1. Mechanics Biomechanics

1.1 Medical reference and goals of the experiment

The human body consists largely of soft tissue. In its natural environment and under the influence of gravity, the body could neither retain its form nor exert forces. The skeleton, whose rigid structures are connected by joints and ligaments, performs the supporting function.
Muscles, that are connected by tendons to the individual bones, allowing body parts to move in relation to each other and thus to exert forces on the environment.

Biomechanics models the complex interplay of muscle groups and analyses the forces and torques that appear in movements. This knowledge is needed in various fields such as sports medicine, orthopaedics, rehabilitation therapy after accidents, and last but not least in the development of prosthetics.

As seen over the course of this experiment, already for simple movements very large forces in muscles and the involved joints occur. Forces in joints, bones and their connecting pieces (cartilage, ligaments, etc.) are compensated by inner stress. This stress causes deformation of the body parts and is up until a certain threshold, the threshold of elastic range, reversible. If the strain goes over this threshold, the body part is deformed plastically and thus long-term damaged.
Very high inner stress eventually leads to the fracture or rupture of the corresponding body part (fracture stress). These thresholds are individually very different and depend on many factors, e.g. the bone mass, the degree of mineralisation (nutrition!) and age. At around the age of 35, bone deterioration starts – until the age of 80, the human body loses around a third of its whole bone mass. The reparability of the bones deteriorates in the same way. Illnesses, e.g. osteoporosis, additionally aggravate the elastic properties of the bones. All of this increases the probability that a harmless accident can have serious consequences for older people (fracture of the femur neck). All clinical or biomechanical studies in that area require profound knowledge of the mechanics of the musculoskeletal system and the elastic properties of all body parts.
Due to the abundance of involved muscle groups and the complex relations of forces in joints with

multiple degrees of freedom, biomechanical studies are complex and require great effort: High speed cameras are filming the probands executing defined movements on force measurement plates (see Fig. 1.1.1).
From this information, the acting forces and the torsional momenta can be determined using mathematical models and computer programs.

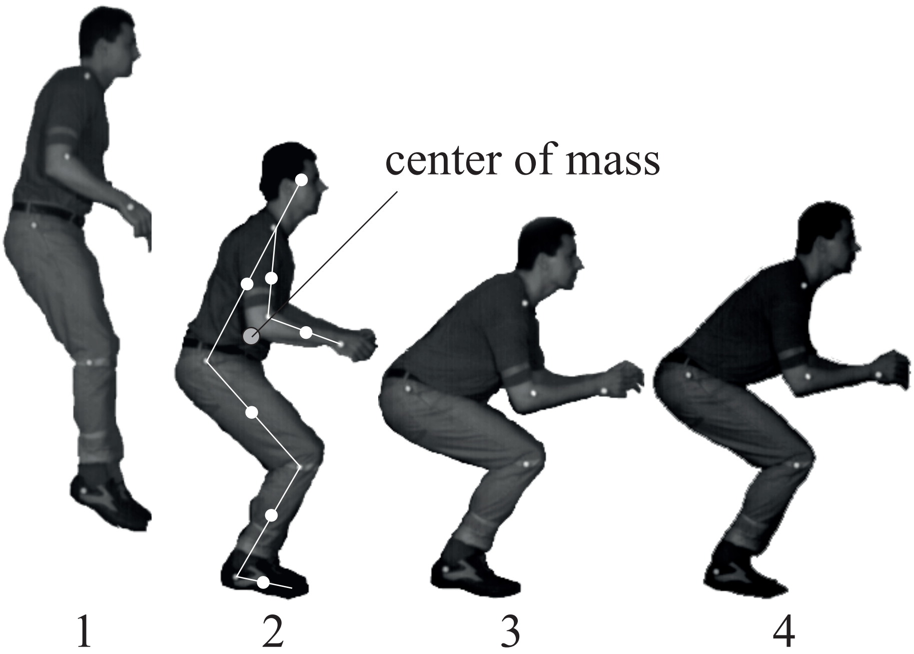


Figure 1.1.1: *High speed capturing of the motion sequence of ground touching after a jump. The positions of the joints are marked (pictures courtesy of Prophysics, Oerlikon). The individual stages correspond to those of a jump as performed in the experiment: 1 – free fall, 2 – ground touching, 3 – shock absorption, and 4 – straighten up.*

In this experiment, you will get to know the basis of biomechanics. For the observation of (human) motion sequences, there are two types of forces:

External forces: All forces that are exerted on the body from the environment: Gravitational force, normal forces, and friction forces. Usually, forces exerted from the basis (the ground) are called ground reaction forces.

Internal Forces: All forces exerted by the muscles, bones, joints, and connectors such as ligaments and tendons.

Every movement can be separated into two parts, where the center of mass has a central role: The movements of the individual body parts relative to the Körperschwerpunkt and the movement of

the Körperschwerpunkt itself. The former includes all forces exerted by the muscles (e.g. pulling or extending arms or legs). They have *no* direct influence on the movement of the Körperschwerpunkt. Accelerations of the center of mass (jump, free fall, walking) can only be produced by external forces. Directed movement is only possible by controlling the reaction forces (external forces) created by exerting forces on the environment (e.g. the ground) in such a way that a corresponding center of mass movement is induced.
In the first part of the experiment: After a short introduction of the terms **force** and **torque**, you will measure the occurring external forces of various motion sequences using a force measurement plate (jump from 50 cm height and basic jump). With the understanding of the external forces ad the torques during jumping, the internal forces can be estimated. These tests require familiarity from school and the physics lecture with physical quantities such as position, velocity, and acceleration. A short repetition can be found in Part 1.3, *Physikalische Grundlagen.*
The second part of the experiment is about the **elastic properties** of the tendons and the bones (only long bones such as in the thigh (femur) or in the upper arm (humerus) will be covered). Long bones consist of a long, hollow middle piece (diaphysis) filled with bone marrow and two thickened end pieces of porose bone mass (epiphysis) that have cartilage bearing areas of the joints at their end. These elastic properties are determined mainly by the external, hard part of the bone, the so-called compacta or bone cortex. This kind of long bone form gives the skeleton optimal mechanical stability, while keeping the bone mass at a minimum. You will work out the relationship of mass distribution and the mechanical properties using poles and tubes made of steel.

Note: Please read the chapter *Usage of the computer*, (only in german manual), in advance!

