

# The spin structure of the proton beyond NLO

Ignacio Borsa

In collaboration with Daniel de Florian, Rodolfo Sassot, Marco Stratmann and Werner Vogelsang  
Based on [PhysRevLett.133.151901](#)

Theoretical Particle Physics Seminar  
Zürich- October 15th

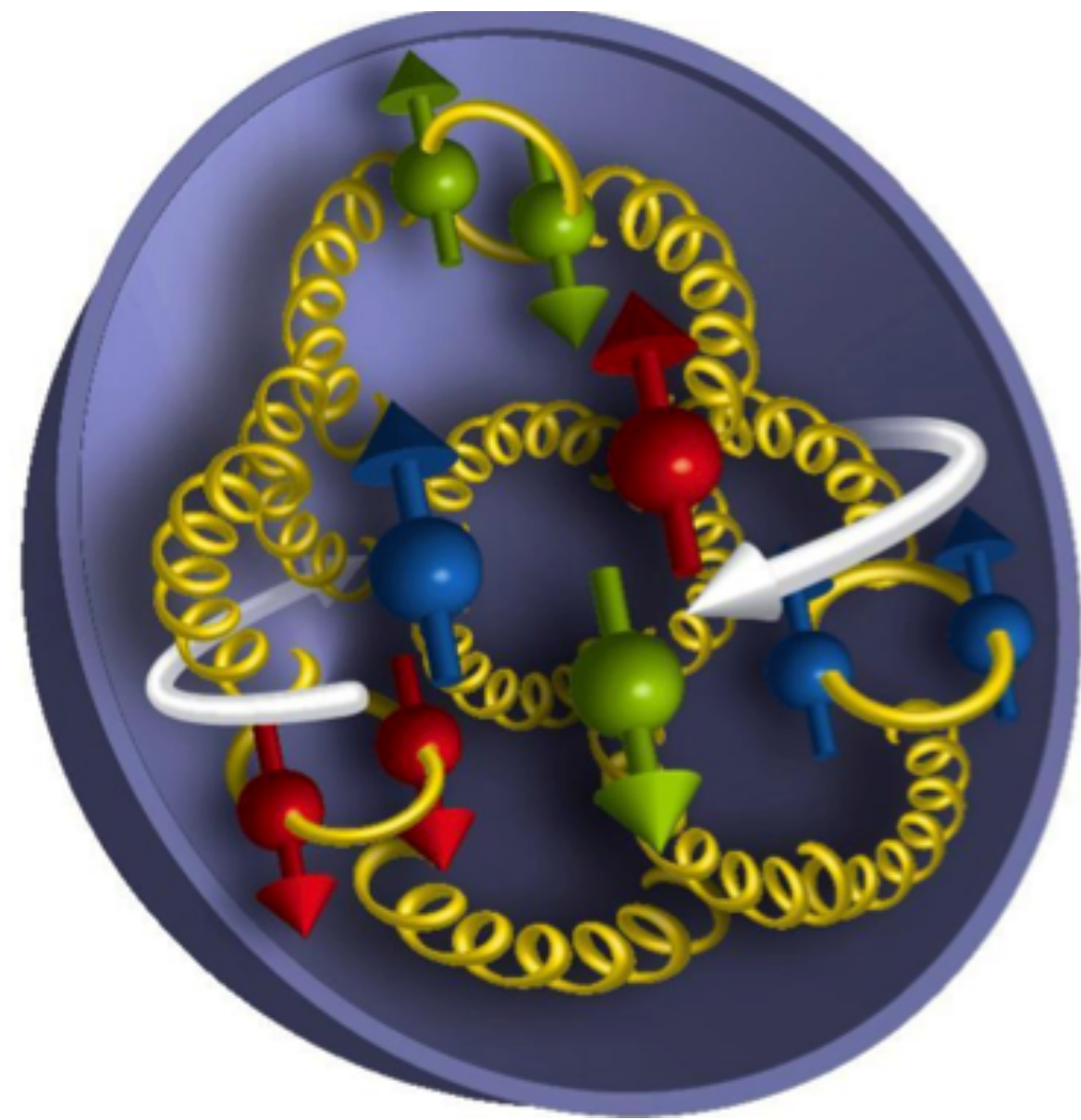
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# Introduction

# Motivation - The proton's spin structure

How is the proton's spin distributed among its constituents?



