AUTOMATED CALCULATION OF HIGH-PRECISION SCATTERING AMPLITUDES

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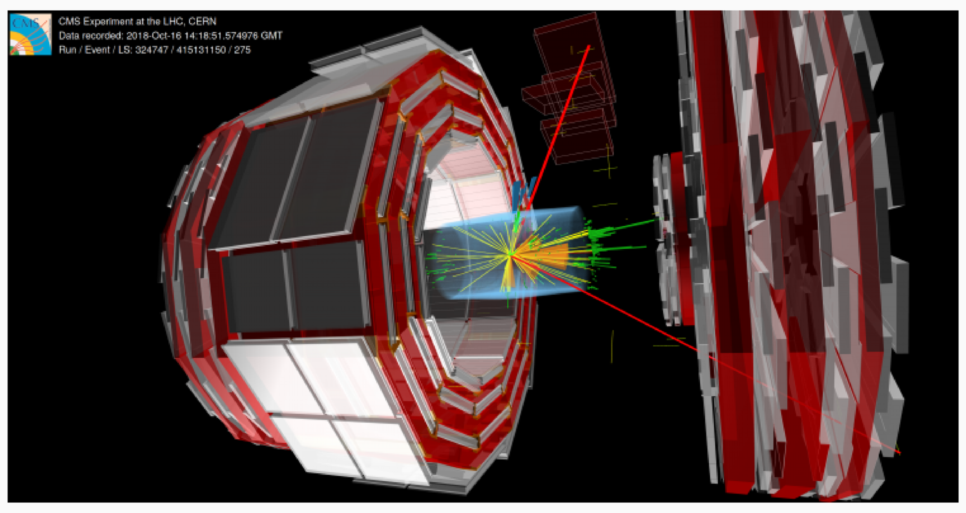
Testing the Standard Model at the LHC in search of new physics

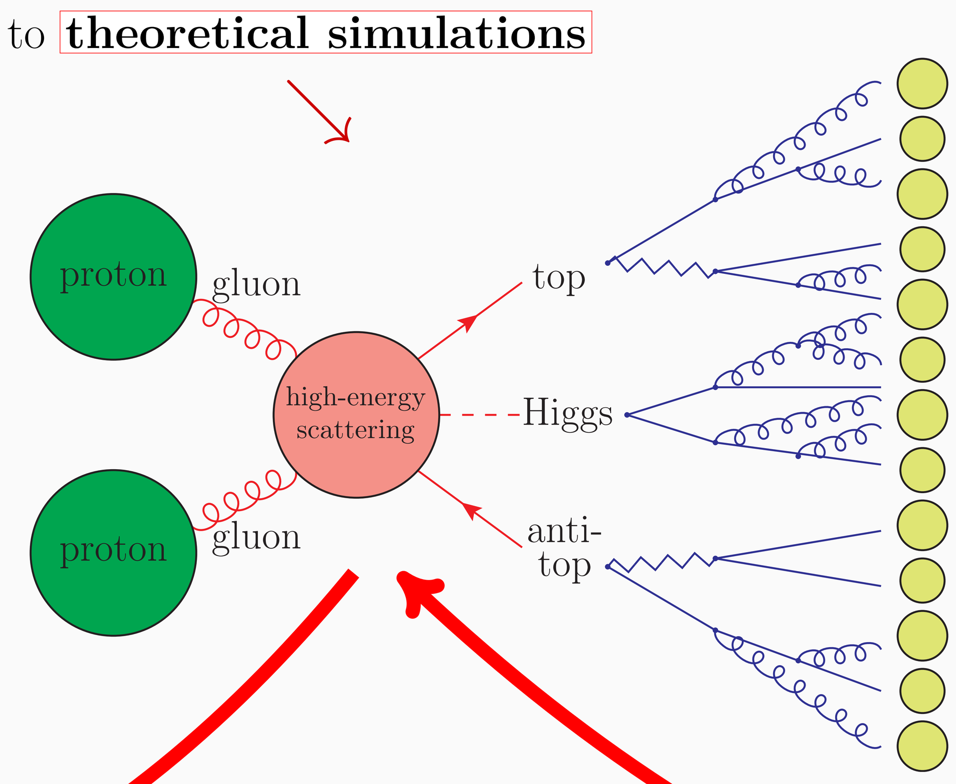
High-energy collisions of protons (14 TeV) produce **huge amount of particles**