1.2 Experimental procedure

1.2.1 Organisation

The experiment consists of two large parts:

1. The measurement of the forces and the torques by statistical experiments (1.2.2, p. 7) and the measurement of the forces in motion sequences (1.2.3, p. 11)

2. Elasticity test (1.2.4, S. 23).

The parts of the experiment can be performed independently. The assistant will divide the six groups such that e.g. four groups work on the elasticity test and the others with the scale and the plate.

1.2.2 Forces and torques

1.2.2.1 Forces and the Newtonian axioms

Every body is influences at every point in time by external forces. In order to calculate which motion a body will perform in the next moment, all influencing forces must be determined. Conversely,

with knowledge of the motion (e.g. path of a body, state of rest) conclusions on the influencing forces can be drawn.
Forces are so-called vectorial quantities, signified by point of action, direction, and absolute value. All forces sum up to the **resulting** force $\vec{F}_{res}$. It is directly related to the acceleration by the II. Newtonian axiom:

$$\text{II. Newtonian Axiom: } \vec{F}_{res} = m \cdot \vec{a}. \tag{1.1}$$

And thus the unit of force:

$$\text{Force } \vec{F} = m \cdot \vec{a} \text{ ; Unit } [\vec{F}] = 1 \text{ kg} \cdot \frac{\text{m}}{\text{s}^2} = 1 \text{ N (Newton)} .$$

Additionally, the III. Newtonian Axiom is needed, which states:

> *When body A exerts a force on a second body B, the second body B simultaneously exerts a force equal in magnitude and opposite in direction on the first body A. In other words: actio = reactio.*

You will verify these laws in the following exemplified by the standing on two legs (Fig. 1.2.1). For that purpose, two scales are available.

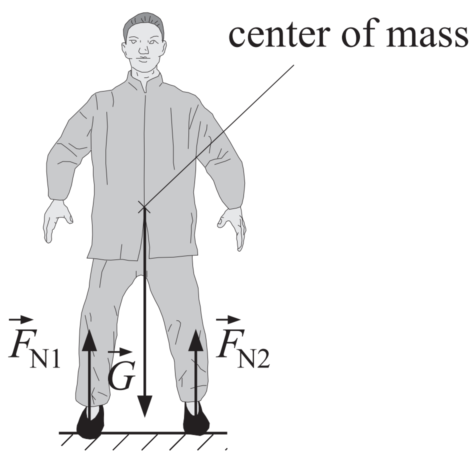


Figure 1.2.1: *External forces while standing on two feet.*

In Fig. 1.2.1, the external forces that act on a human body when standing still are shown. In every case, the weight force $\vec{G}$ inside the body shows towards the center of the earth. The weight force, which exerts force on each every individual part of the body, can be summed up into a point. This for each body exactly defined point is called **center of mass**. Since the man is in a state of rest, the total acceleration and thus the resulting force have to be zero (**balance of forces**). There has to be a force, which compensates the gravitational force. Only the force that is exerted by the ground onto the man can be a possibility. It is called the **normal force**, since it is orthogonal (normal) to the contact surface.

Since the man is standing on two feet, there are actually two contact surface and thus two normal forces $\vec{F}_{N1}$ and $\vec{F}_{N2}$.

A simple measuring device for such normal forces is the scale. It measures the force that the scale has to exert to create a balance of forces. You have access here to two types of scales.

◇ Align the two scales approximately at shoulder length. Stand at first with both feet onto one, then onto both scales with one foot each. Because the scales are gauged to kilograms, you have to multiply the readout values by the gauge factor g (You can use the rounded value of 10 m/s^2), to determine the force. Note down the displayed values each time:

Standing on one scale: $|\vec{F}_N| =$

Standing on two scales: $|\vec{F}_{N1}| =$ $|\vec{F}_{N2}| =$

◇ Compare the value that you get for standing on one scale with the values for two scales. What do you conclude?

In the first case (Standing on one scale), the scale has to exert a force that compensates the whole weight force of the body. If the body is in a state of rest, their absolute values are thus equal and the scale shows the weight force (its intended use). In the second case, each scale has to exert approximately half of the weight force. The normal force is thus a reaction force (identical to the force of the ground you are standing on): You exert a force on the scale that corresponds to your weight force and due to *actio* = *reactio*, the scale exerts an equally large, opposite force on you.

Thus the balance of force for the above case can be summarised into a formula:

$$\text{From } \vec{F}_{res} = 0 \text{ follows } \vec{F}_{res} = 0 = \vec{G} + \vec{F}_{N1} + \vec{F}_{N2} \text{ or } -\vec{G} = \vec{F}_{N1} + \vec{F}_{N2}.$$

Thus for the absolute values:

$$G = F_{\mathrm{N1}} + F_{\mathrm{N2}}.$$

1.2.2.2 Torques

As seen above, the normal force can be divided into both legs. If you shift the weight between both legs, the normal forces will change (Fig. 1.2.2). The law behind the division of the normal forces is the next matter of interest.

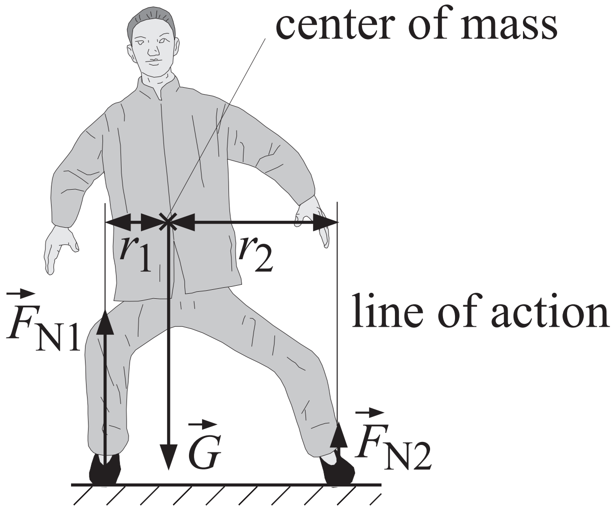


Figure 1.2.2: *Change of force distribution due to weight shifting.*

◇ Align the two scales again approximately at shoulder length. Stand on both scales and shift your weight from one leg to the other, as seen in Fig. 1.2.2. Look at the display of the two scales. What do you observe? How does the displayed value of a scale change if you shift your weight on that leg? In doing so, how does the center of mass of your body move?

◇ Look at the sum of the two normal forces each time – is the balance of forces preserved?

In Fig. 1.2.2, not only the forces and the center of mass, but also the line of action are drawn. This line runs from the point of action of the force in the direction of the force. The horizontal distances r_1 and r_2 correspond to the distances of the center of mass (approx. the belly button) to each of the lines of action. These distances are also called lever arms, the product of force and lever as **torque**.
For its absolute value[1] and unit, the following relations are apply:

[1]Here, the true vector relations are simplified. We are satisfied with just the absolute value and the direction from the resulting rotation from this torque – a more rigorous definition can be found in Part 1.3, *Physikalische Grundlagen*.

