

# Achievements

## Lecture 1: VESTA plotting of crystal structures

## Lecture 2: How to describe a crystal structure

- Crystal lattice
- Basis

## Lecture 3 +4: How to resolve crystal structures

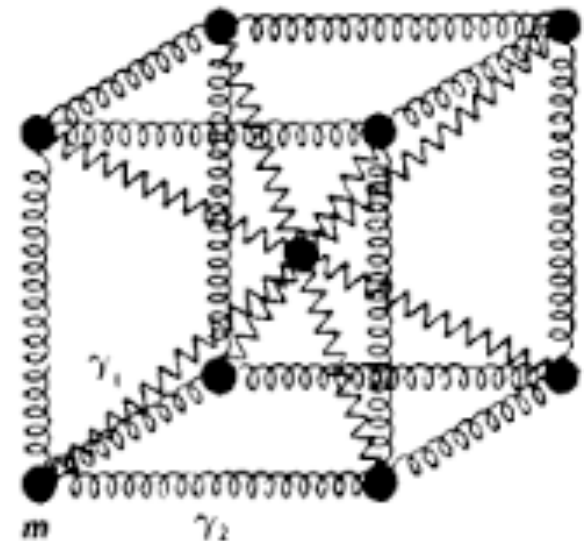
- Reciprocal space
- Scattering theory (Form and Structure Factor)
- Resolving the crystal structure of a superconductor

## Lecture 5: How to crystals bind together

- van der Waals, ionic, covalent crystal bindings

## Lecture 6-7: Crystal vibrations (phonons)

- Tasks
- Why is phonons important
- Theory & concepts
- How to measure phonons



# Tasks

## **(1) Read chapter 5**

- Phonon heat capacity (12 pages)
- Anharmonic crystal interactions (2 pages)
- Thermal conductivity (5 pages)

## **(2) Who is summarizing next week?**

## **(3) Solve exercise sheet 5**

# Exercises

## **Exercise 1** *Elastic waves in lattices and continuous media*

In continuous media the 1D wave equation reads

$$\frac{\partial^2 \xi(x, t)}{\partial t^2} = v^2 \frac{\partial^2 \xi(x, t)}{\partial x^2}, \quad (1)$$

with the speed of sound  $v = \sqrt{E/\rho}$ , elastic modulus  $E$ , and density  $\rho$ . For a linear chain of atoms with distance  $a$ , mass  $m$ , and spring constant  $C$  we get

$$m \frac{\partial^2 \xi_n}{\partial t^2} = C (\xi_{n+1} + \xi_{n-1} - 2\xi_n). \quad (2)$$

Show that in the limit of continuous media ( $\lambda \gg a$ ) equation (2) transitions into equation (1). Calculate  $E$  as a function of  $C$ ,  $m$ , and  $a$ .

## **Exercise 2** *Linear chain of atoms with different spring constants*

Calculate the dispersion relation  $\omega(k)$  for a linear chain of identical atoms of mass  $m$ , distance between atoms  $d = a/2$ , and alternating spring constants  $C_1$  and  $C_2$ . (The unit cell with two identical atoms has thus a lattice constant of  $a$ .) Draw  $\omega(k)$  for  $C_1/C_2 = 1.0, 0.6, 0.3$ , and  $0.1$ .

## **Exercise 3** *Acoustic and optic waves in 2D*

Sketch the longitudinal and transverse waves for optic and acoustic modes in a 2D NaCl structure with lattice constant  $a$ . The wavevector with  $\lambda = 4a$  is in the  $[1\ 0]$  direction.

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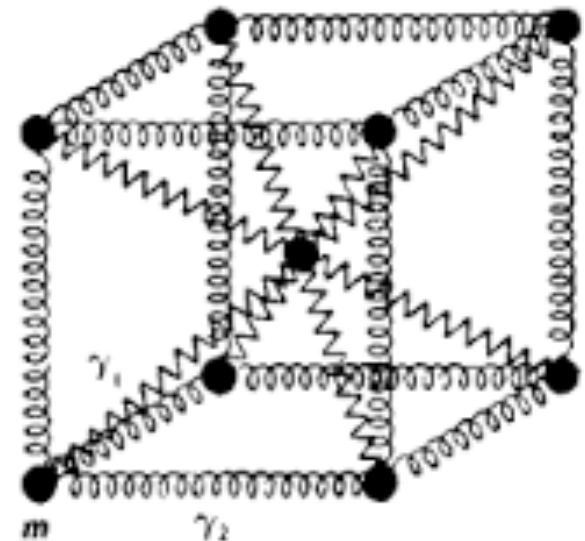
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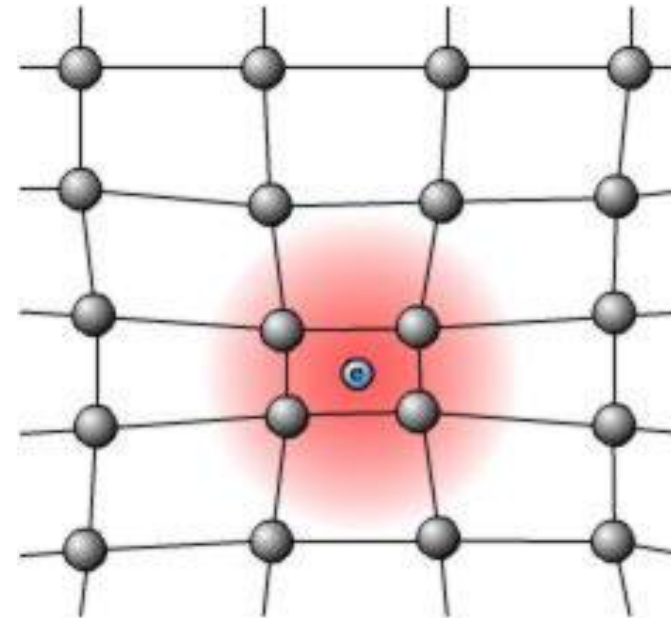
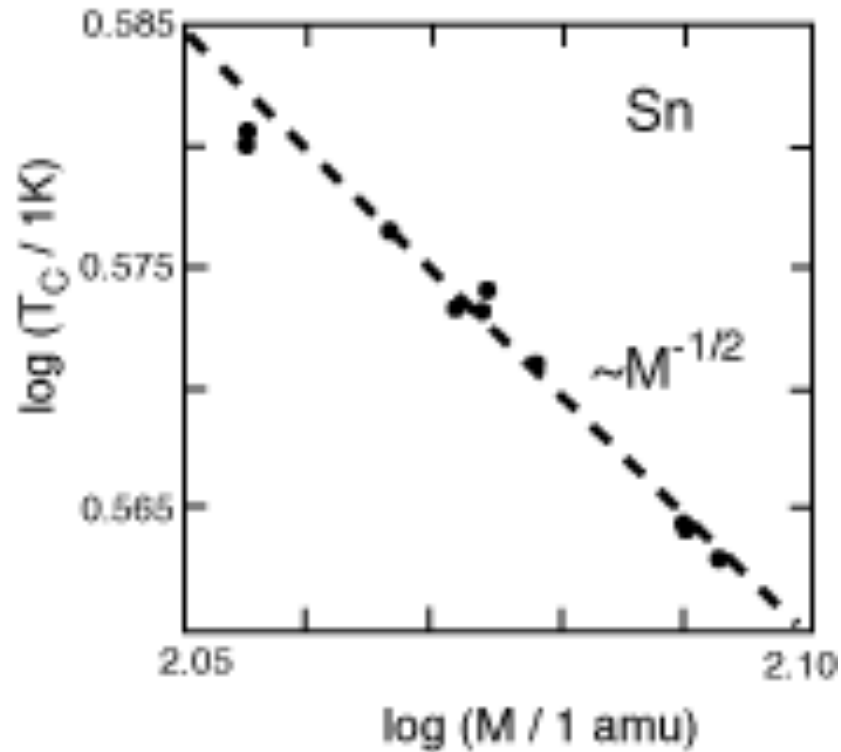
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- Tasks
- Why is phonons important
- Theory & concepts
- How to measure phonons



