Course Plan

<u>STANDARD PENSUM</u>

24.02 – Introduction + Crystal structures------------------ (DONE)
02.03 – Reciprocal space + Form & Structure Factor -------- (DONE)
09.03 – Scattering theory -------------------------------- (DONE)
16.03 – Crystal binding energies------------------------- (DONE)
23.03 – Lattice vibrations ------------------------------ (DONE)
06.04 – Phonons measurements ---------------------------- (DONE)
13.04 – Phonons specific heat---------------------------- (DONE)
20.04 – Free electron gas ------------------------------- (DONE)
27.04 – Band structure ---------------------------------- (DONE)
04.05 – Fermi surfaces
11.05 – Semi conductors

<u>BONUS PENSUM</u>

18.05 – Magnetism
25.05 – Superconductivity + Charge density wave order
01.06 – Repetition + Examples of Exam Questions

09.06 & 10.06 ORAL EXAM (Room: Y-36-H-48**)**

Fermiology - continued

1. Band structure calculations

   Tight – binding approach

   Definitions of band width, Fermi surface, electron mass

2. Quantum Oscillations

   Landau Levels

   Onsager Relation

3. Measurements of Fermi surfaces

   Fermi surface area

   Electronic mass

IMPORTANT: 11th of May – LECTURE AND EXERCISE IN 36J33

Summary of previous lecture

(1) Bloch's Theorem:

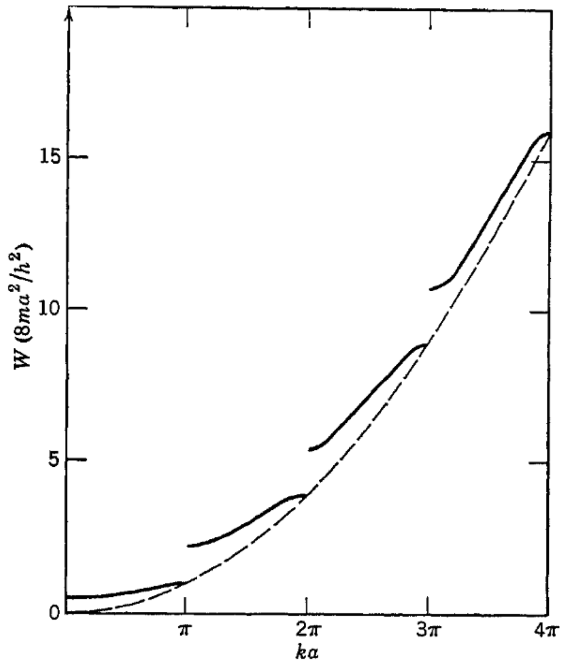
An electron in a periodic potential (crystal) has wavefunction of the form:

$$\psi(\mathbf{r}) = \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{r}} u(\mathbf{r})$$

Where $u(\mathbf{r})$ is a periodic function.

(2) Electronic band structure:

A periodic potential modifies the electron dispersion.
A band gap opens at the zone boundaries.



University of Zurich[UZH]

Solid State Physics
Exercise Sheet 10

FS16
Prof. Dr. Johan Chang

Discussion on 4^{th} May | Due on 11^{th} May

Exercise 1 *Kronig-Penney Model*

With this exercise, we will solve the Kronig-Penney model in small steps. This model starts with the assumption that the potential that the electron feels near the atomic positions can be approximated by a δ-function. The periodic potential can thus be written as $U(x) = Aa\sum_s \delta(x - sa)$ (in one dimension) where A is a constant and a is the lattice spacing.

(a) Show that in the Fourier series $U(x) = \sum_G U_G e^{iGx}$ we get for the coefficients $U_G = A$.

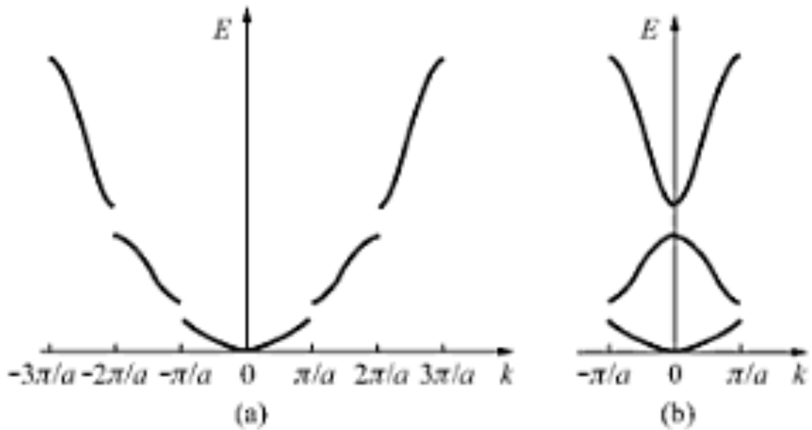
(b) Let $\psi(k)$ be the Fourier coefficients of the electron wave function. Use the central equation (derived in the lecture or found in text books and on the web) to show that

$$(\lambda_k - \epsilon)\psi(k) + A\sum_n \psi(k - 2\pi n/a) = 0 \qquad (1)$$

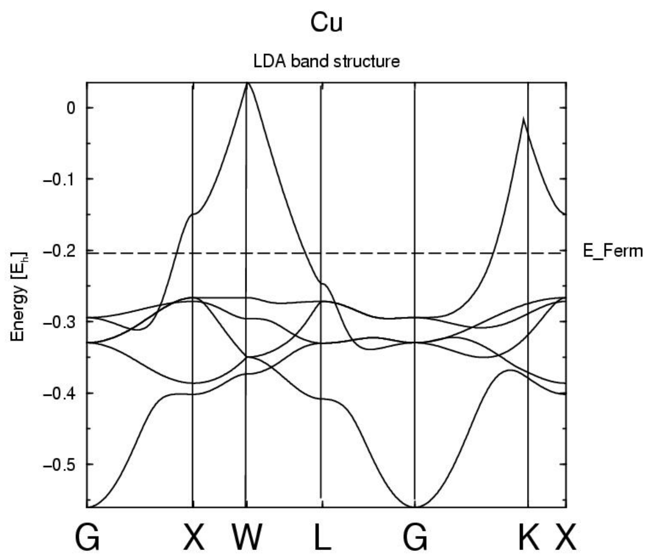
where $\lambda_k = \hbar^2 k^2/2m$.

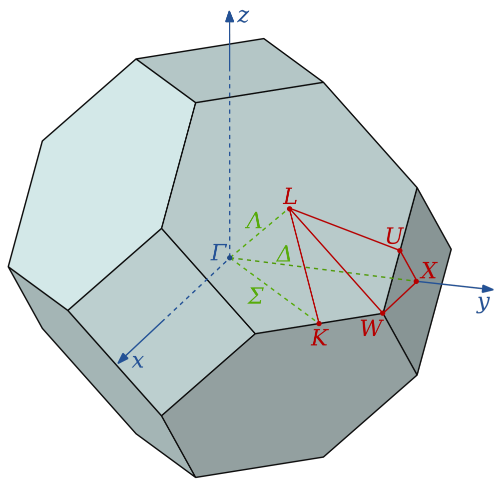
Summary of previous lecture

(3) Reduced zone scheme

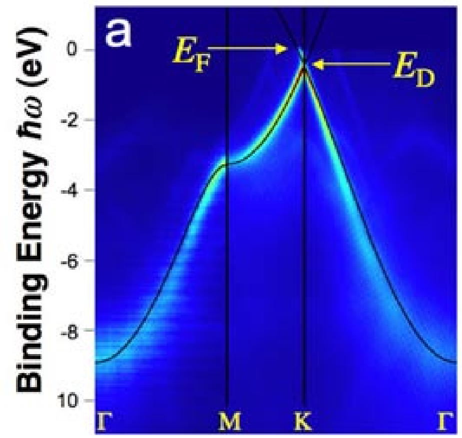


(4) Full band structure





Angle-resolved photoemission spectroscopy (ARPES)