Absolute value of the torque $M = F \cdot r$; Unit $[M] = 1\,\mathrm{N} \cdot \mathrm{m} = 1\,\mathrm{Nm}$ (Newtonmeter),

where r the lever arm, meaning the shortest distance between line of action and the center of rotation. For the torque, the following rules apply:

1. The torque can be calculate in relation to any arbitrary point, its direction corresponds to the rotational axis and its sense of direction the sense of rotation.
2. All points acting on a body sum up (vectorially) to a resulting torque.
3. A **balance of torques** requires the resulting torque to be zero. If this is not the case, the body starts to rotate.
4. A force that acts in a specific point, exerts *no* torque on that point (lever arm zero).

Let us analyse the above case: In Fig. 1.2.2, the force $\vec{F}_{N1}$ exerts a torque on the man in relation to the center of mass, trying to rotate the man in clockwise direction. A second torque is exerted by force $\vec{F}_{N2}$, acting in opposite direction (counter-clockwise). The gravitational force exerts no torque in relation to the center of mass. Since the man is not rotating (Hopefully, you aren't either!), these two torques of the normal forces must thus cancel, which means:

$$M_1 = r_1 \cdot F_{\mathrm{N1}} = -M_2 = r_2 \cdot F_{\mathrm{N2}},$$

where the sense of direction of the torques is signified by the leading signs.
Torques thus play a central role in the balance of people. Both normal forces need to be inversely proportional to the distance to produce a balance of torques:

$$\frac{F_{\mathrm{N1}}}{F_{\mathrm{N2}}} = \frac{r_2}{r_1}.$$

1.2.3 Measurement of forces during movements

1.2.3.1 External forces

For the following observations, all vectorial quantities are reduced to its component in vertical direction. Furthermore, the whole body mass is seen as unified inside the center of mass:
In part 1.2.2, the contact forces (especially the normal force) acting e.g. between a body and the surface were being analysed in statistical equilibrium situations. In these situations, the partaking forces are constant in time. In movements, the contact forces can change very rapidly. Your measurement is thus more complex and will be demonstrated in the following. Example curves, taken from the high speed film[2] in Fig. 1.1.1 and analysed using anthropometric[3] models, are collected in Fig. 1.2.3.

[2] "High speed" means here approx. 500-1000 images per second.
[3] Anthropometrics is the science of measuring body parts. It relates size and weight of individual body parts to body size or body weight and where their centers of mass are located.

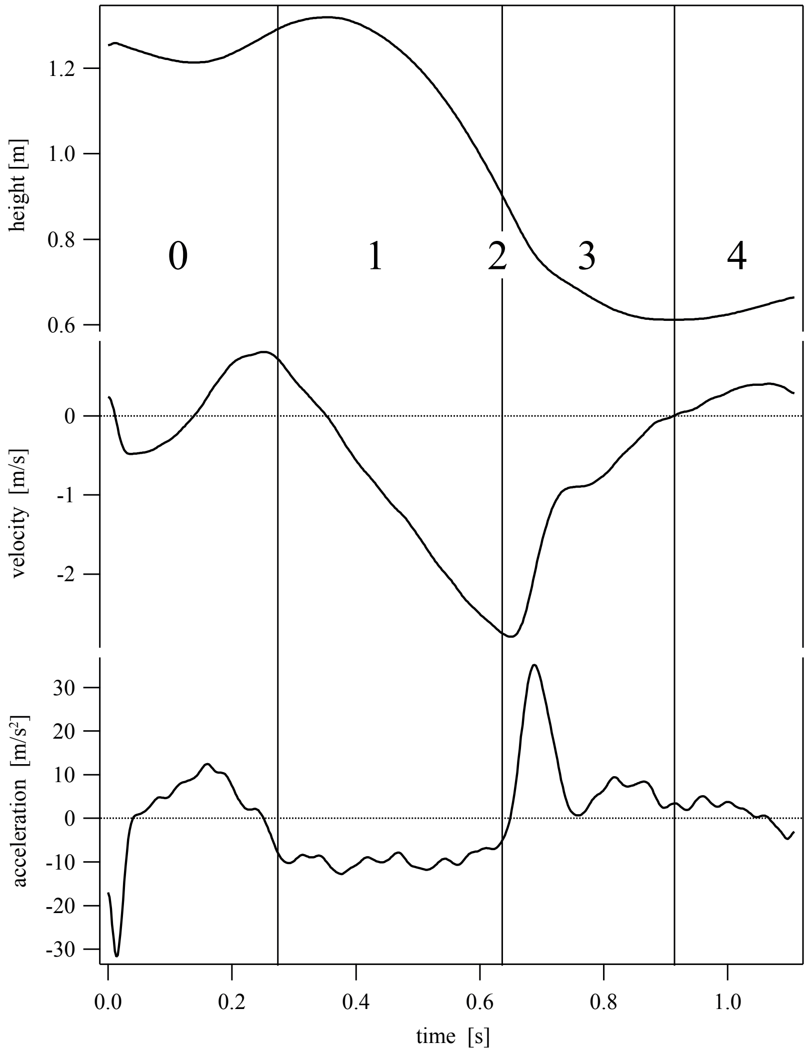


Figure 1.2.3: *Recording of the height (top), velocity (middle), and acceleration (bottom) of the center of mass for the depicted jump in Fig. 1.1.1; the individual phases of the jump are numbered: 0 = jump, 1 = free fall, 2 = ground contact, 3 = deceleration, 4 = straightening.*

The process you will look at now can be described as follows – the numbered phases correspond to the phases in Fig. 1.1.1 and Fig. 1.2.3:

Phase 0 Standing approx. 0.4 m height above ground: Gravity is compensated exactly by the normal force from the ground. The resulting force on the body is therefore zero and the body experiences no acceleration.

Phase 1 Jump: Due to gravity, the vertical component v_z of the velocity and thus the momentum $p_z = m \cdot v_z$ (free fall), where z here the coordinate in vertical direction and m the mass of the.

Caution: Since the height is measured upwards (Fig. 1.2.3), while the acceleration is directed downwards, the velocity increase has a *negative* sign!

Phasen 2 and 3 Ground contact: As soon as the feet touch the ground (or the force measurement plates), the body exerts a force on it. The reaction force of the ground (normal force) acts upwards and thus contrarily to gravity – the resulting force becomes smaller and switches sign if the normal force becomes larger than the gravitational force. The body is decelerated.

