



Mechanics

Exercise Sheet 5

HS 2026
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<https://www.physik.uzh.ch/en/teaching/PHY311/HS2026.html>

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Exercise 1: Three point particles

The system of Fig. 1 consists of three point particles, P_1 and P_2 of mass m and G of mass $2m$. All three points move frictionlessly in the $y = 0$ plane, and G is constrained to stay in the center of mass of the P_1, P_2 system. P_1 and P_2 interact by means of a conservative force with potential energy

$$V_{\text{int}}(\mathbf{r}_1, \mathbf{r}_2) = a|\mathbf{r}_1 - \mathbf{r}_2|^3 - b|\mathbf{r}_1 - \mathbf{r}_2|^2 \quad (1.1)$$

with $a, b > 0$. G is connected to the origin O by means of a spring of zero rest length and spring constant k . All three points are subject to gravity ($\mathbf{g} = -g\mathbf{e}_z$).

- a) How many generalised coordinates are required to describe the system, and why? Write the Euler-Lagrange equations of the system, using as free coordinates the x, z coordinate of G , the distance r of P_1 from G and the angle θ between $\mathbf{r}_1 - \mathbf{r}_G$ and the x -axis, as in Fig.1.
- b) Write the explicit expressions of *i)* the total energy, *ii)* the y -component of the total angular momentum of the system computed with respect to G and *iii)* the x -component of the total momentum of the system.
Are these quantities conserved? State why/why not on the basis of the Lagrangian symmetries. Do your conclusions change if $k = 0$?
- c) Find the equilibrium configurations, i.e. configurations x_0, z_0, r_0, θ_0 such that $\ddot{q} = \dot{q} = 0$, disregarding the trivial case $r_0 = 0$. Show that they are all unstable.
Hint: Study the behaviour of perturbations around θ_0 .

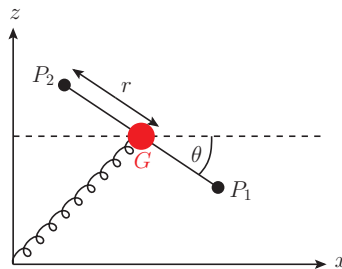


Figure 1: System of exercise 1.

Exercise 2: Circular orbits of the Yukawa potential. (3+2 points)

Consider a particle of mass m moving in the central potential

$$V = -k \frac{e^{-ar}}{r} \quad (2.1)$$

with the constants $k > 0$ and $a > 0$.

- a) In analogy to the Kepler problem, study the effective potential and prove:

$$\text{Stable circular orbits exist if } r < \frac{(1 + \sqrt{5})}{2a}. \quad (2.2)$$

- b) Study the circular orbits for small values of r by expanding the minimizing condition of the effective potential around $r = 0$ up to (and including) $\mathcal{O}(r^3)$. Which values of ℓ^2 (with ℓ being the angular momentum) do the circular orbits take in terms of r and the given constants a, k, m ? Compare your result to the values found for a gravitational potential.