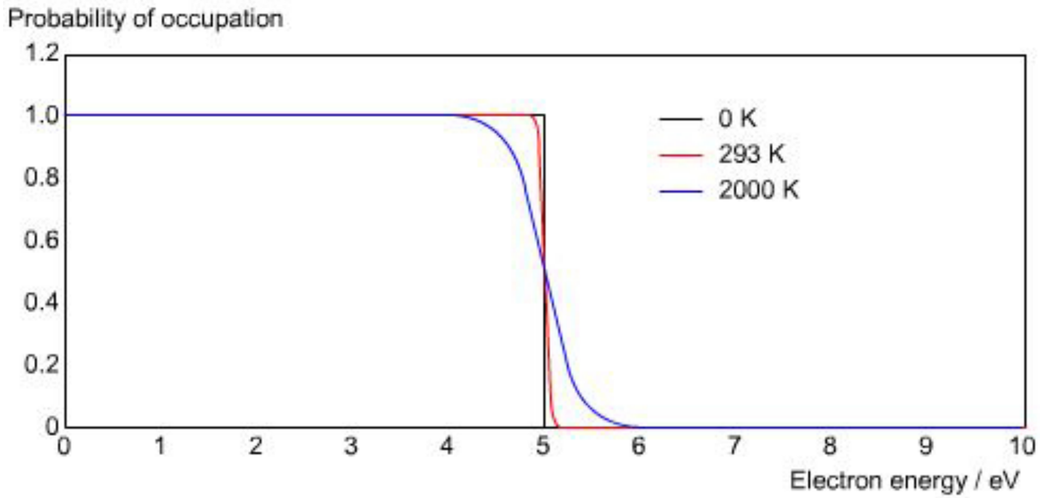
Fermi surface

<u>FROM WIKIPEDIA:</u>

In condensed matter physics, the **Fermi surface** is an abstract boundary in reciprocal space useful for predicting the thermal, electrical, magnetic, and optical properties of metals, semimetals, and doped semiconductors.

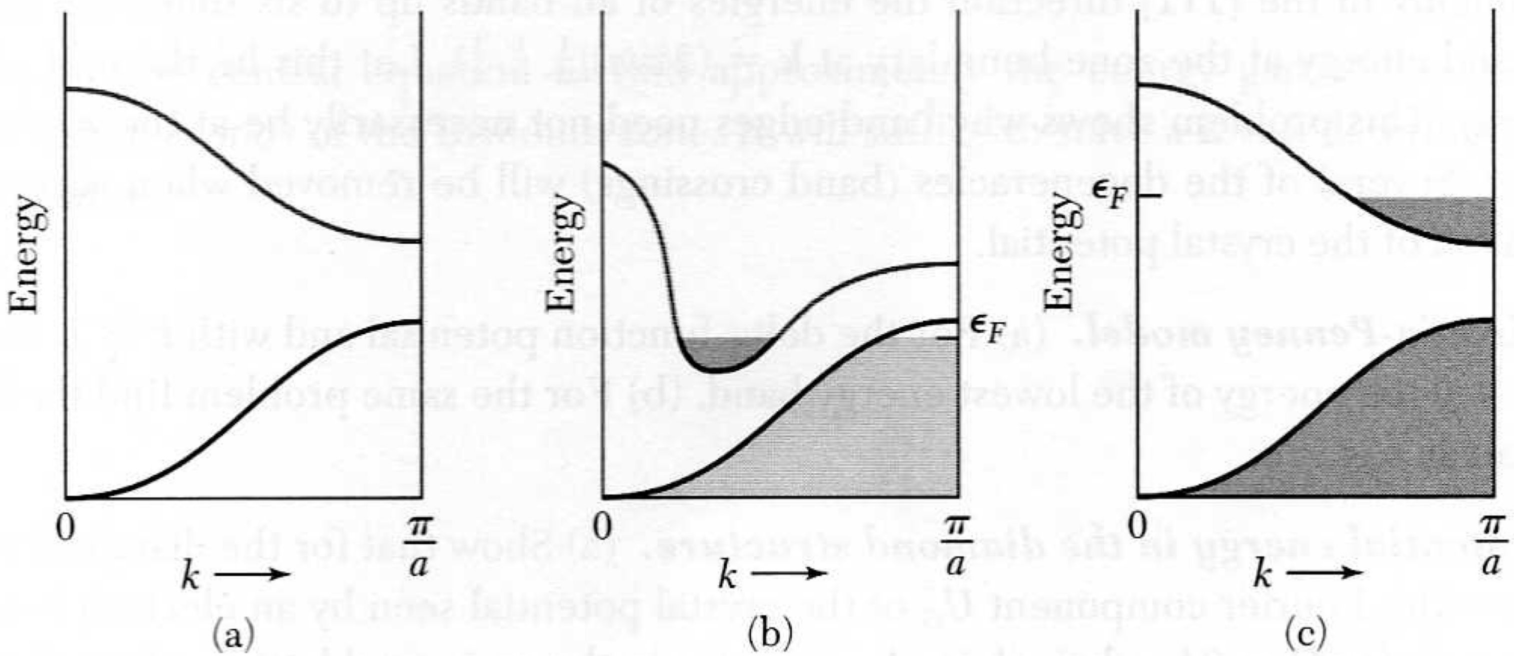
The shape of the Fermi surface is derived from the occupation of electronic energy bands.

The existence of a Fermi surface is a direct consequence of the Pauli exclusion principle, which allows a maximum of one electron per quantum state.



Fermi-Dirac distribution for several temperatures

Metals, Band Insulators, Semi metals

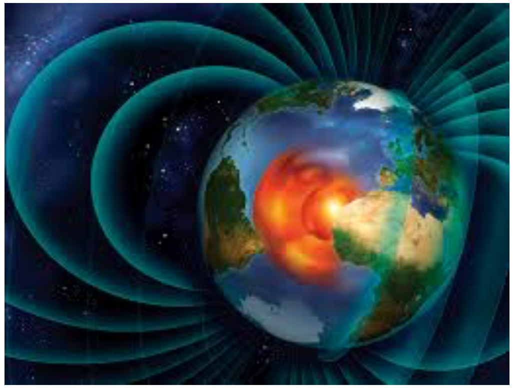


Magnetic field

Human Brain

1 nG to 10 nG

Earth



0.25 - 0.65 Gauss

Fridge Magnets

50 Gauss

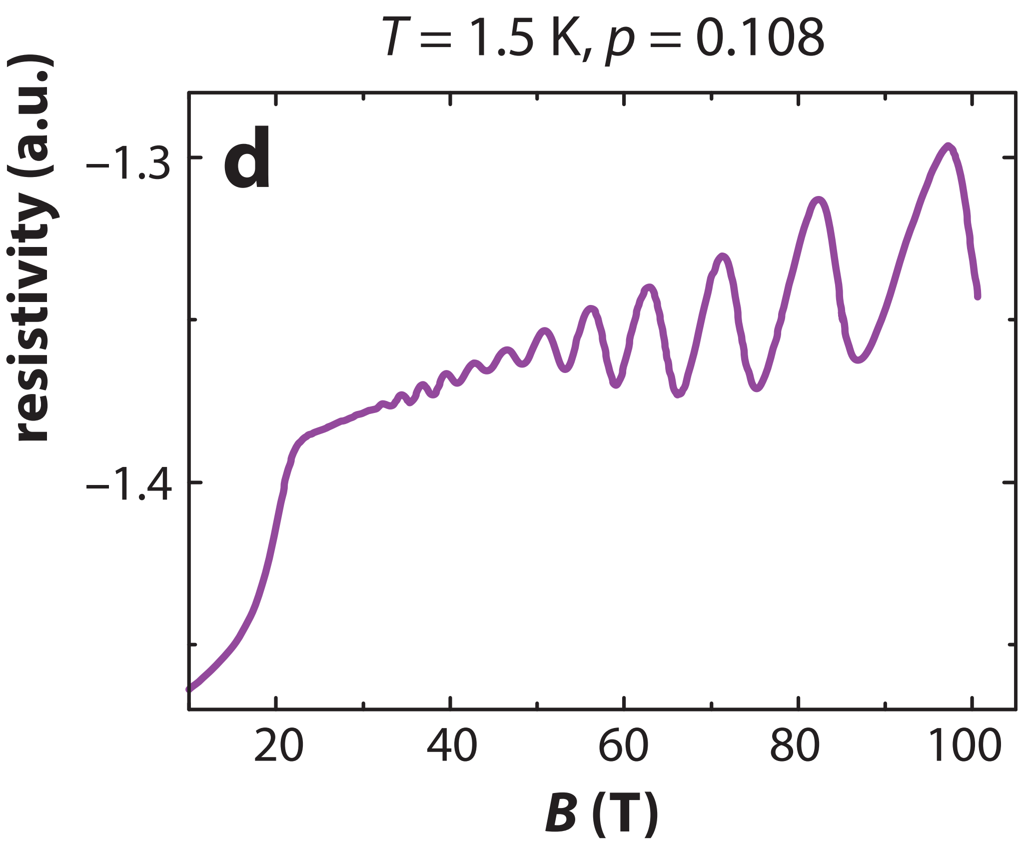
Neodymium – iron – boron
$Nd_2Fe_{14}B$ Magnet