Experimental measurements compared to **theoretical simulations**



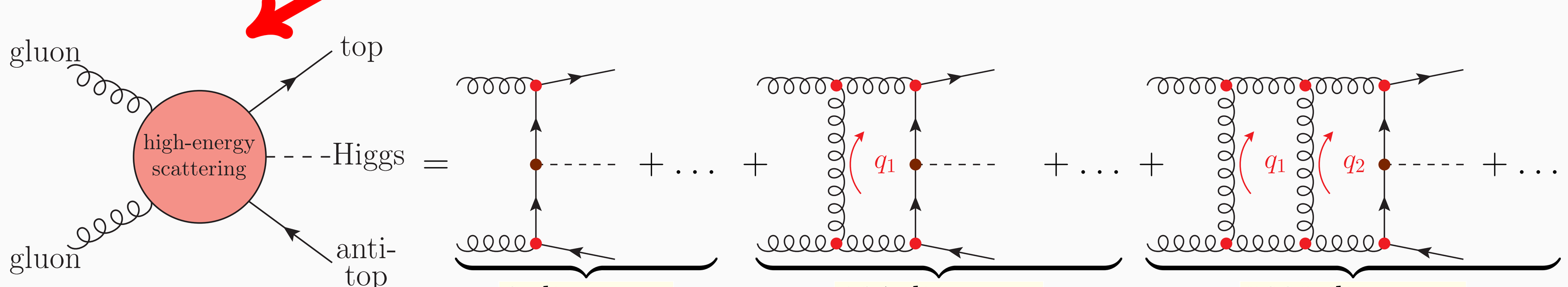
[https://cmsexperiment.web.cern.ch/sites/cmsexperiment.web.cern.ch/files/field/image/display1_hd_0.png]

↔



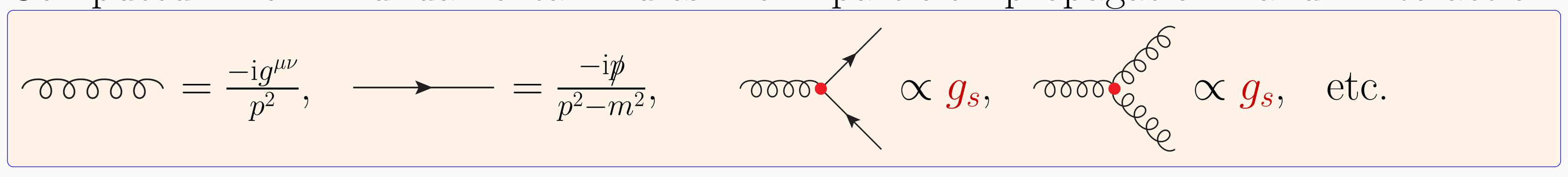
High-energy scattering probabilities calculated at high precision for a wide range of processes are crucial in new physics searches.

Scattering probabilities in perturbation theory



8 diagrams	164 diagrams	5335 diagrams
↓	↓	↓
Leading order (LO)	**Next-to-leading order (NLO)**	**NNLO**
↓	↓	↓
~ 100% **uncertainty**	~ 10% **uncertainty**	~ 1% **uncertainty**