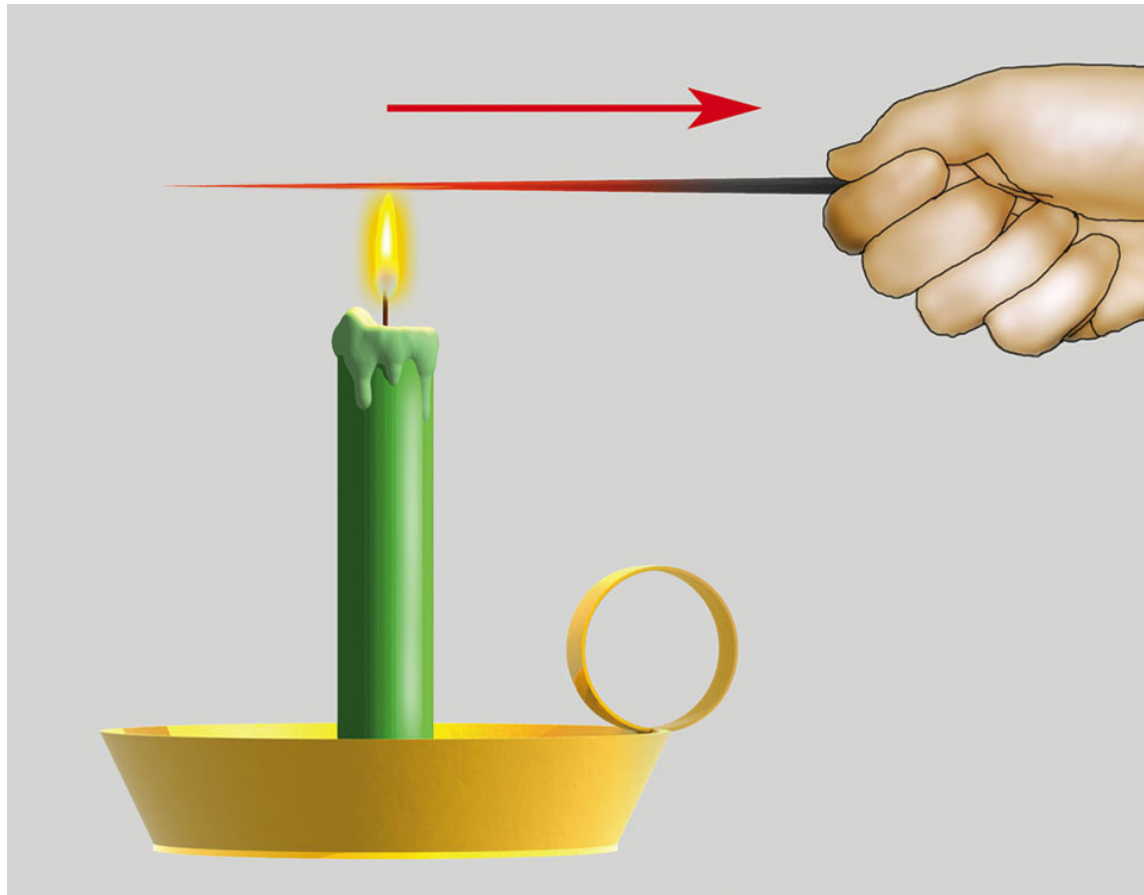
# Phonons can make superconductivity



E. Maxwell, Phys. Rev. **86**, 235 (1952) and  
B. Serin et al., Phys. Rev. B **86** 162 (1952))

<http://www.chm.bris.ac.uk/webprojects2000/igrant/theory.html>

# Phonons can conduct heat



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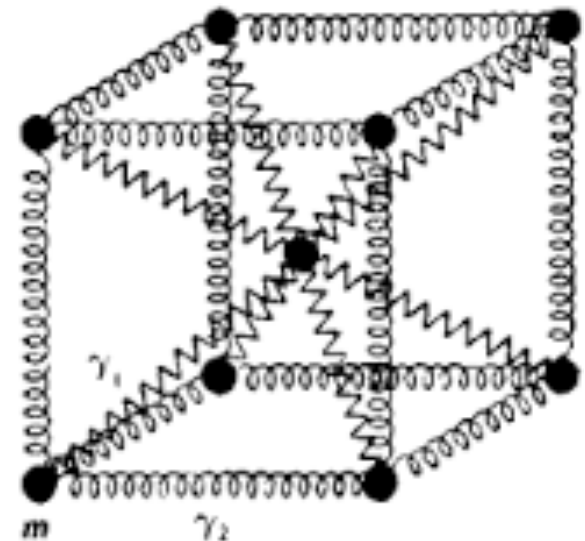
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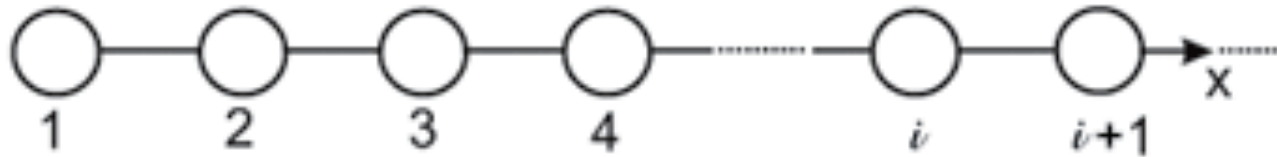