10000 G = 1 Tesla

Static 45 –Tesla
Hybrid magnet

100 Tesla
Pulsed magnet

QUANTUM OSCILLATIONS:



Resistivity measurement of a high-temperature superconductor: $YBa_2Cu_3O_{6.51}$ (YBCO)

http://www.annualreviews.org/doi/pdf/10.1146/annurev-conmatphys-030212-184305

Outline:

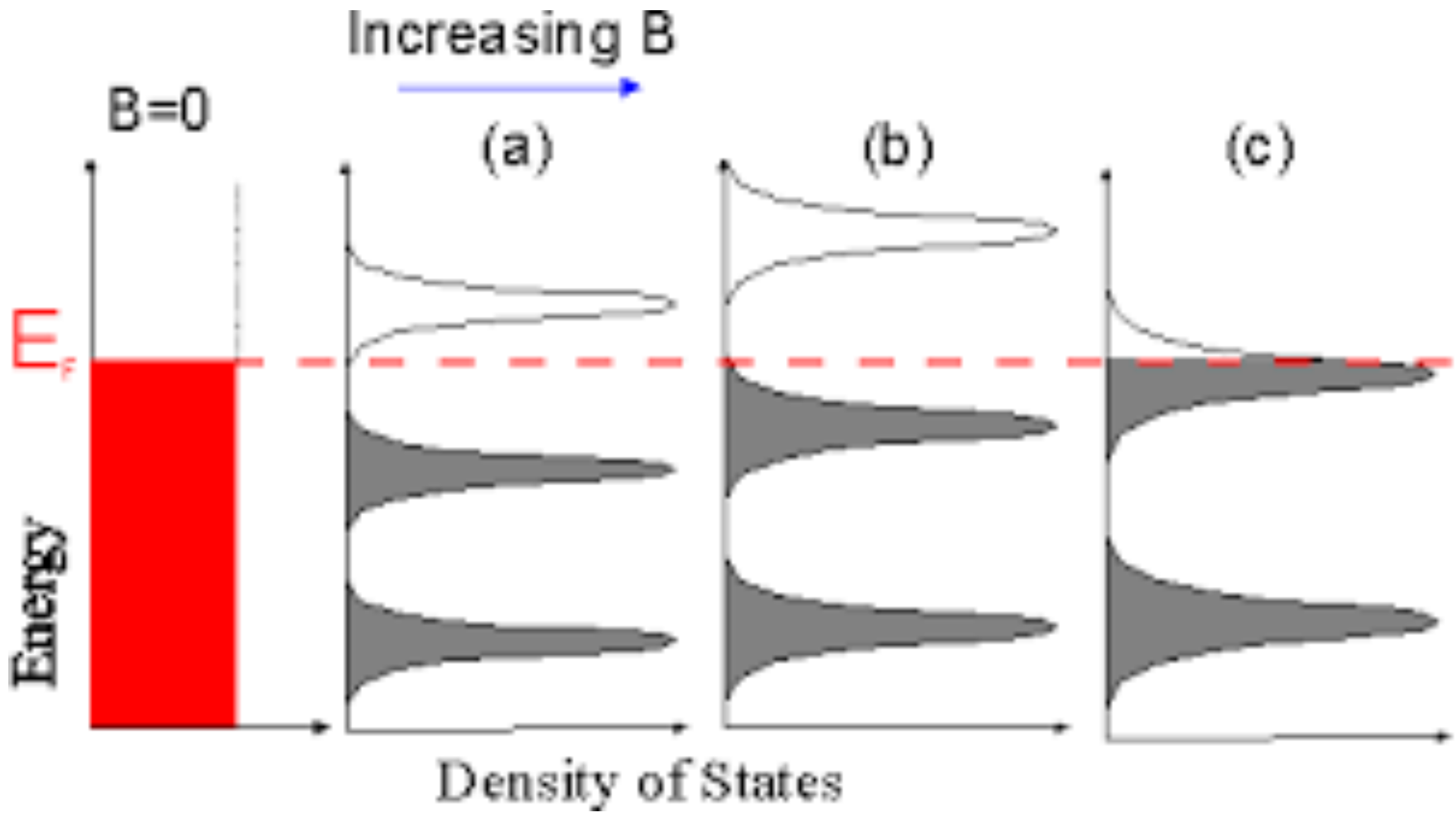
(1) Landau quantization / Landau levels:

Gives an understanding of why quantum oscillations exist.

(2) Onsager relation:

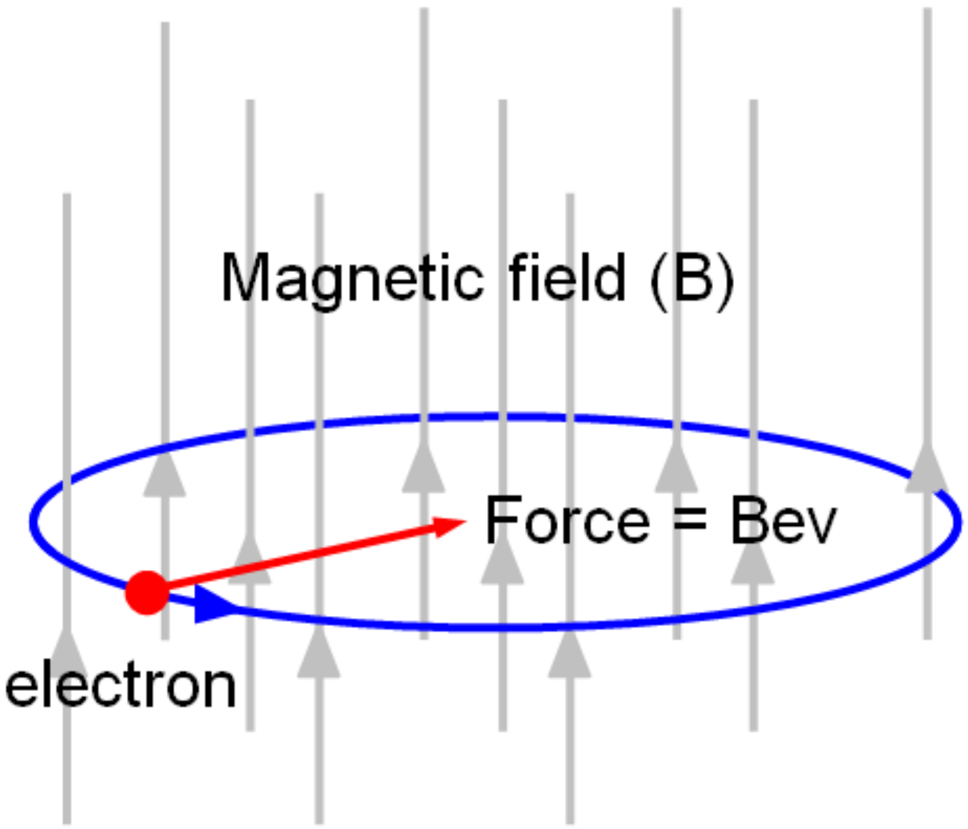
Gives a quantitative relation between the oscillations
and the Fermi surface area

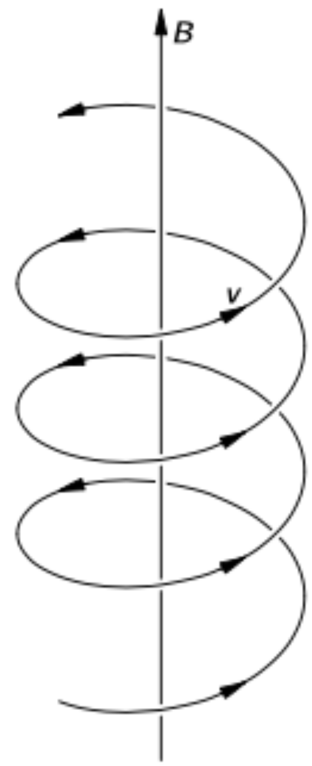
Landau Quantization



Density of states in two dimensions (2D).

