University of Zurich[UZH]

Solid State Physics
Exercise Sheet 5

FS17
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Discussion on 29th March

Due on 5th April

Exercise 1 *Elastic waves in lattices and continuous media*
In continuous media the 1D wave equation reads

$$\frac{\partial^2 \xi(x,t)}{\partial t^2} = v^2 \frac{\partial^2 \xi(x,t)}{\partial x^2}, \tag{1}$$

with the speed of sound $v = \sqrt{E/\rho}$, elastic modulus E, and density ρ. For a linear chain of atoms with distance a, mass m, and spring constant C we get

$$m \frac{\partial^2 \xi_n}{\partial t^2} = C\left(\xi_{n+1} + \xi_{n-1} - 2\xi_n\right). \tag{2}$$

Show that in the limit of continuous media ($\lambda \gg a$) equation (2) transitions into equation (1). Calculate E as a function of C, m, and a.

Exercise 2 *Linear chain of atoms with different spring constants*
Calculate the dispersion relation $\omega(k)$ for a linear chain of identical atoms of mass m, distance between atoms $d = a/2$, and alternating spring constants C_1 and C_2. (The unit cell with two identical atoms has thus a lattice constant of a.) Draw $\omega(k)$ for $C_1/C_2 = 1.0$, 0.6, 0.3, and 0.1.

Exercise 3 *Acoustic and optic waves in 2D*
Sketch the longitudinal and transverse waves for optic and acoustic modes in a 2D NaCl structure with lattice constant a. The wavevector with $\lambda = 4a$ is in the [1 0] direction.