Phase 4 Straightening: After the jump, you stand on the ground and the normal force compensates the gravitational force. During straightening, the center of mass is accelerated upwards for a short period of time and the normal force thus becomes larger.

Calibration

First, we will look at the third phase, the landing on the ground, in more detail. The goal is to perform a jump on a force measurement plate while measuring the occurring normal force as a function of time. The plate is connected to an amplifier giving out an electric voltage proportional to the normal force exerted by the plate. The voltage is measured using a computer in intervals of approx. 100 μs and then plotted as a function of time. Since only the absolute value of the force is relevant here, the voltage measurement has to be **calibrated** first (the necessary equipment and settings of the amplifier are available at the measuring station). The calibration is performed using 5 kg mass piececs:

- Start up the computer and open up the program *Mechanics* on the desktop.
- First you have to perform the calibration, the programme will be automatically opened in **Calibration** mode. You can only access the other modes once the calibration has been completed.
- To the left of the blank graphic, the number of weights that are on the scale must be specified (0 - 5).
- To calculate the force that the weights exert on the plate, the mass m of a single weight and the acceleration due to gravity g must be entered in the respective fields provided. In the calibration curve, both the weight force G of the mass and the normal force F_{N}, calculated as $G = F_{\mathrm{N}} = m \cdot g$, are plotted against the stress.
- Make sure that under **Number of weights** „0“ is entered and the measuring plate is unloaded. Click on **Start**. The computer now measures the voltage at the output of the measuring amplifier for 2-3 s and determines the average value. *Make sure that the measuring plate is not touched during this time!*
- As soon as the measurement is completed, a point appears in the graph on the right, with which the programme assigns the mass 0 kg and thus the force 0 N to the measured voltage value.

- Increase **Number of weights** and increase the mass on the force plate by 5 kg at a time until the total mass is 25 kg. The graph should contain 6 points in the end that follow approximately a line. If that is not the case, replace obviously wrong measuring points or click **Restart Calibration** and restart.
- If the measurements were successful, click on **Fit Curve**. The computer will perform the calibration on its own by fitting a straight line through the measurement points and determining its slope and its axis position.
- The calibration curve, as well as the slope and the ordinate section of the corresponding function are now displayed in the graph. With the help of this function, the programme will from now on calculate the corresponding force directly from a voltage.
- Print the graph by clicking on the printer icon on the left. When printing for the first time, you must enter a name or a group designation; this will appear in the footer of the printout and serves to identify the printout. After confirming the entry, a preview appears on which the graph can be seen twice. For more on printing, see the chapter *operating the computer*, S. 3.

Now, the measuring plate is calibrated and the computer will display data in the unit Newton. You can now test the function of the measuring plate. *Important! From now until the end of the experiment, do not change the settings of the amplifier! Or you will have to repeat the calibration!*

Calibration of force versus voltage of the force measuring plate.

Momentum

- Switch to the **Impulse** mode on the left. A new graph appears in which the measured force is displayed as a function of time. In this measurement mode, the signal is displayed continuously after the start. A measurement can be stopped at any time by pressing **Stop**, after 2 minutes it will be stopped automatically. Make sure that the option **Trigger enabled** is ***not*** selected.
- Stand quietly on the force plate and click on **Start**. A continuous signal is displayed, the measured values run across the screen from right to left. The plate must now apply its weight force, and the value on the screen should approximately correspond to this (in Newton).

◇ Seesaw up and down. Is the normal force larger or smaller than your weight if you accelerate your center of mass upwards or decelerate it during the downward motion? Observe the signal curve during the measurement.

Acceleration upwards:

Deceleration during downward motion:

◇ Try to explain your observations (Think of the individual motion of the body's center of mass):

In the following, the force is to be measured as a function of time during a jump. Activate the option ***Tigger enabled***. In the field **Duration Measurement**, enter a value of 2-3 (seconds), this will record the voltage only over the short time during which you perform the jump from a stool to the jump plate.Click **Start** to initiate the measurement, you must make the jump within the next 2 minutes. **Note:** Before the continuous signal is displayed, the program will establish the base value for the trigger for about 1 second, the load on the plate **should be as constant as possible during this time**.[4]. The signal recording starts 0.5 seconds before your feet touch the plate or before you jump off, the recording duration must be adjusted if necessary.[5]

Important! Try to land as softly as possible by bending your knees when landing. knees when landing. The occurring forces will be a multiple of the weight force even with a soft landing. of the weight force. A hard landing can cause permanent damage to the joints!

[4] Significant load changes during trigger calibration may cause the trigger not to fire

[5] In fact the whole measurement is recorded, but via a trigger the program remembers the position where a certain value is exceeded and determines the section that is displayed.

◇ Jump from 0.4 m height onto the force plate (soft landing!). After deceleration, try to straighten up calmly and standing still until the measurement is over. – Compare the measured curve with the model curves shown in Fig. 1.2.3: Does the here measured curve in phase 3 and 4 resemble more the curve height of the center of mass, of the velocity, or of the acceleration?

◇ What does that mean for the force to the corresponding quantity? (Think about the Newtonian axioms!)

◇ Note down the measured maximum values displayed on the left side:

Maximum value $F_{\mathrm{N\ max.}} =$

◇ Which part of the curve represents the weight? Estimate from the curve, how big the maximum force in units of its weight force G was.

Maximum value $F_{\mathrm{N\ max.}} =$ G.

- Click on the printer symbol on the left, print out the curve and glue it on the next page of your manual.

- Start again the measurement and make a basic jump on the force plate with a soft landing.

- Print again the curve and glue it in your manual.

◇ You should see on the curve two large peaks. Compare the areas under the two peaks – what kind of observation can you make?

Normal force of the ground versus time during jump from 0.4 m height.

Normal force versus time during basic jump on the force measuring plate.

◇ The area under this curve, above the constant line $\vec{F}_N = G$ gives you the momentum change your body receives due to the acceleration (principle of force impact, Part 1.3, *Physikalische Grundlagen*).Since your vertical movement during the jump is only influenced by the constant force of gravity, you have the same but opposite momentum shortly before decelerating on the force plate as shortly after the jump. What consequence does this have for the change in momentum and thus the areas under the deflections?

Heartbeat

Switch to the mode **Heartbeat** on the left. In this mode the voltage scale on the y-axis is regauged, this happens in the first 1-2 s of each measurement.It is important that you stand still on the plate, after gauging a value around 0 N is displayed for the current weight force.