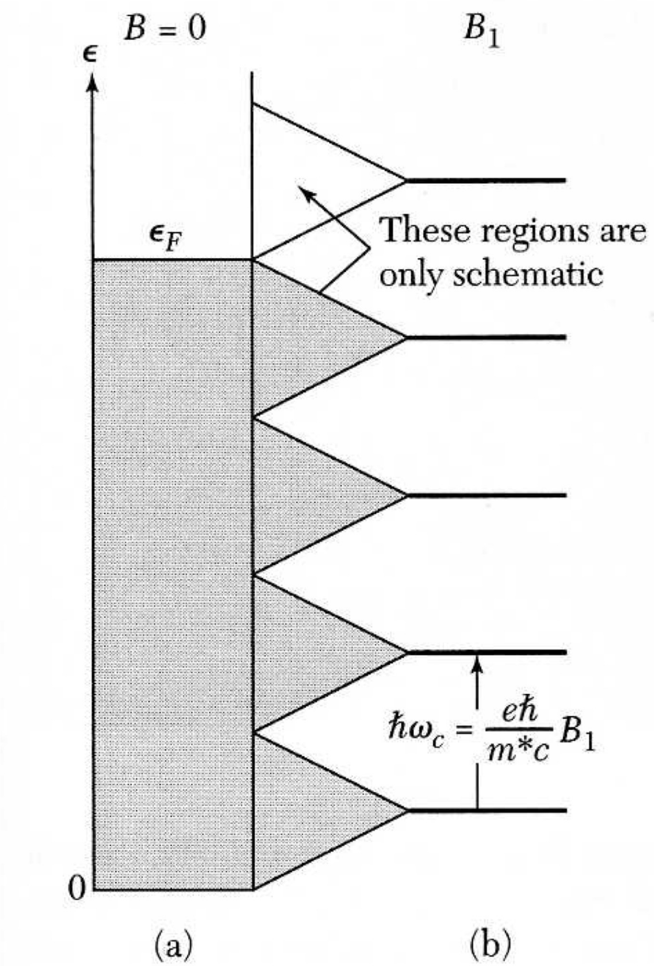
Electron in a magnetic field

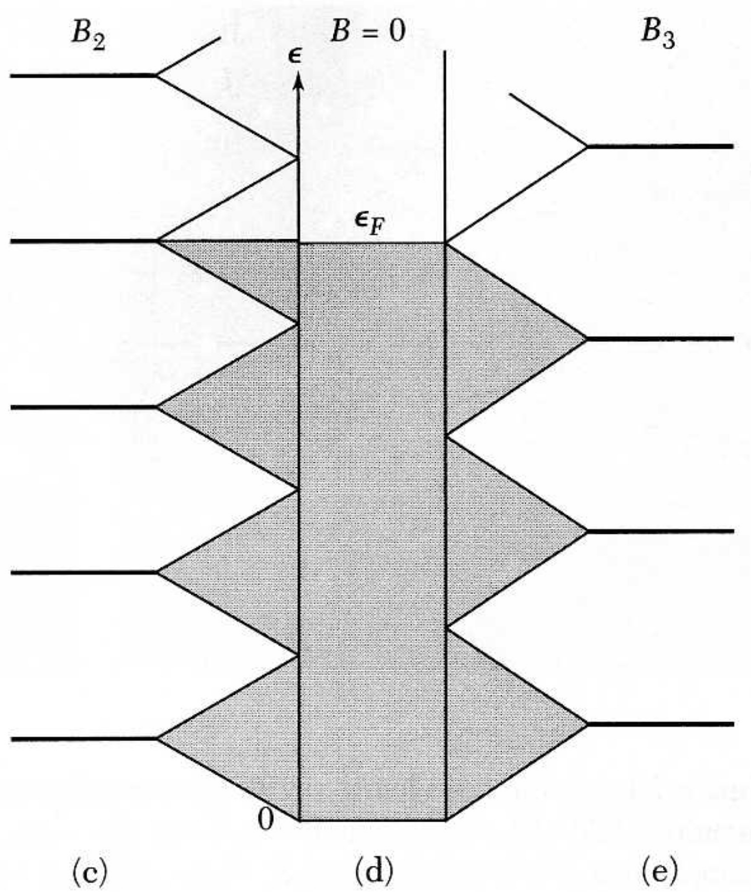




Lorentz Force: **F** = e **v** x **B**

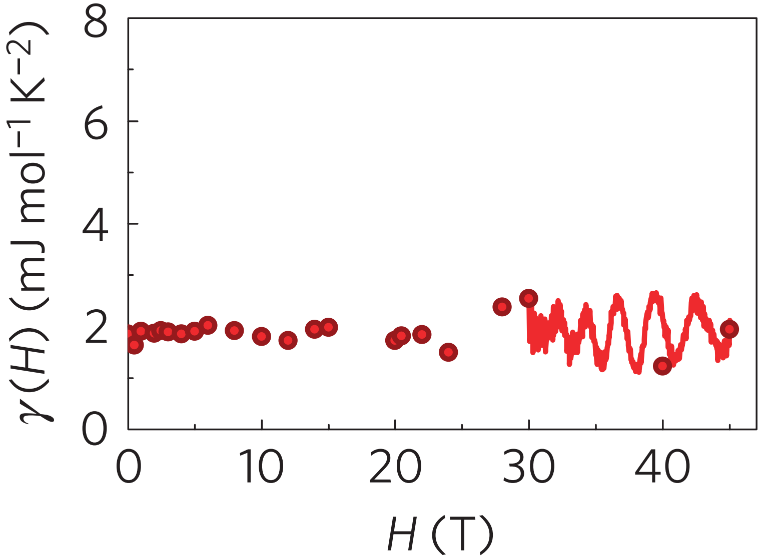
Landau Quantization:





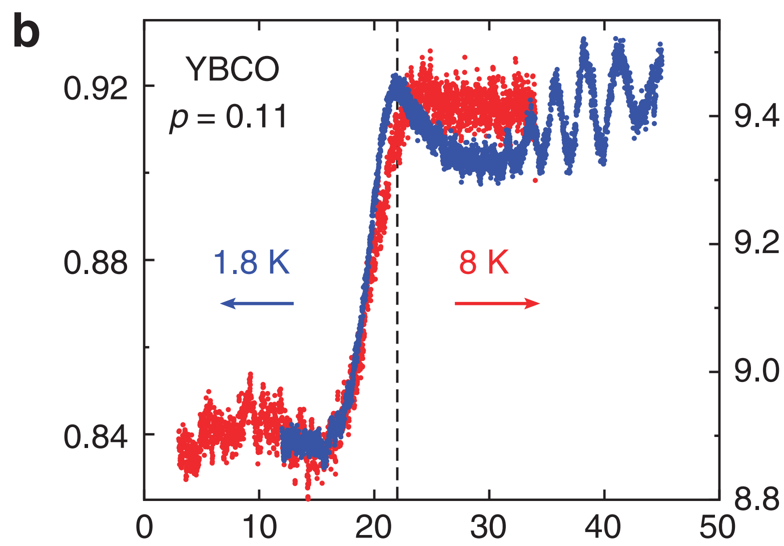
QUANTUM OSCILLATIONS: YBCO

Heat Capacity



Nature Physics **7**, 332–335 (2011)