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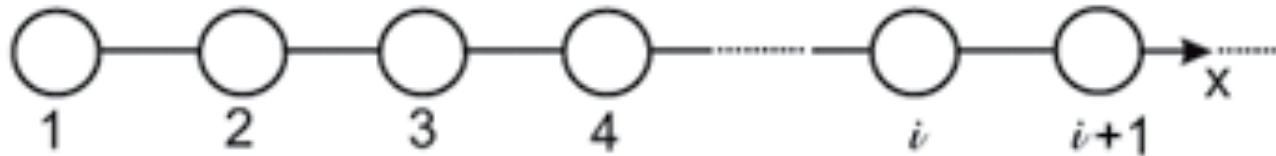
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# Linear chain -Models

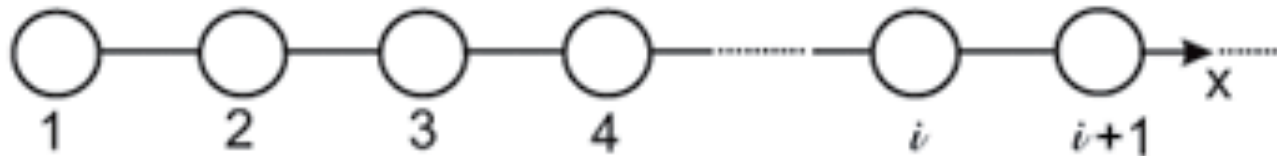


**Structure factor:  $S = \sum_i e^{-iqr_i}$**



**Madelung constant:  $\alpha = 2 \ln(2)$**

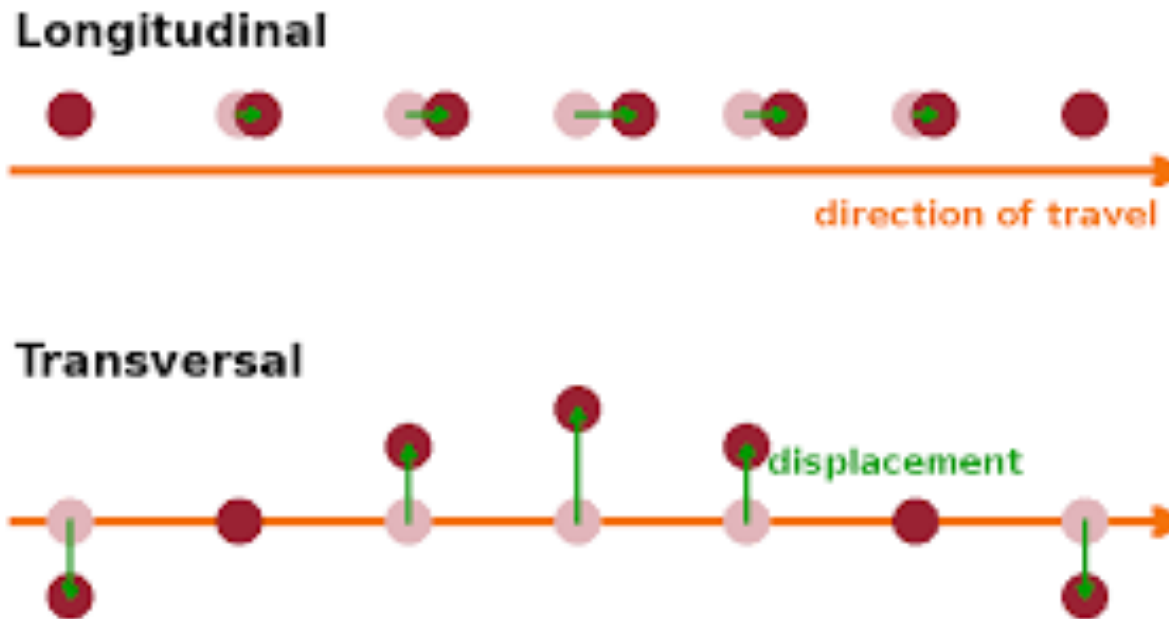
**Distortion Energy :  $E = 0.5 * \text{constant} * \delta^2$**



