University of Zurich[UZH]

Solid State Physics
Exercise Sheet 8

FS16
Prof. Dr. Johan Chang

Discussion on 20th April

Due on 27th April

Exercise 1 *Chemical potential in two dimensions*
(a) Show that the chemical potential of a Fermi gas in two dimensions is given by:

$$\mu(T) = k_B T \ln\left[\exp\left(\frac{\pi n \hbar}{m k_B T}\right) - 1\right]$$

Hint: Use $N = \int_0^\infty D(\epsilon) f(\epsilon) d\epsilon$ and remember that $D(\epsilon)$ is a constant in two dimensions.
(b) In two dimensions, derive the Fermi energy E_F and express $\mu(0)$ in terms of the Fermi energy E_F.

Exercise 2 *Fermi energy E_F and Fermi temperature T_F*
The atom ^{3}He has spin 1/2 and is a fermion. The density of liquid ^{3}He is $0.081\,\mathrm{g/cm^3}$ near $T = 0\,\mathrm{K}$. Calculate the Fermi energy E_F and Fermi temperature T_F.

Exercise 3 *Kinetic energy of an electron gas*
Show that the kinetic energy of a three-dimensional gas of N free electrons at 0 K is $U = \frac{3}{5} N E_F$.

Exercise 4 *Occupation of states*
Plot $D(E) \cdot f(E,T)$ against the energy E in units of the chemical potential μ, where $D(E)$ is the electronic density of states in three dimensions and $f(E,T)$ is the Fermi-Dirac distribution. You may neglect all the pre-factors for your plot. Use the temperatures (in units of μ) $k_B T = 10^{-4}$, 10^{-3}, and 10^{-2}. In the legend of your plot, write the corresponding temperatures in Kelvin. If you need any material parameters for your plot, take the ones for copper.