Thermal conductivity

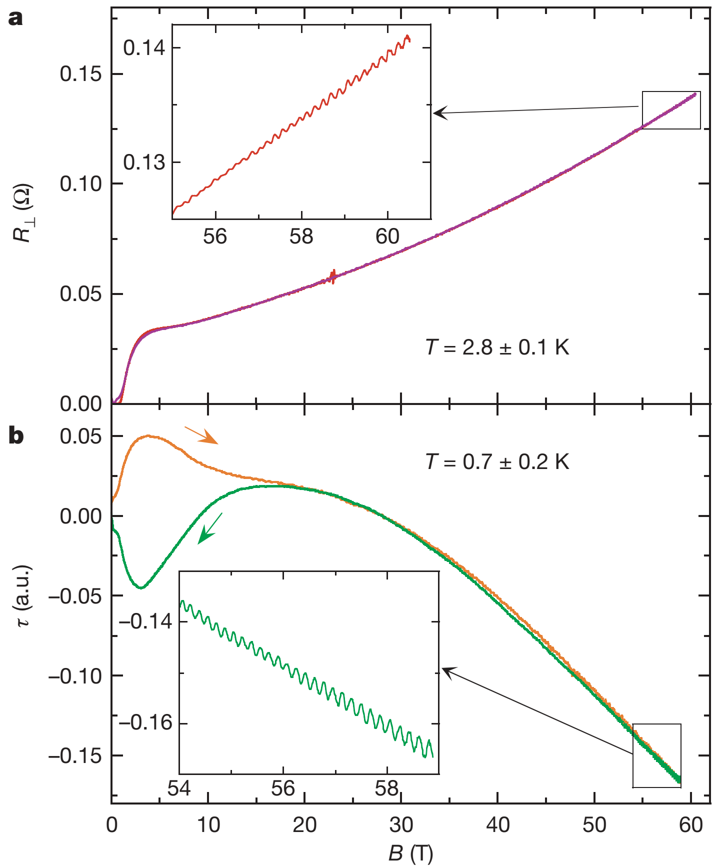


Nature Communications 5:3280 (2014)

Shubnikov-de Haas effect = Quantum oscillations with resistivity

De Haas–van Alphen effect = Quantum oscillations with magnetic susceptibility

QUANTUM OSCILLATIONS:



Nature **455**, 952 (2008)

$Tl_2Ba_2CuO_{6+x}$

Shubnikov-de Haas effect =

Quantum oscillations with resistivity

De Haas–van Alphen effect =

Quantum oscillations with magnetic susceptibility

QUANTUM OSCILLATIONS: Gold

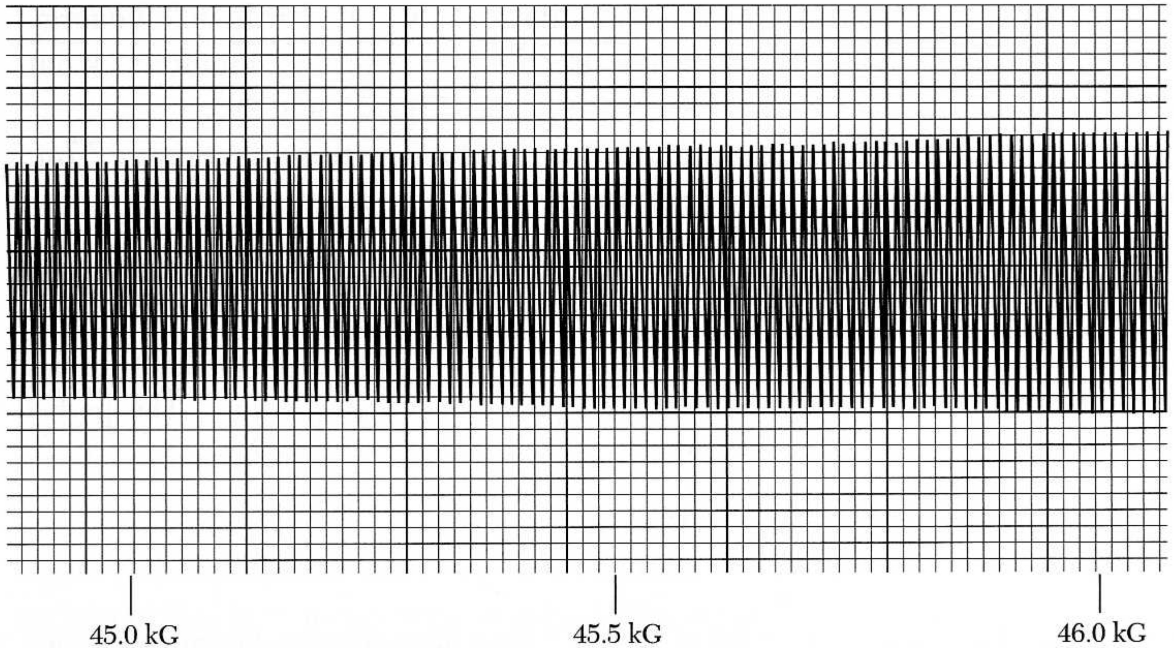


Figure 31 De Haas-van Alphen effect in gold with **B** || [110]. The oscillation is from the dog's bone orbit of Fig. 30. The signal is related to the second derivative of the magnetic moment with respect to field. The results were obtained by a field modulation technique in a high-homogeneity superconducting solenoid at about 1.2 K. (Courtesy of I. M. Templeton.)

From Kittel.

Outline:

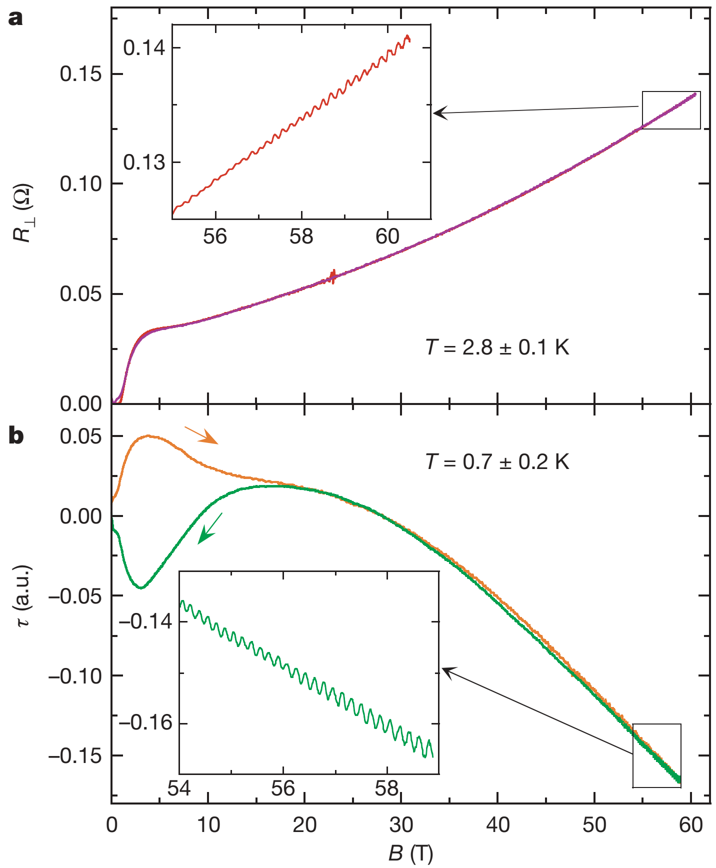
(1) Landau quantization / Landau levels:

Gives an understanding of why quantum oscillations exist.

