1 dx \; x(1-x)\Big[\underbrace{\cancel{\ln(x(1-x))}}_{O(1)} + \underbrace{\ln\left(\frac{m_\mu^2}{Q^2}\right)}_{\text{big log.}}\Big]$$

$\Rightarrow$ which could be reabsorbed in the coupl. constant

$$\begin{cases} \mu > m_\mu & \Rightarrow \; 2 \text{ flavours} \\ \mu < m_\mu & \Rightarrow \; 1 \text{ flavour} \end{cases}$$



$$\left[\; \alpha^{(MS)}_{N_f=2}(\mu = m_\mu) = \alpha^{(MS)}_{N_f=1}(\mu = m_\mu) \;\right]$$

$\Downarrow$

- Well behaved pert. theory
- matching scale acting as
  UV cut off of the low-scale th.
  IR " " " " high- " th.

$\Downarrow$

Simplest example of matching between 2 effective theories

The matching procedure in generic EQFT (with non-renormalizable operators) is slightly more complicated :

N.B.: Matching with mass-independent renormaliz. is by far more practical with respect to "integrating-out" at the functional level

$$\mathcal{L}_{eff}(\phi_L) = \mathcal{L}_{ren}(\phi_L) + \sum_i c_i \frac{O_i^d}{\Lambda^{d-4}}$$

Write-down all the ops. allowed by the symm. up to the desired order in $\frac{1}{\Lambda^n}$

$$\langle T\{\phi_L(x_1)\ldots\phi_L(x_n)\}\rangle_{\substack{full\\ \mu\lesssim\Lambda}} = \langle T\{\phi_L(x_1)\ldots\phi_L(x_n)\}\rangle_{\substack{eff\\ \mu}}$$

n. of condition needed = n. of unknown $c_i(\mu)$ $\Rightarrow$ $c_i(\mu)$ fixed

N.B. One can use any kind of I.R. regulator and any kind of ext. state in the 2 Green functions since the I.R physics is the same $\Rightarrow$ I.R. sensitivity cancels out $\Rightarrow$ $\boxed{\text{good choice } p_i \gg m_{light}}$

$\Downarrow$

$C_i(\mu)$ encode the short-distance physics

2 big classes of EFT
- matching procedure possible (at least in part) $\Rightarrow$ Fermi, HQET
- matching not possible $\Rightarrow$ CHPT, SM