



Mechanics

Exercise Sheet 2

HS 2026
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Exercise 1: Skier on a bump

- a) Consider spherical polar coordinates r, ϕ, θ ($r > 0, \phi \in (-\pi, \pi), \theta \in (0, \pi)$) referred to cartesian coordinates x, y, z , as in Fig. 1.

Express the velocity and the acceleration of a point particle using polar coordinates and the unit vectors $\mathbf{e}_r, \mathbf{e}_\phi$ and $\mathbf{e}_\theta$. Prove that

$$\begin{aligned}\mathbf{v} &= \dot{r} \mathbf{e}_r + r \dot{\theta} \mathbf{e}_\theta + r \dot{\phi} \sin \theta \mathbf{e}_\phi, \\ \mathbf{a} &= (\ddot{r} - r \dot{\theta}^2 - r \dot{\phi}^2 \sin^2 \theta) \mathbf{e}_r + (r \ddot{\theta} + 2 \dot{r} \dot{\theta} - r \dot{\phi}^2 \sin \theta \cos \theta) \mathbf{e}_\theta \\ &\quad + (r \ddot{\phi} \sin \theta + 2 \dot{r} \dot{\phi} \sin \theta + 2 r \dot{\phi} \dot{\theta} \cos \theta) \mathbf{e}_\phi.\end{aligned}$$

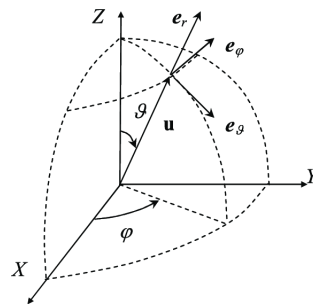


Figure 1: Spherical polar coordinates.

- b) A skier of mass m is skiing on a bump represented by a sphere of radius R (Fig. 2). He slides without initial speed from the top of the bump ($\theta = 0$).

Assuming the skier to be a point-like particle and neglecting friction, determine the speed of the skier as function of the angle θ .

Compute the reaction force $\mathbf{N}$ as a function of θ and find the angle at which the skier will take off.

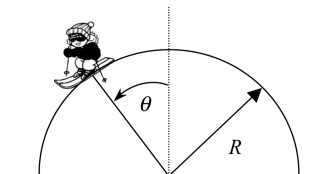


Figure 2: Skier on a bump.

Exercise 2: Conservation laws for a system of particles.

The profile of Fig. 3 has mass M , curvature radius R and lies on a horizontal, frictionless plane. A point-like body of mass m can move without friction on the curved surface of the profile. At t_i the body is at rest on top of the profile.

- Are the total linear momentum and mechanical energy of the system (profile + body) conserved? Why? Find the x -component of the velocity of the body v_x and the profile V_x once the body has reached the basis of the profile.
- Find the velocity of the center-of-mass of the system as a function of the coordinate θ describing the position of the body on the profile. Check that $(v_{CM})_y = 0$ for $\theta = 0, \pi/2$.



Figure 3: System of exercise 2.

Exercise 3: Atwood machine

Consider the following device under the action of gravity :

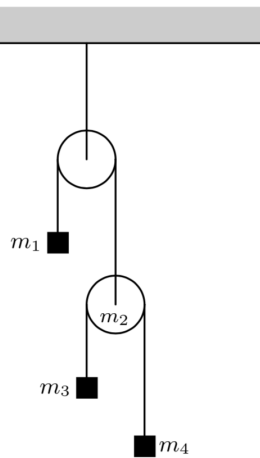


Figure 4: Atwood machine

- Write Newton's equations for the 4 masses and give the constraints of the system.
Hint: What do the finite lengths of the strings imply?
- Choose a_1 , a_3 , T_1 and T_2 as variables (where T_1 and T_2 are the tension of the cables) and solve their equation of motion.
Hint: Write $[(m_1 + m_2)(m_3 + m_4) + 4m_3m_4]v_i$ for each variable $v_i = a_1, a_3, T_1, T_2$.
- What can we tell about a_1 if m_1 and m_2 are the same? And if $m_1 = m_2 + m_3 + m_4$?
- What are the limits of this description?