(2) Onsager relation:

Gives a quantitative relation between the oscillations
and the Fermi surface area

QUANTUM OSCILLATIONS:



Nature **455**, 952 (2008)

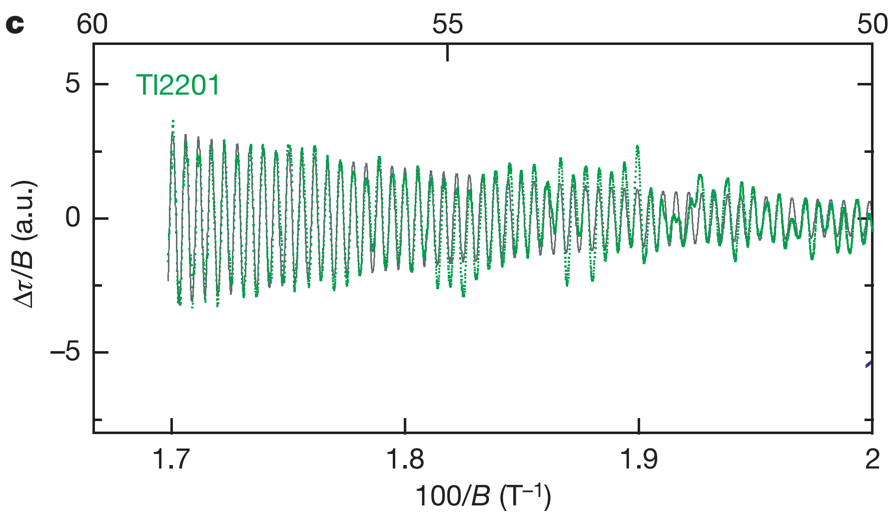
$Tl_2Ba_2CuO_{6+x}$

Shubnikov-de Haas effect =

Quantum oscillations with resistivity

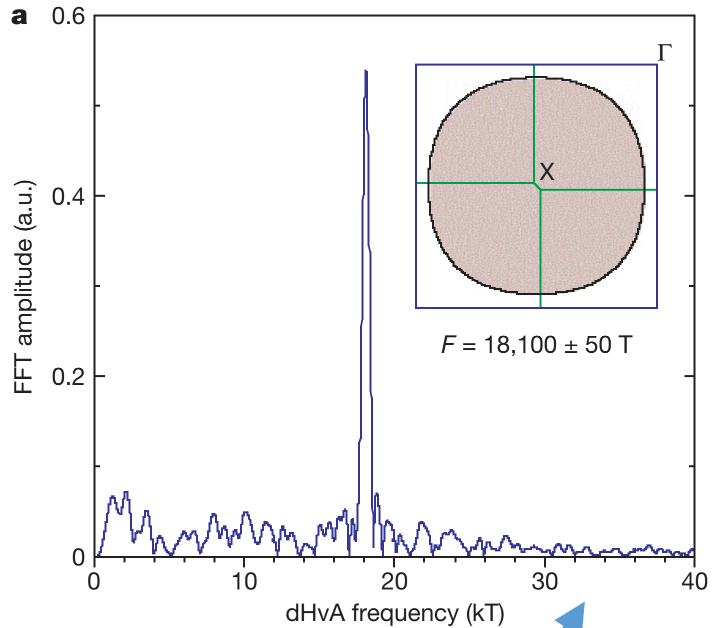
De Haas–van Alphen effect =

Quantum oscillations with magnetic susceptibility

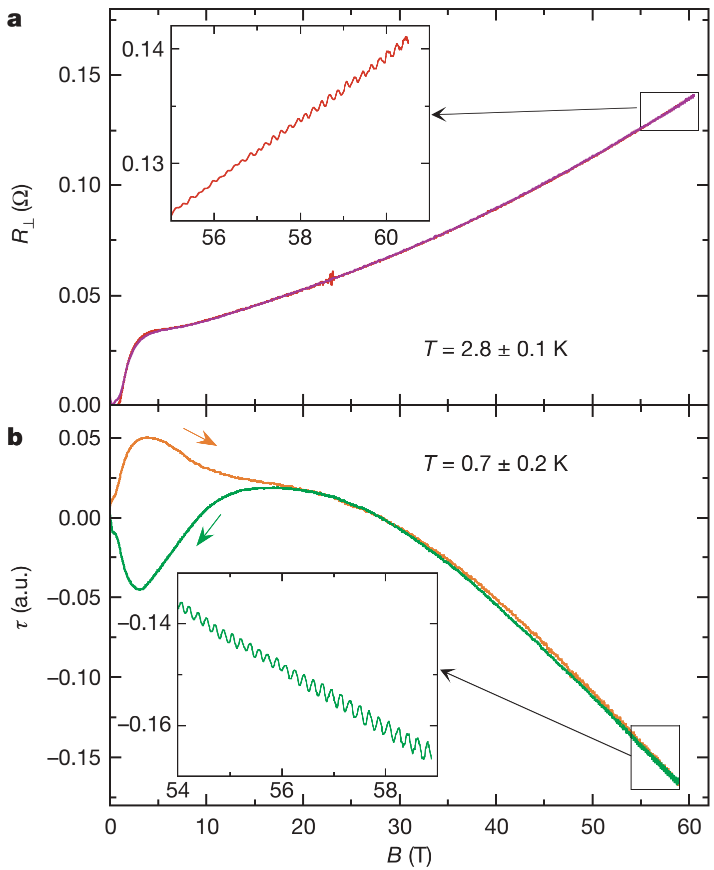


QUANTUM OSCILLATIONS:

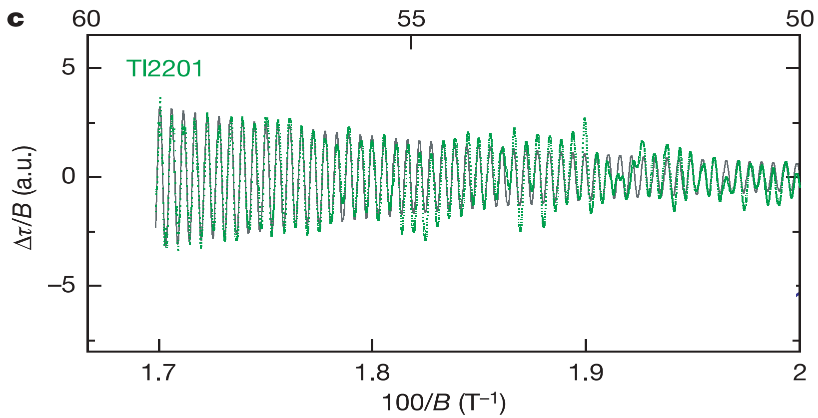
(c) Fourier Transform



(a) RAW DATA



(b) OSCILLATIONS VERSUS 1/B

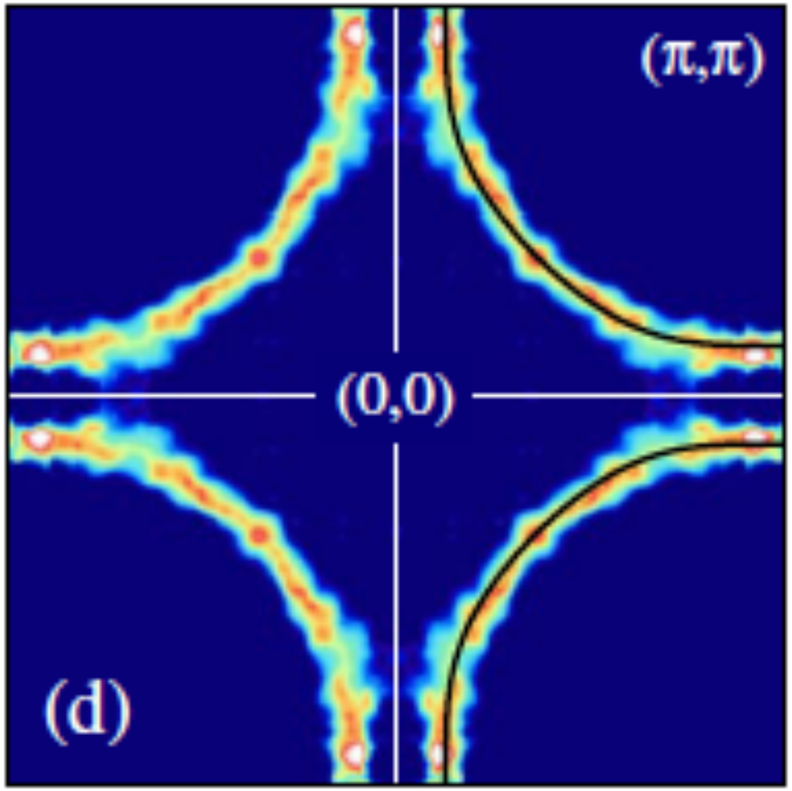


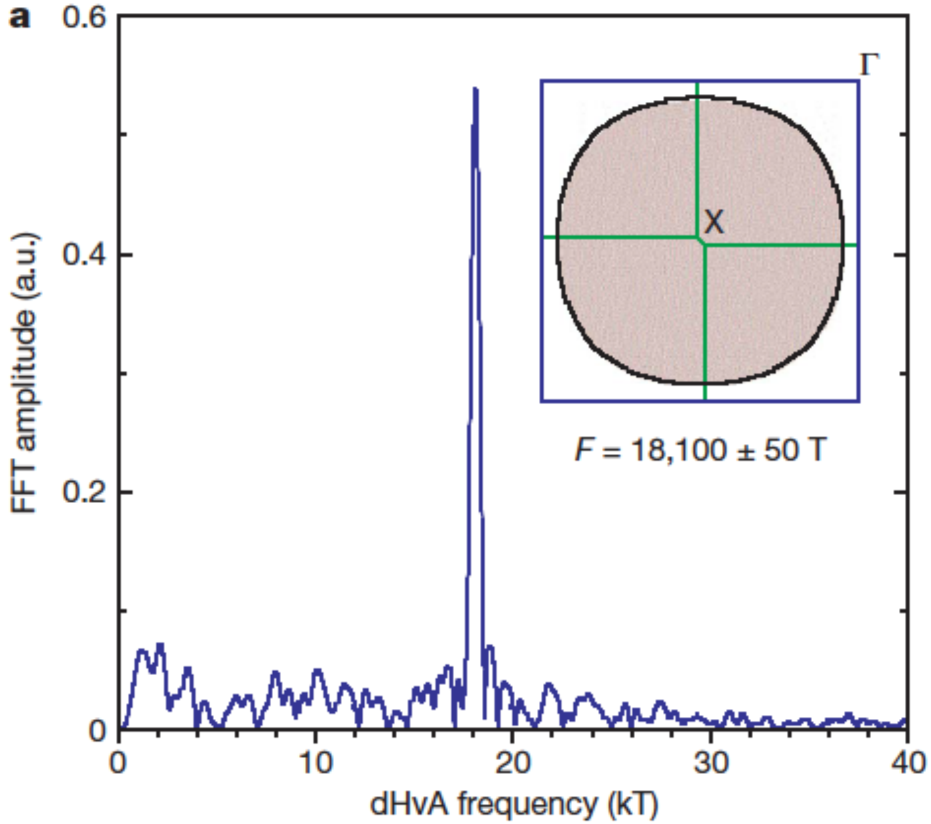
Nature **455**, 952 (2008)

$Tl_2Ba_2CuO_{6+x}$

Fermi surface:

ARPES vs Quantum Oscillations

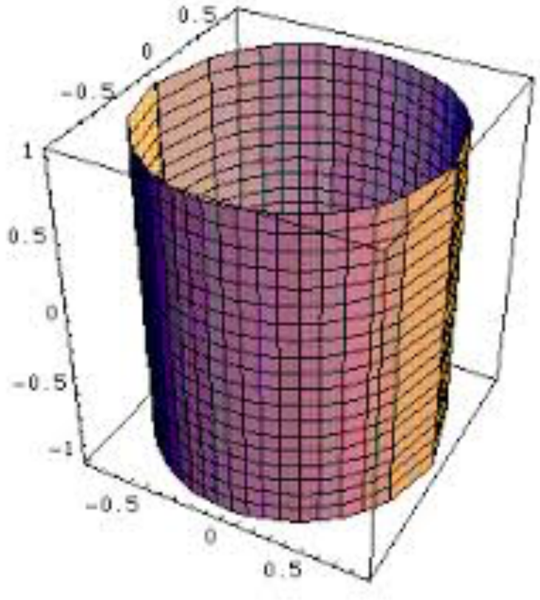




PRL **95**, 077001 (2005)
Data taken @ Swiss Light Source

Nature 455, 952 (2008)
$Tl_2Ba_2CuO_{6+y}$ (Tl2201)

2D and 3D Fermi surfaces



Potassium Lithium Copper

QUANTUM OSCILLATIONS: Gold

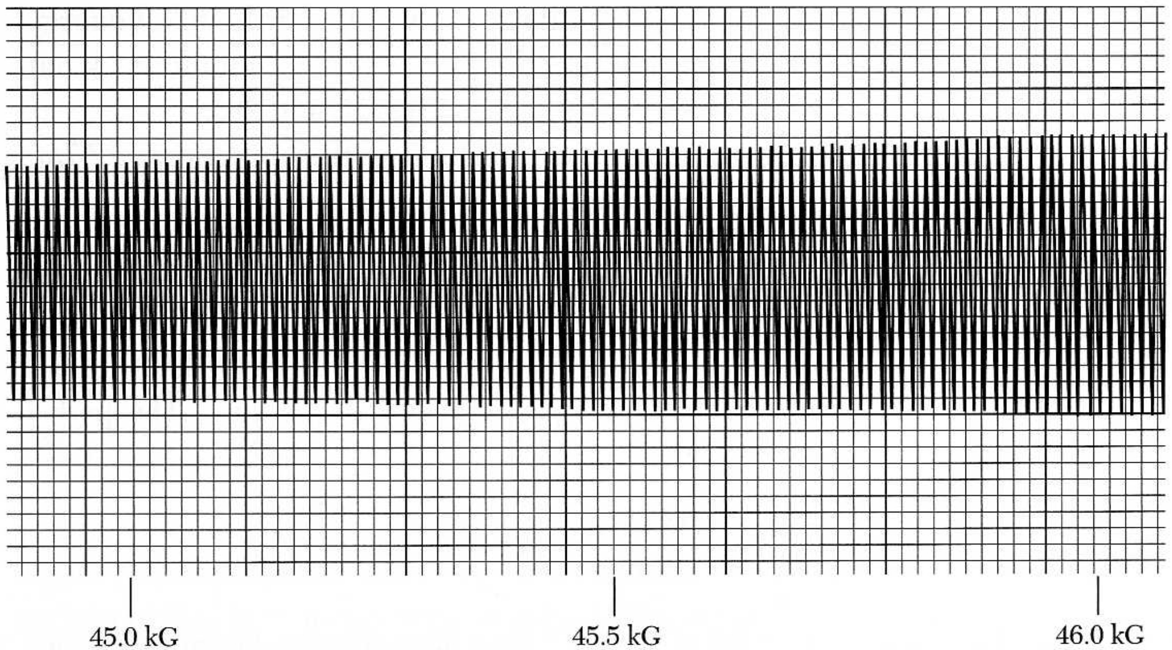


Figure 31 De Haas-van Alphen effect in gold with **B** ‖ [110]. The oscillation is from the dog's bone orbit of Fig. 30. The signal is related to the second derivative of the magnetic moment with respect to field. The results were obtained by a field modulation technique in a high-homogeneity superconducting solenoid at about 1.2 K. (Courtesy of I. M. Templeton.)

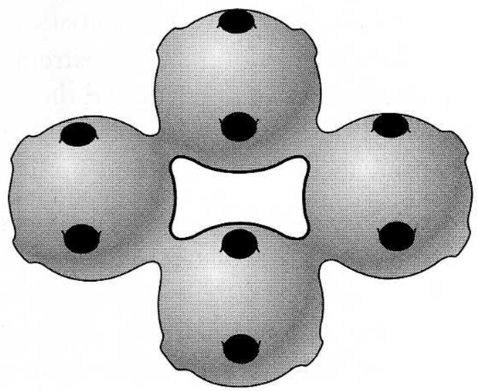
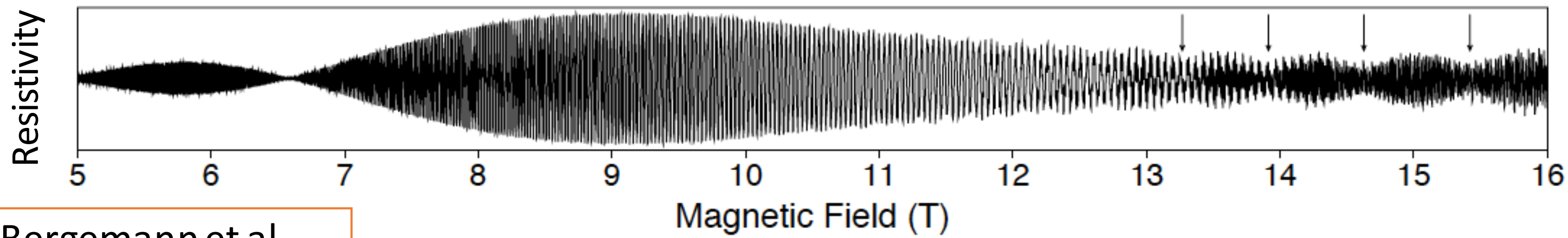
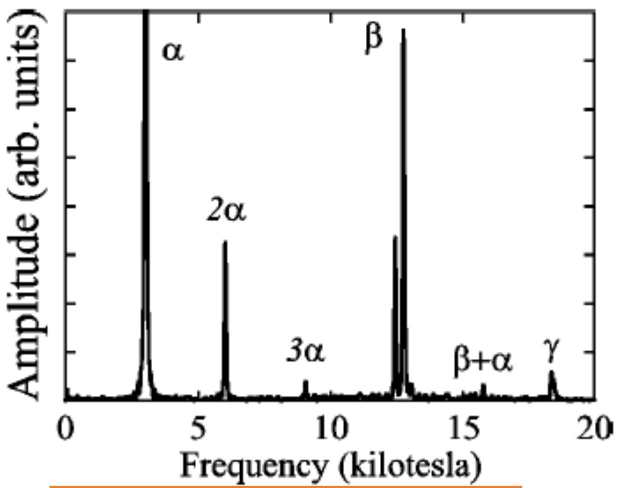