**Phonon dispersion:  $\omega =$**



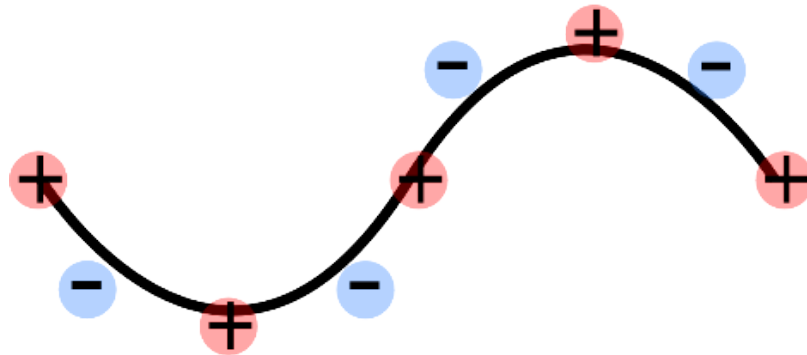
# Longitudinal and Transverse Phonons



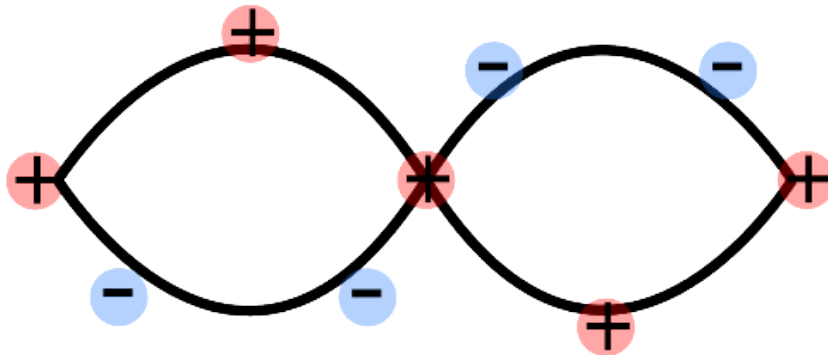
LA = Longitudinal Acoustic  
LO = Longitudinal Optical  
TA = Transversal Acoustic  
TO = Transversal Optical

# Acoustic and optical modes

Acoustical Mode



Optical Mode



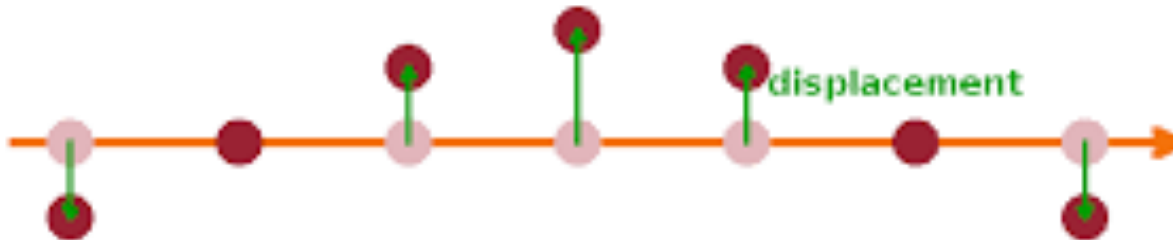
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# Number of phonon branches

## Longitudinal



## Transversal



LA = Longitudinal Acoustic  
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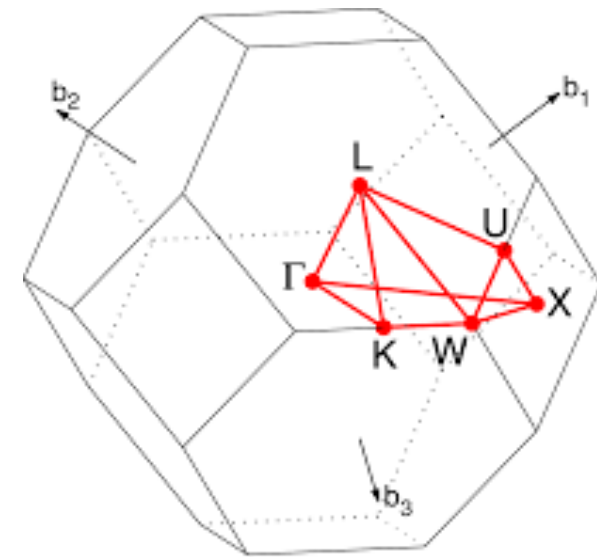
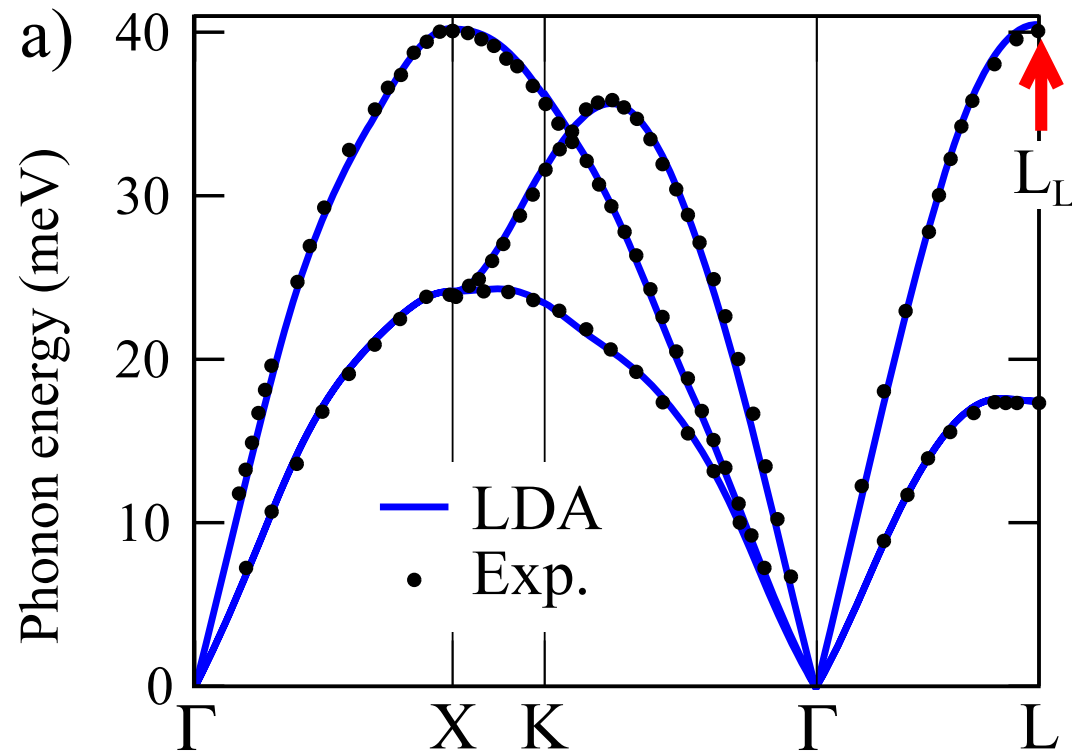
$p$  = number of atoms in the primitive cell