◇ Click on **Start**, try to stand very still and hold your breath.Stop the measurement after about 15 to 20 seconds. You should see an area of small short fluctuations about 1 s apart in the displayed graph – this is your heartbeat.This shows you that the acceleration of the blood (and the associated associated mass) causes an acceleration of the body's centre of mass. The force required for this must be provided by an external force, the normal force.

◇ Select a range containing 10-20 deflections by moving the red and blue lines with the mouse and then clicking **Ajust X-Y Range**.Via **Reset X-Axis** or **Reset Y-Axis** you can zoom out again if necessary. When you have set a suitable section, print out the graph and glue it in the notebook.

Weight force differences due to the pulse

- ◇ Now place the blue lines on the right and the left as exactly as possible on a deflection.To the left of the graph, the times corresponding to the positions of the lines are shown in the fields t_1 and t_2, dt indicates the time difference between them, Determine your pulse P with the help of the graph and dt.

$$P =$$

- ◇ Pick a single heartbeat and set one of the red lines to the maximum and the other to the minimum. Read the value for the force difference dF on the left.

$$dF = \qquad\qquad \mathrm{N}$$

The difference in force dF in this section of the curve results from two events, a filling phase (diastole) and an ejection phase (systole). In the following, we will only refer to the systemic body circulation.[6]

- ◇ Estimate the maximum acceleration a that the blood experiences during the ejection phase. Assume that during a heartbeat about 70 cm^3 of blood is pumped into the artery and that the two phases have the same amplitude ($\rho_{blood} \approx 10^3\,\frac{kg}{m^3}$). Give the result in $\frac{m}{s^2}$ and in units of g.

$$a = \qquad\qquad \frac{m}{s^2} = \qquad\qquad g$$

- ◇ Using the difference in force between diastole and systole, estimate the difference in pressure in Pa and mmHg. The aorta has a diameter of about 2 cm.

$$\Delta p = \qquad\qquad \mathrm{Pa} = \qquad\qquad \mathrm{mmHg}^7$$

- ◇ Compare your result with your blood pressure values, how good is this estimation?

1.2.3.2 Internal forces

Now, we will look at internal and external forces on the example of a foot during the jump on the jump measuring plate. A quasi-static case is assumed (compare with squats): The foot is assumed

[6] The pulmonary circulation is neglected because the directions of blood flow are approximately perpendicular to gravity, whereas in the systemic circulation they are parallel or antiparallel to it.

[7] Blood pressure values are usually given in mmHg: 1 mmHg ≈ 1.3 mbar = 1.3 hPa

to be at rest during deceleration, that means that the equilibrium conditions are met for a short moment – thus one can see from Fig. 1.1.1 that the foot does practically not change its position during the phases 2 and 4. Under this assumption, the force relations in the foot are in the following way:

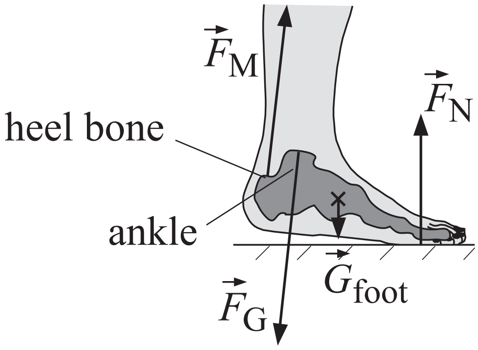


Figure 1.2.4: *Forces acting on the foot (qualitative); the forces acting inside the foot are ignored here – their main goal is to maintain the arc of the foot, regarded here as rigid. The bones of the boots are simplified in this model.*

$\overrightarrow{G}_{\text{Foot}}$ – the weight of the foot acting on its center of mass,

$\vec{F}_N$ – the normal force of the ground, transmitted by the foot and the ankle joint on the rest of the body,

$\vec{F}_M$ – the force that the calf exerts through the heel bone on the foot, and

$\vec{F}_G$ – the normal force (joint force) that the rest of the body exerts through the ankle joint on the foot.

Since this is only an approximation of the forces, we will ignore the weight of the foot. (The feet together make up around 4% of the total body mass.) The task is more complex since the joint force?s absolute value and direction is unknown at first. For the foot, all force as well as all torques (meaning the products of the forces and their levers) have to be determined. But because we assume that the foot is at rest and in particular does not rotate, one can determine the torques in relation to an arbitrary point and assume equilibrium conditions for said point. It is advantageous to choose the ankle joint as a point of reference, since the lever of the joint force acting in the ankle joint is zero. It thus drops out of the calculation and the situation is as depicted in Fig. 1.2.5.

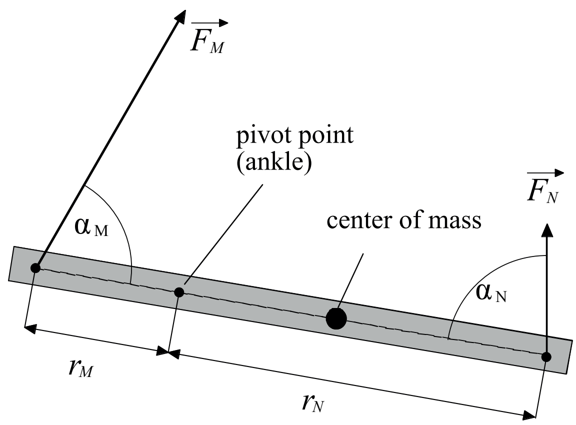


Figure 1.2.5: *Simplified illustration of the forces and levers in the foot – the weight of the foot is ignored in the assumption.*

The equilibrium condition for the torques is thus (cf. Part 1.3, *Physikalische Grundlagen*):

$$F_{\mathrm{M}}\, r_{\mathrm{M}}\, \sin\alpha_{\mathrm{M}} \;-\; F_{\mathrm{N}}\, r_{\mathrm{N}}\, \sin\alpha_{\mathrm{N}} \;=\; 0.$$

The angles α_{M} and α_{N} between the muscle force (along the lower leg), the normal force, and the foot axis can be determined from the film recordings. They are approximately:

$\alpha_{\mathrm{M}} \;=\; 70°$ and $\alpha_{\mathrm{N}} \;=\; 80°$.