Figure 30 Dog's bone orbit of an electron on the Fermi surface of copper or gold in a magnetic field. This orbit is classified as holelike because the energy increases toward the interior of the orbit.

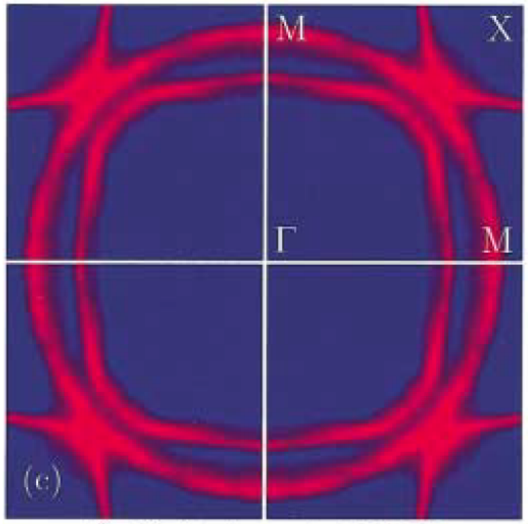
Multi – band metals



Bergemann et al,
PRL 84, 2662 (2000)

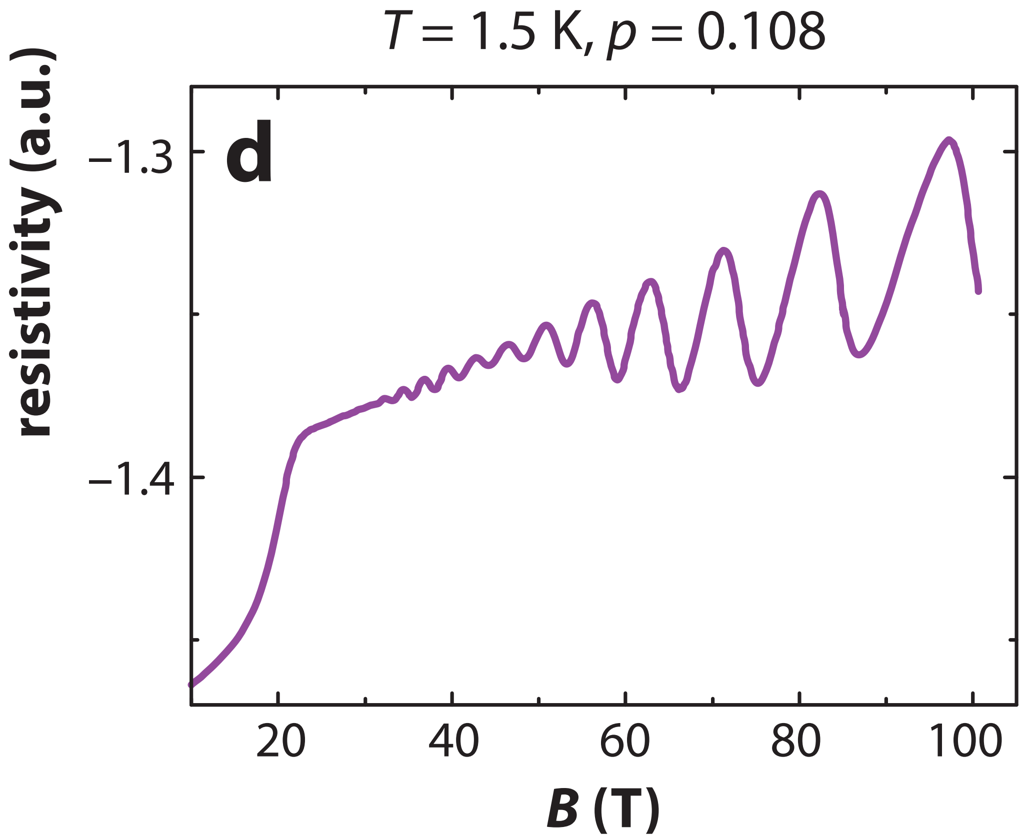


A.P. Mackenzie et al,
JPSJ 67, 385 (2003)



A. Damascelli et al,
PRL 85, 5194 (2000)

QUANTUM OSCILLATIONS:



OSCILLATION AMPLITUDE

$$\propto \; e^{\left(\frac{-\pi \hbar k_F}{eB\ell}\right)}$$

Where ℓ = mean free path

Resistivity measurement of a high-temperature superconductor: $YBa_2Cu_3O_{6.51}$ (YBCO)

http://www.annualreviews.org/doi/pdf/10.1146/annurev-conmatphys-030212-184305

QUANTUM OSCILLATIONS:

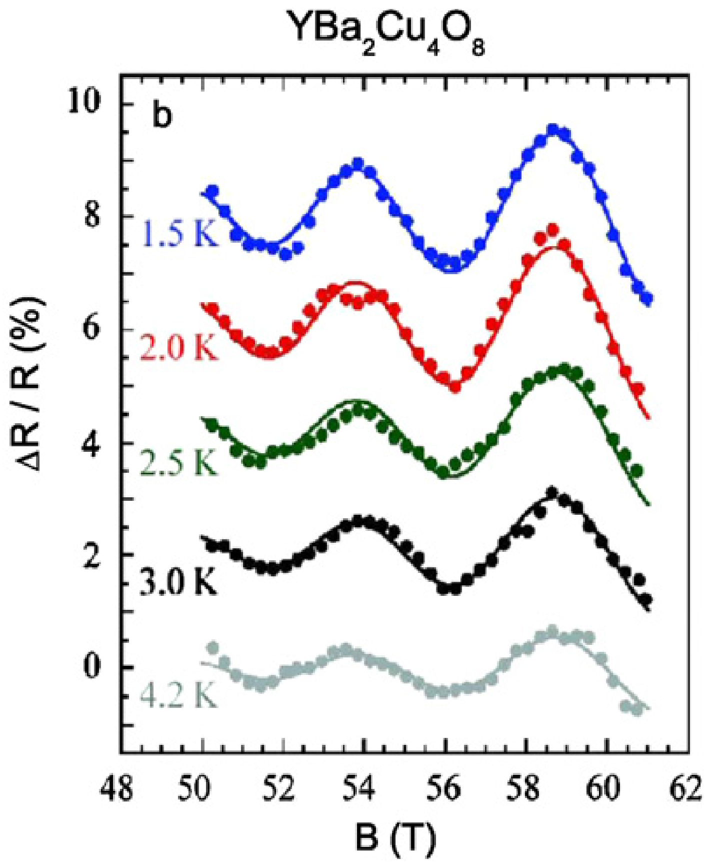
Temperature dependence

Thermal Condition:

$$\hbar\omega_c > k_B T$$

Landau level splitting > thermal energy

$$\omega_c = \frac{eB}{m}$$



Temperature dependence of the oscillatory amplitude yield information about the electronic mass.