3 acoustic branches

$3p-3$  optical branches

Total  $3p$  phonon branches

# Phonons in aluminium

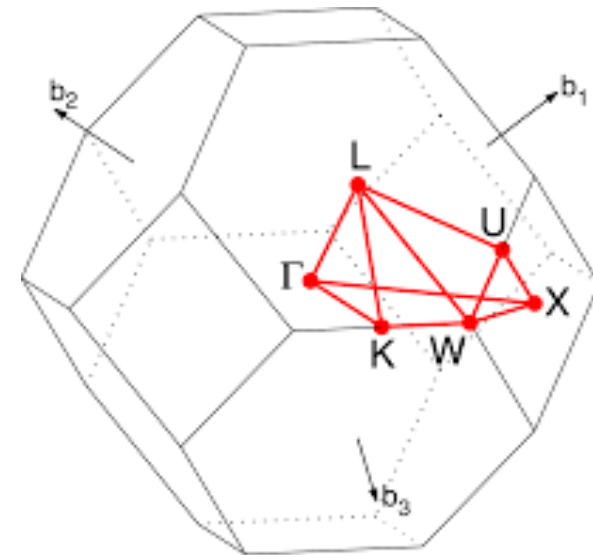
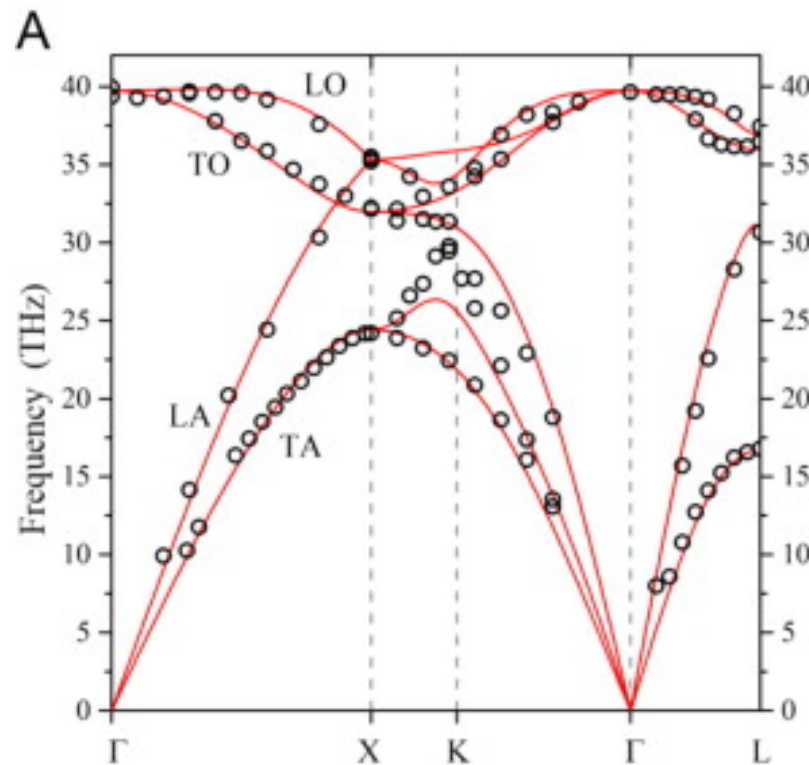


FCC path:  $\Gamma$ -X-W-K- $\Gamma$ -L-U-W-L-K|U-X

[Setyawan & Curtarolo, DOI: 10.1016/j.commatsci.2010.05.010]

<http://iopscience.iop.org/article/10.1088/0953-8984/24/5/053202>

# Phonons in diamond



FCC path:  $\Gamma$ -X-W-K- $\Gamma$ -L-U-W-L-K|U-X

[Setyawan & Curtarolo, DOI: 10.1016/j.commatsci.2010.05.010]

