Applications of General Relativity

Sheet 6

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Exercise 1 [Gravitational field of a moving particle] (5 points)

Consider a particle of mass M moving with constant velocity $|\boldsymbol{V}| \ll c$. Calculate the gravitomagnetic potential and the equation of motion for a test particle m in the field generated by M to order $\mathcal{O}(v/c)$. *Hint: Start in the M particle rest frame Σ' and then perform a coordinate transformation to the global frame Σ. Since we are working at $\mathcal{O}(h)$ the transformation is nothing than a* Lorentz *boost, which can be further approximated since $V \ll c$.*

Exercise 2 [Particles in the field of a gravitational wave] (3 points)

Show that the curves $r = r(\varphi)$ described by

$$r^2(\varphi) = R^2 \begin{cases} 1 - 2h\cos(2\varphi)\cos(\omega t) \\ 1 - 2h\sin(2\varphi)\cos(\omega t) \end{cases} \tag{1}$$

for $h \ll 1$ are ellipses. How is the eccentricity ϵ related to h? *Hint: Start from the expression of an ellipse in polar coordinates and assume $\epsilon \ll 1$.*

Exercise 3 [Gravitational Bremsstrahlung] (6 points)

The gravitational wave analogue of Bremsstrahlung can be generated by a small mass m scattering off a large mass $M \gg m$ with impact parameter b. Assume that the large mass sits at $(0, 0, 0)$, that $E = 0$ (parabolic orbit) and that the orbit lies in the x-y-plane. The parabolic trajectory of the particle m in polar coordinate is described by:

$$r(\varphi) = \frac{2b}{1+\cos(\varphi)} \qquad\qquad \dot{\varphi} = \sqrt{\frac{M}{8b^3}}\,[1+\cos(\varphi)]^2\,. \tag{2}$$

Calculate the gravitational wave amplitude $h_{\mu\nu}$ starting from[1]:

$$\gamma_{ij}(t, \boldsymbol{x}) = \frac{2G}{r}\frac{\mathrm{d}^2 I_{ij}(\varphi(t_\mathrm{r}))}{\mathrm{d}t^2}, \tag{3}$$

where $\gamma_{\mu\nu} = h_{\mu\nu} - 1/2\; h\eta_{\mu\nu}$, t_r is the retarded time and $I_{ij}(t) = \int T_{00}(t) y^i y^j \,\mathrm{d}y^3$ is the quadrupole tensor. *Hint: It is not necessary/required to express the final $h_{\mu\nu}$ as a function of time, it is enough to show the dependence on φ, but use the expression for $\dot{\varphi}$ to simplify the evaluation of the time derivative in equation (3).*

[1] See Caroll for derivation and restrictions of this formula, `http://arxiv.org/abs/gr-qc/9712019v1`.