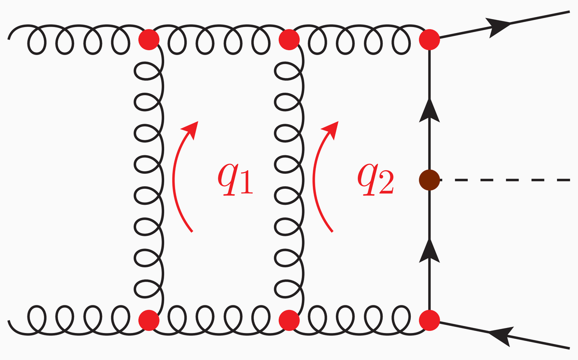
- Scattering probability related to **scattering amplitude**
  = Sum of all quantum field interactions leading from initial state to final state
- Computed from fundamental rules for particle propagation and interaction:



- Expansion in small couplings, e.g. g_s for strong interaction.
  Higher precision ≙ higher order in coupling(s) ≙ more loops

Complexity grows rapidly with number of loops and external particles
⇒ **Automated numerical tools**, e.g. OpenLoops **2** @ NLO [Buccioni, Lang, Lindert, Maierhöfer, Pozzorini, Zhang, M.Z. 2019] enable studies of **large number of processes**.

Our goal: Fully automated high-precision calculations @ NNLO



$$= \sum_{r=0}^{4}\sum_{s=0}^{2}\left(\sum_{\mu_1=0}^{3}\cdots\sum_{\mu_r=0}^{3}\sum_{\nu_1=0}^{3}\cdots\sum_{\nu_s=0}^{3} \mathcal{N}_{\mu_1\cdots\mu_r\nu_1\cdots\nu_s} \times \int \mathrm{d}^D q_1 \int \mathrm{d}^D q_2 \frac{q_1^{\mu_1}\cdots q_1^{\mu_r} q_2^{\nu_1}\cdots q_2^{\nu_s}}{\mathcal{D}(q_1,q_2)}\right) + \int \mathrm{d}^D q_1 \int \mathrm{d}^D q_2 \frac{\mathcal{N}(q_1,q_2)\big|_{D=4-2\varepsilon} - \mathcal{N}(q_1,q_2)\big|_{D=4}}{\mathcal{D}(q_1,q_2)}$$

Recursive calculation of coefficients
$\mathcal{N}_n = \mathcal{N}_{n-1}\cdot\mathcal{K}_n$
exploiting factorisation of amplitude into fundamental building blocks $\mathcal{K}_n$
⇒ Highly efficient and fully general algorithm
[Pozzorini, N.S., M.Z. 2022] ✓

Treatment of divergences and their effects
→ expressed through universal subtraction terms

- of UV origin: available for QED and QCD corrections [Lang, Pozzorini, Zhang, M.Z. 2020, 2020, 2021] ✓
- of IR origin: under investigation

[Picture created by Zhang, F.L. using ChatGPT, DALL·E; final result plot: Devoto, Grazzini, Kallweit, Mazzitelli, Savoini 2024]

Loop integrals

↓ **1. Recursive reduction** [F.L., M.Z.] ✓

$$\sum_{\Omega} C_1^{\mu_1\ldots\mu_r}(\Omega) C_2^{\nu_1\ldots\nu_s}(\Omega)$$

$$\int \mathrm{d}^D q_1 \int \mathrm{d}^D q_2 \frac{1}{\mathcal{D}_\Omega(q_1,q_2)}$$

(simpler scalar integrals)

↓ **2. Reduction** with `Kira` [Maierhöfer, Usovitsch, Uwer 2017; Klappert, F.L., Maierhöfer, Usovitsch 2020; F.L., Usovitsch, Wu 2025]

$$\sum_{\Omega_M} C_{\Omega,\Omega_M}$$

$$\int \mathrm{d}^D q_1 \int \mathrm{d}^D q_2 \frac{1}{\mathcal{D}_{\Omega_M}(q_1,q_2)}$$

(small set of master integrals)

↲ **3. Master integral evaluation** (external)

Conclusions

Automated tools @ NLO have played a key role in the success of the LHC. Similar tools @ NNLO are highly desirable to meet the precision demands of the next years. While there has been huge progress in this field, new methods still need to be developed by our group and others to reach this goal.