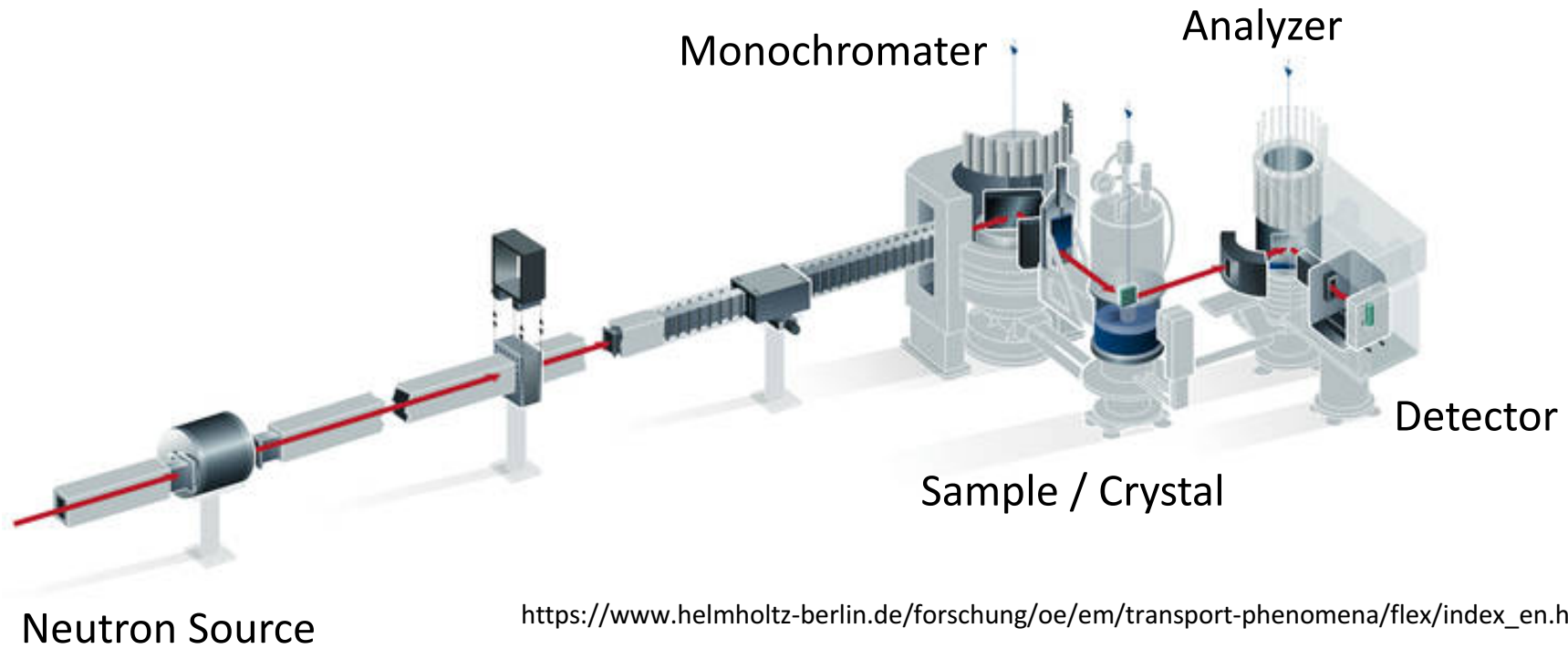
1 THz = 4.14... meV

$p$  = number of atoms in the basis of the primitive cell

3 $x$  $p$  phonon branches

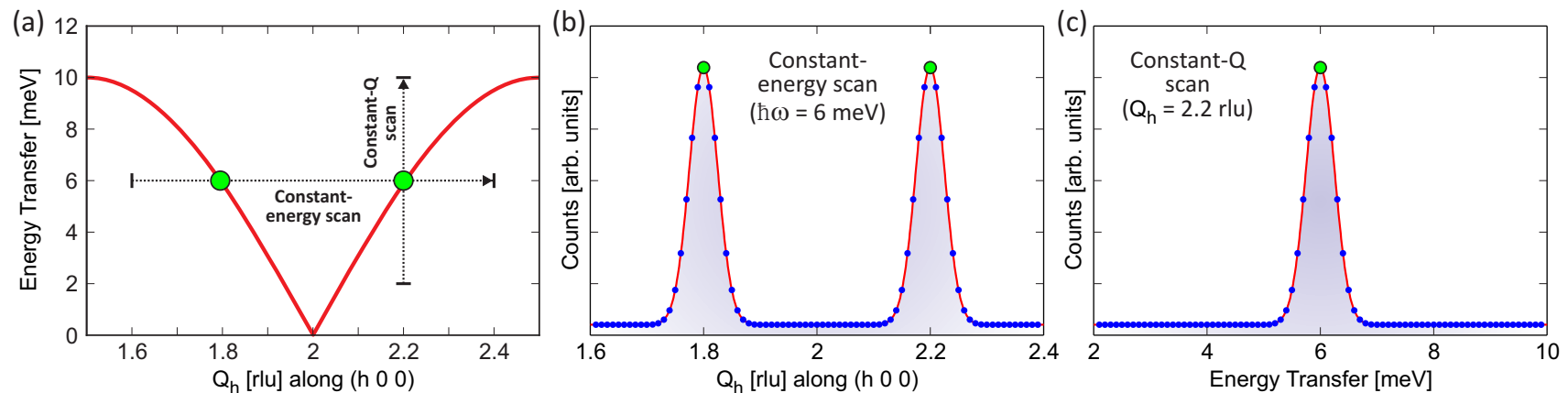
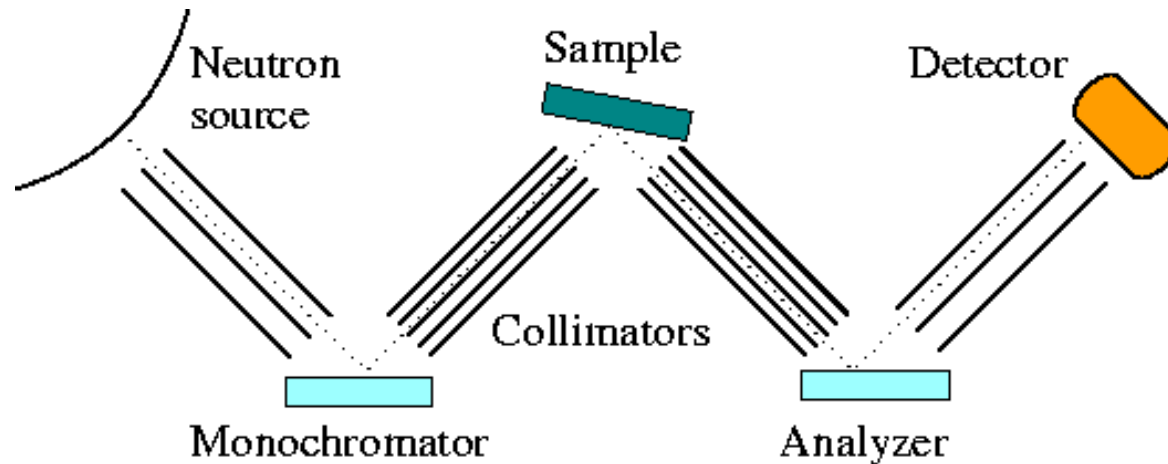
3 Acoustic branches and 3 $p$ -3 optical branches

# Triple axis spectrometer



**The Nobel Prize in Physics 1994**  
Bertram N. Brockhouse, Clifford G. Shull

# Triple axis spectrometer



**Figure 5:** (a) Schematic view of how two points of the phonon dispersion curve can be measured using either (b) constant-energy scan or (c) constant-Q scan. By performing multiple scans it is possible to map out the complete dispersion (see below).