- ◇ Estimate on yourself the distance between the attack points of the normal force and the muscle force in relation to the ankle joint (approximately at the ankle):

  $r_{\mathrm{M}} \approx$ distance heel bone–ankle $\approx$

  $r_{\mathrm{N}} \approx$ distance ball of the foot–ankle $\approx$

- ◇ Plug in the above values and the maximum normal force $F_{\mathrm{N\ max.}}$, measured during the jump, in the above equation for the torque equilibrium and calculate the maximum occurring muscle force. If you did not perform the jump yourself, transfer the value in units of the weight and multiply it with your own weight. The factor 1/2 comes from the calculation of pressure per foot.

  $F_{\mathrm{M\ max.}} \approx \frac{F_{\mathrm{N\ max.}} \cdot r_{\mathrm{N}} \cdot \sin(80^\circ)}{2 \cdot r_{\mathrm{M}} \cdot \sin(70^\circ)} \approx$

- ◇ A rough estimation of the joint force can be determined by the knowledge that the joint force has to compensate the muscle and normal force, both of which show in approximately the same direction (force equilibrium). What is thus the result for $\vec{F}_G$?

  $F_{\mathrm{G\ max.}} \approx 0.5 \cdot F_{\mathrm{N\ max.}} + F_{\mathrm{M\ max.}} \approx$

  and in units of the weight:

  $F_{\mathrm{G\ max.}} \approx \qquad G$

  with a *soft* landing!

- ◇ To demonstrate which forces occur during a hard landing, start a new 3–second measurement and hit the measuring plate short and hard (but not with a lot of strength) with the available hammer. Estimate the value of the force peak in units of the weight of the hammer (mass 1 kg):

  $F_{\mathrm{N\ max.}} \approx \qquad \mathrm{N}$

  $F_{\mathrm{N\ max.}} \approx \qquad G.$

1.2.4 Elastic behaviour

If one sums up all forces (vectorially) that act on a body, one obtains the resulting force which causes an acceleration of the Körperschwerpunkt. But what happens to the forces or force fractions that compensate each other? They build force pairs of equally large, opposite forces, which act on the body. Until now, we assumed that the body is rigid. In reality, each body acted upon by a force pair is being deformed. Possible deformations include extension, compression, all side compression, bending, torsion, and shear strain. Usually, multiple deformations take place at once. We will look at two types of these deformations, the extension and bending.

1.2.4.1 Stress and extension

Imagine a cylindrical object, e.g. in form of a tendon or a thigh bone (femur) in your body. On its ends, the force pairs act along the cylinder axis. Such a situation is depicted in Fig. 1.2.6 for the case of an Achilles tendon.

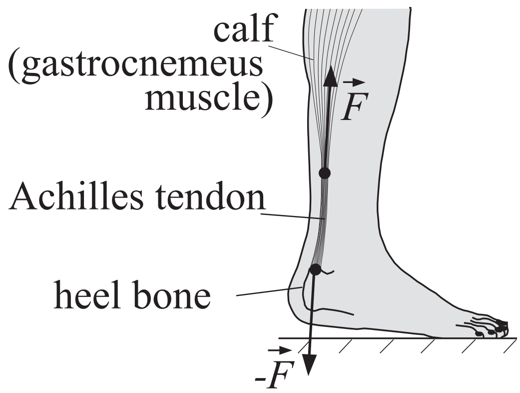


Figure 1.2.6: *Forces that act on the Achilles tendon.*

The force $\vec{F}$, exerted by the calf muscle on the foot, is transferred similar to a rope by the Achilles tendon onto the heel bone. Due to *actio = reactio* the heel bone exerts an equal, but opposite force -$\vec{F}$ on the tendon. Thus a force pair exists that extends and stretches the tendon. The size of the extension is not only dependent on the absolute value of the acting force, but also of the cross section the forces are distributed on: The bigger the cross section, the smaller the extension of the tendon and vice versa.

This comes into the calculation by introducing a new quantity: the **(mechanical) stress** σ, given by the ratio of force and cross section: The Achilles tendon thus experiences a stress $\sigma = F/A$, where A is the cross section.[8] The stress has the same unit as the pressure and is given in units of 1 Nm^{-2} = 1 Pa (Pascal).

[8]The sign is chosen such that a negative value gives a compressional stress.

◇ Estimate on yourself the diameter of the Achilles tendon and calculate the cross section A in m^2:

$$A =$$

◇ The maximum force, transferred from the calf muscle through the tendon onto the heel bone during the jump on the force measuring plate, has been calculated in part 1.2.3 already. If you have not yet done that experiment, insert here 4-times your weight:

$$F = F_{\mathrm{M\ max.}}\ (\text{or } 4 \cdot G) =$$

◇ Using these values, calculate the maximum stress the tendon experienced during the jump:

$$\sigma = \tfrac{F}{A} =$$

For small stress, the extension is proportional to the acting stress (Hooke?s law). The extension ϵ is usually given as the relative length variation, meaning the length variation Δl divided by the length l of the body. Thus the unit m/m = 1 is used. Hooke?s law is therefore written as follows:

$$\sigma = E \cdot \epsilon = E \cdot \frac{\Delta l}{l}. \tag{1.2}$$

The proportionality constant is called elasticity modulus E. It has the dimension of a stress (since the extension is dimensionless) and is for a tendon [9] around 1 GPa.

◇ Calculate using Eq. (1.2) the extension of the Achilles tendon during the jump.

$$\epsilon = \tfrac{\sigma}{E} =$$

◇ The Achilles tendon is approximately 10–12 cm long (depending on body size). How much was the maximum extension during the jump?

$$\Delta l = \epsilon \cdot l =$$

Tendons consist of collagen and elastin and embedded fibroblasts. The elastic properties are mainly determined by the elastin, the mechanical stability by the collagen, which makes up approx. 70% of the dry mass of a tendon. Generally, the following limits are given for tendons:

[9] The elastic properties of a tendon change rapidly as a function of the stress. Hooke?s law is thus not strictly valid and the given elasticity modulus is to be understood as an approximation.

– The tendon reacts in the range of 0 - 5% elastically, meaning it goes back to its initial shape after experiencing stress (reversible extension).

– After around 5% extension, the plastic range starts. After experiencing the stress, a lasting deformation is retained, that only slowly goes away (relaxation).

– After around 8% extension, the weakest fibres start tearing, the extension limit (tendon rupture) is around 10%.

◇ Compare your value from above with the given limits: In which region was the tendon during the landing after the jump?

1.2.4.2 Bending

As a second case of deformation, we will now look at bending. A beam is fixed on one side while the free side is loaded with a mass piece. As a consequence, the beam starts bending (Fig. 1.2.7). Such a stress is experienced by e.g. the thigh bone during squats or a landing after a jump, since it is fixed on the knee and the other end is loaded with the upper body as a mass.

