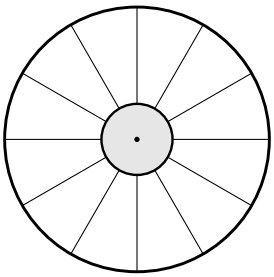
Mechanics
Exercise Sheet 9

HS 2026
Prof. G. Isidori

Issued: 18.11.2026
https://www.physik.uzh.ch/en/teaching/PHY311/HS2026.html Discussed: 25.11.2026

Exercise 1: A moving wheel

A wheel consists of a uniform disk of radius r and mass m, a thin circular rim of radius $R > r$ and mass M, and twelve identical radial spokes connecting the disk to the rim. Each spoke is a thin uniform rod of mass m_s extending from radius r to radius R. The wheel is rigid.



a) Calculate the moment of inertia I of the wheel about the axis through its centre and perpendicular to its plane.

b) The wheel is initially spinning clockwise with angular speed ω_0, while its centre is at rest. It is then placed upright on a rough horizontal surface. During slipping, assume that kinetic friction has constant magnitude $F = \mu M_{\text{tot}} g$, where $M_{\text{tot}} = m + M + 12m_s$ and μ is the coefficient of kinetic friction. Find the time τ at which the wheel begins rolling without slipping, and its centre-of-mass speed for $t \geq \tau$. Neglect rolling resistance once slipping has stopped.

Exercise 2: Earth's free precession

The Earth is approximately an oblate ellipsoid with equal equatorial semi-axes $a = b = 6378\,\text{km}$ and polar semi-axis $c = 6357\,\text{km}$. Model it as a homogeneous rigid body of total mass M. Its rotation axis is slightly misaligned with its symmetry axis. Neglect external torques.

a) Calculate the three principal moments of inertia about the Earth's centre of mass. You may use the change of variables $x = au$, $y = av$, $z = cw$, which maps the ellipsoid to the unit sphere.

b) Use Euler's equations for a freely rotating symmetric body to determine the frequency Ω at which the angular-velocity vector rotates around the symmetry axis in the body-fixed frame. Express Ω in terms of the principal moments of inertia and the component ω_3 along the symmetry axis.

c) Approximate the Earth's spin period by one day and calculate the period of this free precession. The observed Chandler wobble has a period of approximately 433 days. Why is the prediction of this simple model different?