[https://www.psi.ch/Ins/TrainingEN/INS\\_Student\\_Practicum\\_PSI.pdf](https://www.psi.ch/Ins/TrainingEN/INS_Student_Practicum_PSI.pdf)

# Triple axis spectrometer with x-rays

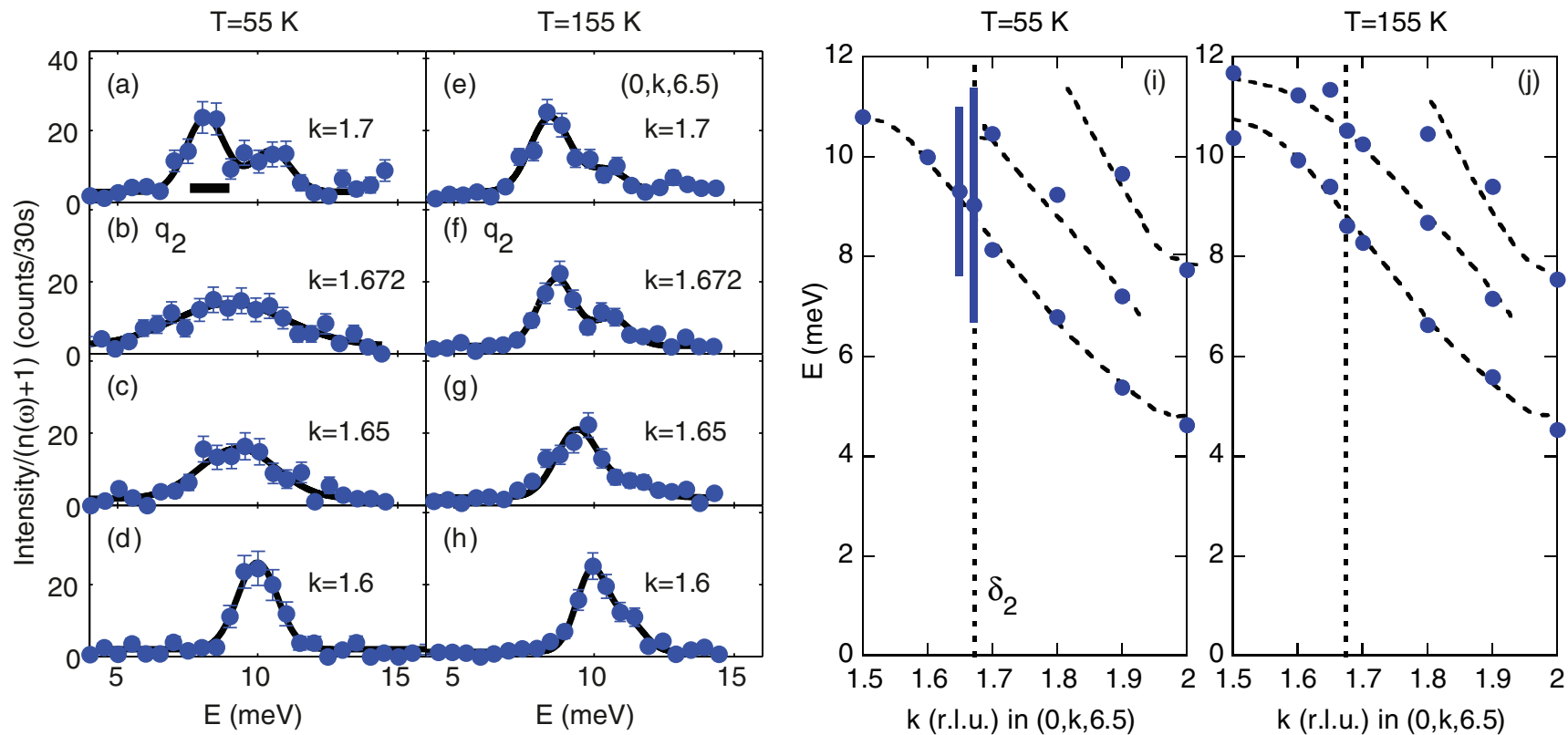
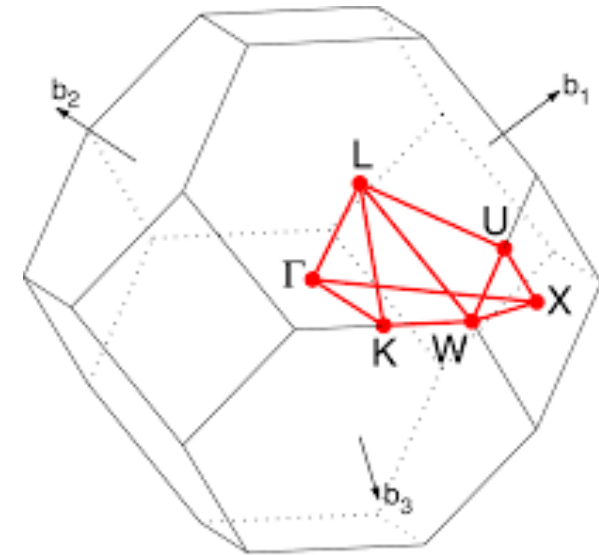
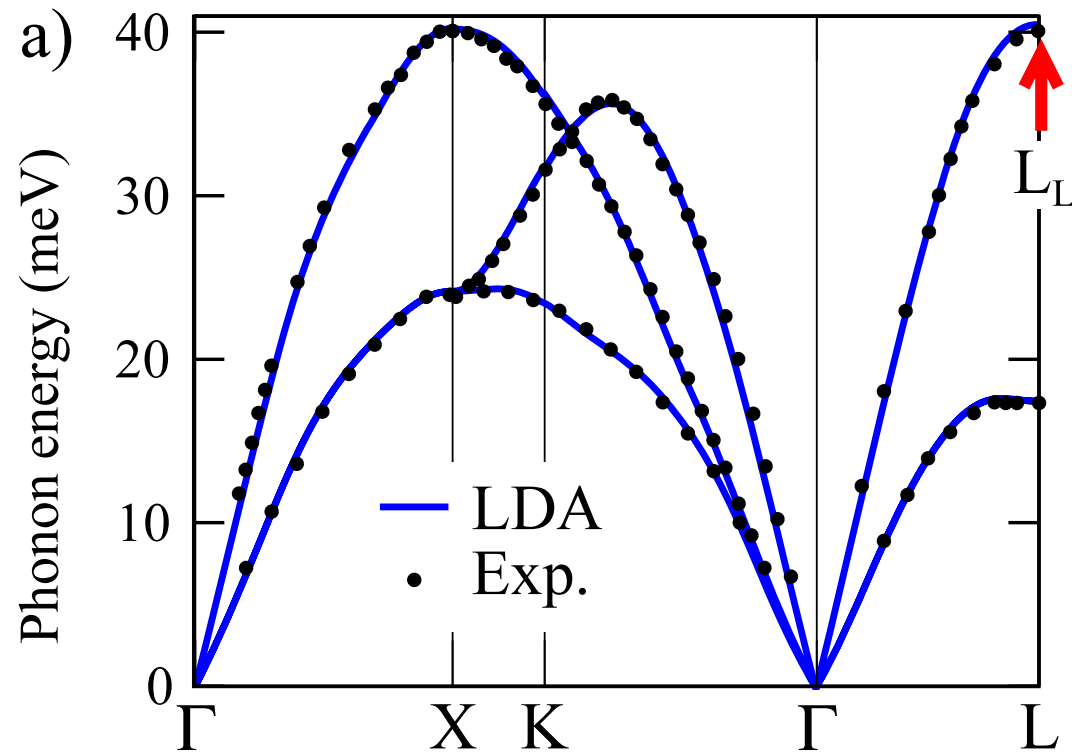