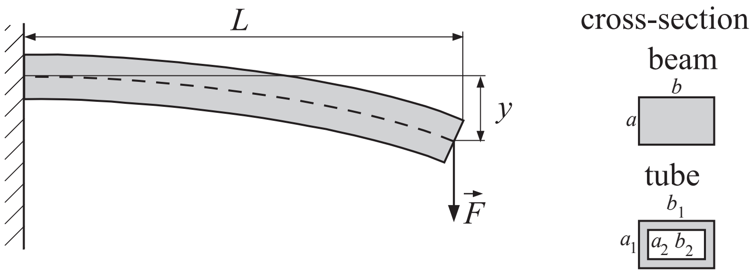


Figure 1.2.7: *Bending of a beam fixed on one side. To the right, the cross section of the beam and the square tube, looked at in the following.*

The goal of this experiment is less about measuring and calculating the bending, but more about the difference between a massive beam and a hollow square tube under stress and deducing the principle of the hollow bone. For that purpose, you have access to the following experimental set-up (Fig. 1.2.8):

– Different beams – Choose the (4) and (5). Both of them have equal length 300 mm and are made of steel. The cross sections are written on the Fixtures.

– A device to fixate the beam with its holder to the left. (Use the middle drill hole)

– A micrometer screw on the right side and a water level.

– A hanging fixture and mass pieces.

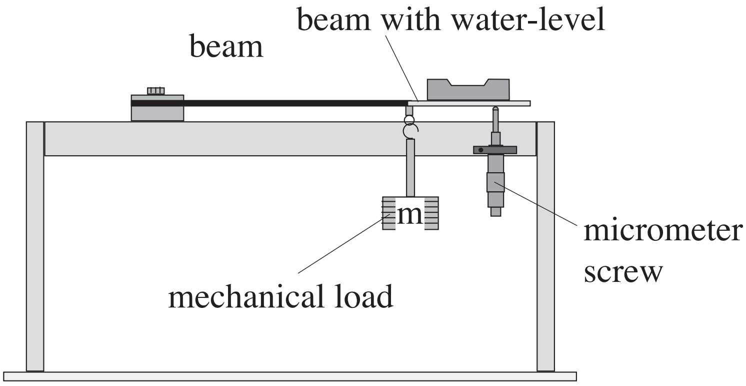


Figure 1.2.8: *Experimental set-up for the bending.*

Build up the experiment using Fig. 1.2.8 as follows:

- Screw in the beam (4) on the left of the holder.
- Hang up the empty hanging fixture (mass 250 g) on the free end of the beam. It is advantageous to start the experiment with a small load directly.
- Hang up the water level between end of the beam and the micrometer screw. Make sure that the water level lies stable in its holder.

Since the bending of a steel beam will be very small, it has to be measured very precisely: In order to achieve that, bring the water level into the horizontal by turning the micrometer screw and read the corresponding value off the micrometer screw (a short instruction is given in Fig. 1.2.9). The zero of the micrometer screw is usually not the position of the unstressed beam. The bending is therefore measured always in relation to a reference point, in our case the position of the beam end with 250 g load.

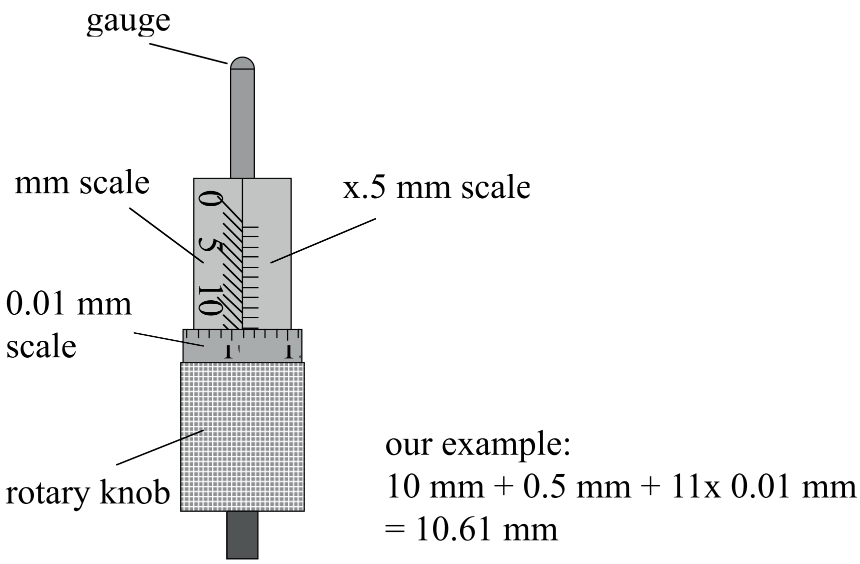
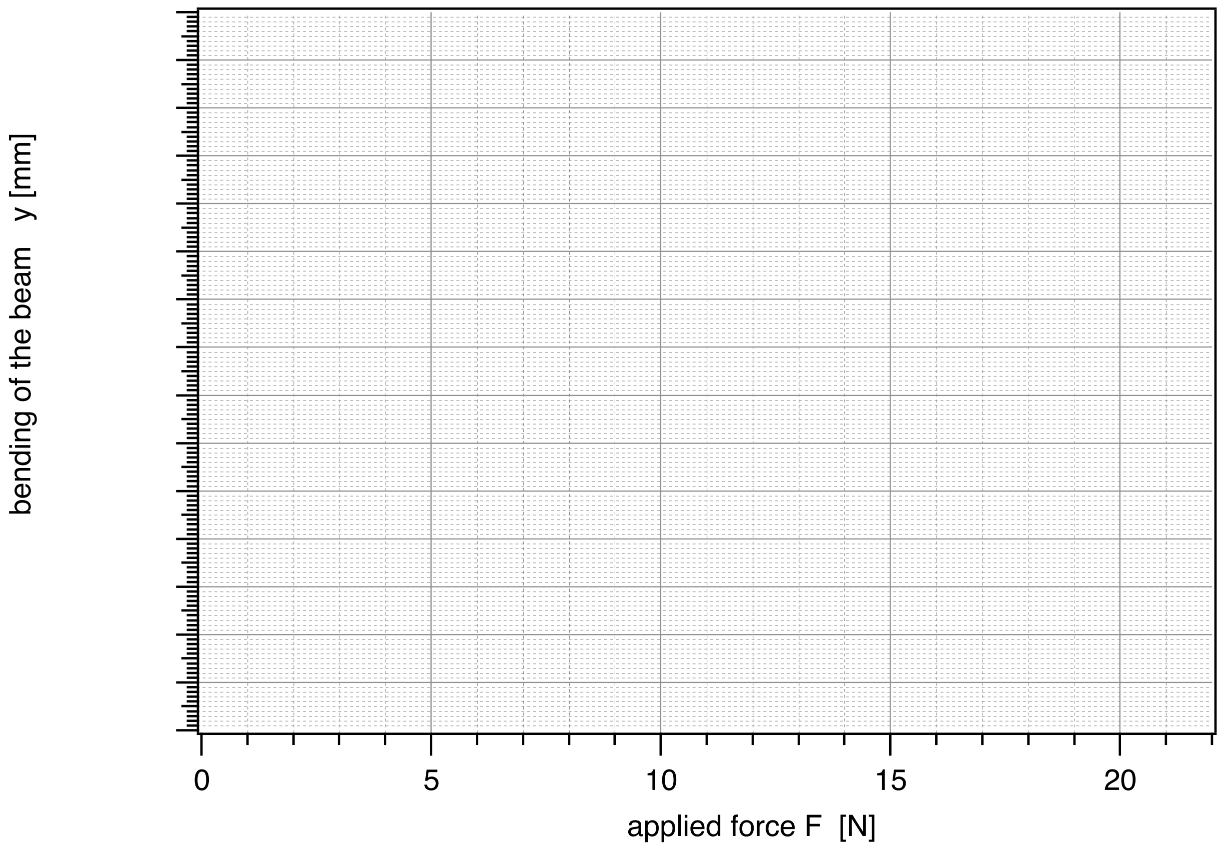