FIG. 5. (Color online) [(a)–(h)] IXS  $E$  scans of the low-energy phonons for wave vectors along the  $(0,k,6.5)$  line. Solid lines are fits to a sum of Gaussian functions. Data have been multiplied by  $1 - \exp[-E/(k_B T)]$  to correct for the Bose factor. The horizontal bar in panel (a) is the instrumental resolution. [(i) and (j)] Phonon dispersion curves along the  $(0,k,6.5)$  line for  $T = 55$  and  $155$  K. The solid circles represent the phonon peak positions determined from fitting data such as that in (a)–(h); the dashed lines are guides to the eye for the different branches. The resolution-deconvolved phonon widths are represented by vertical bars. The vertical dotted line is the CDW ordering wave vector.



# Phonons in aluminium



FCC path:  $\Gamma$ -X-W-K- $\Gamma$ -L-U-W-L-K|U-X

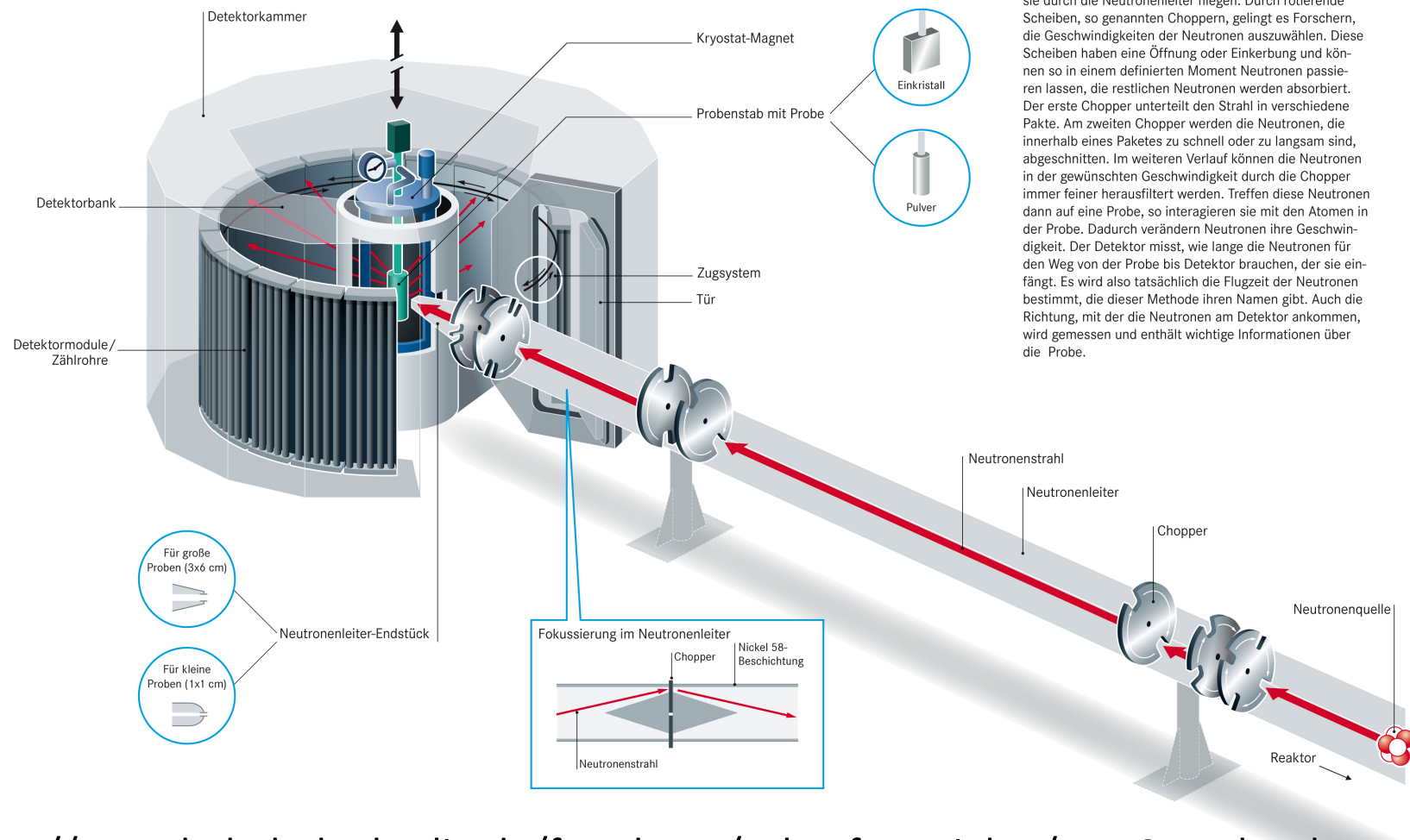
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# Time-of-flight spectrometry

## Flugzeitspektrometer NEAT II

Infografik: E. Strickert



# Acoustic Phonon in $\text{Sr}_2\text{RuO}_4$