Figure 1.2.9: *Reading of a micrometer screw: the mm– and half-mm–scales are read out on the edge of the rotary knob (always the last fully visible line), the fine subdivision in units of 1/100 mm on the scale of the rotary knob. A full turn corresponds to the length change of 0.5 mm.*

◇ Measure the position of the end of the beam for various loads and thus stresses and fill in the table with the values. The first point (250 g) corresponds to the empty holder. Calculate the mass from the force and the bending from the position and write them into the third and fourth column. Draw in the data points as bending versus force in the following graph (First, think about how to divide the y-axis to fill out the graph as much as possible).

Bending of the beam (4)			
Mass [kg]	position of the end of the beam [mm]	Force [N]	Bending [mm]
0.25			0
0.5			
0.75			
1.0			
1.5			
2.0			



An elaborate calculation gives the following relation between force $\vec{F}$ and bending y:

$$y = \frac{L^3}{3EI_z} \cdot F. \tag{1.3}$$

L is here the length of the beam from the point of fixture, E is again the elasticity modulus (material constant) and I_z the so-called geometrical moment of inertia. The latter gives a relation of the material distribution in the cross section. In the case of a massive square tube, it is

$$I_z = \frac{1}{12}\, a^3\, b,$$

where a and b are the lengths of the edge in and orthogonal to the direction of force, respectively. As seen in the definition of the geometrical moment of inertia, the edge length in direction of the force plays a much bigger role than the orthogonal one.[10]
The cross sections of the beam and the tube have the following dimensions:

beam: $a = 4$ mm, $b = 16$ mm,

[10] Take a ruler and try to bend it once along the thing side and once along the thick side – the difference should be evident.

tube: outside $a_1 = 10$ mm, $b_1 = 15$ mm, inside $a_2 = 7$ mm, $b_2 = 12$ mm.

◇ Calculate the geometrical moment of inertia for the beam for the given bending geometry:

$I_z(\text{beam}) = \frac{1}{12} a^3 b =$

Since from Eq. (1.3), it is a straight with equation $y = k \cdot F$, we can equate the prefactor in Eq. (1.3) to the slope k of the measured curve:

$$k = \frac{L^3}{3EI_z}.$$

◇ Draw the best straight through the measurement points, meaning the straight that describes the trend of the data points the best (see chapter *Evaluation of measurement data*, in german manual). Next, draw the slope triangle, which needs to be as large as possible: One side of this rectangular triangle has to be parallel to the abscissa (F) and one parallel to the ordinate (y); the third side comes from the best straight. Calculate the lengths of the sides Δy and ΔF and the slope k using the following formula (Units!):

$k(\text{beam}) = \frac{\Delta y}{\Delta F} =$

◇ Calculate the elasticity modulus of steel using the slope and Eq. (1.3) (Use the right units!). Typical values for steel are between 1.7 x10^{11} and 2.2 x10^{11} Pa (1 Pa = 1 N/m^2) depending on the type of steel.

$E = \frac{L^3}{3I_z \cdot k(\text{beam})} =$

- Unfix the beam and take off the holder and the water level.
- Replace the beam (4) with the tube (5) and put in the holder and the water level again.

◇ Measure the bending of the tube for two different masses and fill in the following table with the results.

Bending of the tube (5)			
Mass [kg]	Position of the end of the tube [mm]	Force [N]	Bending [mm]
0.25			0
0.5			
1.0			

- ◇ Put these data points also into the plot on page S. 29 und compare the bending of the tube with the one of the beam with the same load and stress – what do you observe?

- ◇ Calculate the cross sections of the tube and the beam (Fig. 1.2.7) and consider the possibility that it is the different cross sections (and therefore the different material volumina) that could explain the discrepancy found above.

$$A_{\text{beam}} =$$

$$A_{\text{tube}} =$$

- ◇ Calculate the geometrical moment of inertia for the tube – This is done by subtracting the geometrical moment of inertia of a beam with the inner edge lengths from the one of a massive beam with the outter edge lengths:

$$I_z(\text{tube}) \;=\; \tfrac{1}{12}\left(a_1^3 b_1 - a_2^3 b_2\right) \;=$$

- ◇ Calculate the ratio of the two geometrical moments of inertia and consider using Eq. (1.3) if the different in moments of inertia could explain the different bending behaviours of the beam and the tube:

The moment of inertia indicates the material distribution in the cross section and can be calculated for any kind of bar. The rule is: the farther away the material in relation to the direction of bending, the smaller the bending and the smaller the maximum stress experienced by the bar. In our case, the tube is therefore with comparable material volume much more stable against bending than the beam. Nature has thus found a compromise in the case of the hollow bone between minimum mass and maximum stability. Since the hollow bone has to withstand stress from all directions, its cylindrical shape is optimal (if it can retain its shape under stress and does not buckle, like e.g. an empty hose). Look at the showcased hollow bones more closely – they are filled inside with compact mass, whose absolute value can be neglected, but it still fills an essential function (besides tasks like the generation of bone tissue), the form stability.