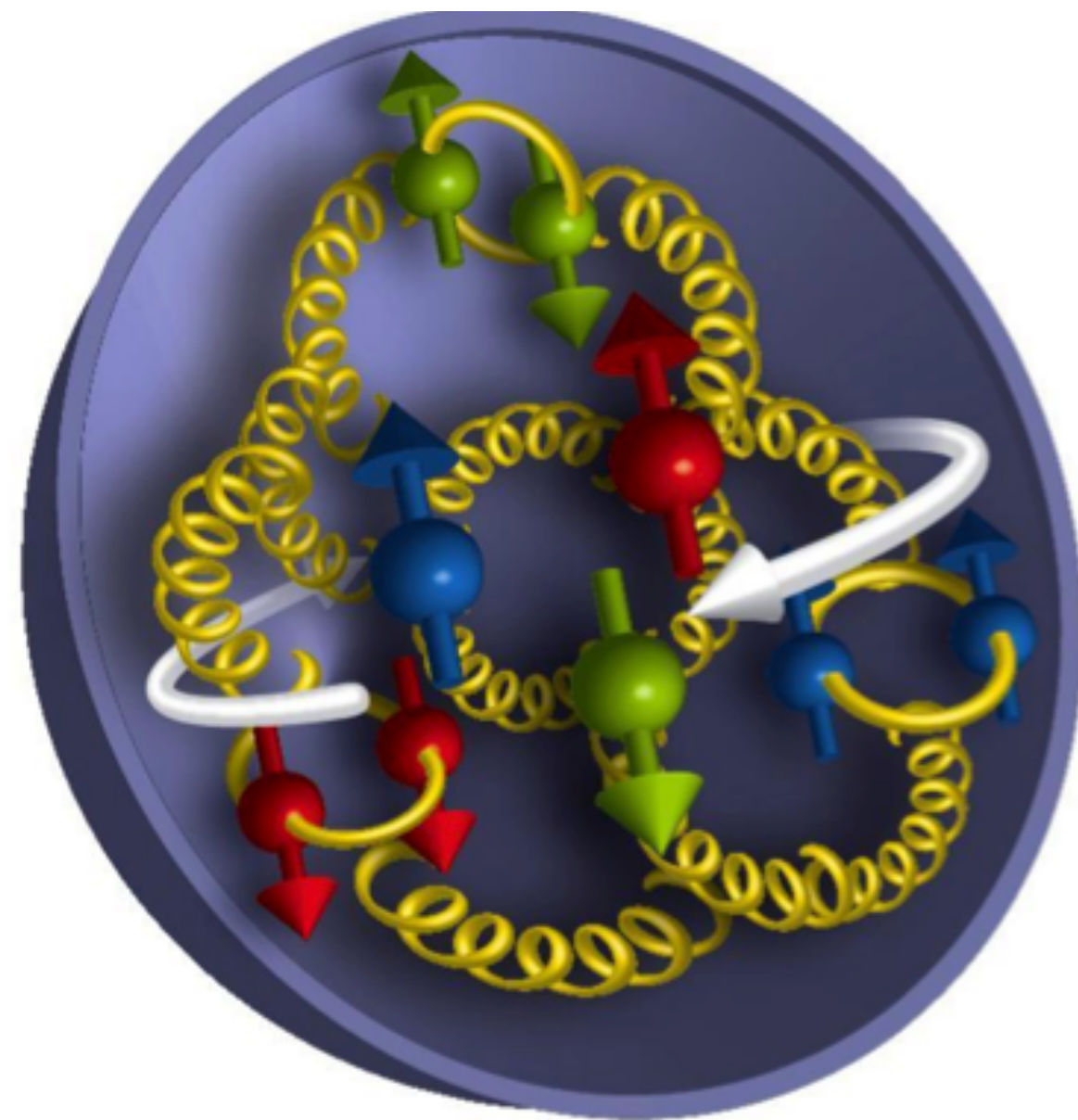
- ▶ Surprisingly low amount of spin carried by intrinsic quarks,  $\Delta\Sigma \sim 0.25 \ll 1$  [European Muon Collaboration (1989)] → “Proton spin crisis”.
- ▶ Significant progress both from experiment and theory [for a review: Aidala, Bass, Hasch, Mallot (2013)]. First evidence of positive polarization of gluons from polarized proton-proton collisions at RHIC [de Florian, Sassot, Stratmann, Vogelsang (2014); Nocera, Ball Forte, Ridolfi, Rojo (2014)].
- ▶ Still, rather incomplete picture of the spin structure in terms of the contribution from gluons or flavor decomposition.

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta g + L_q + L_g$$

Jaffe, Manohar (1990)

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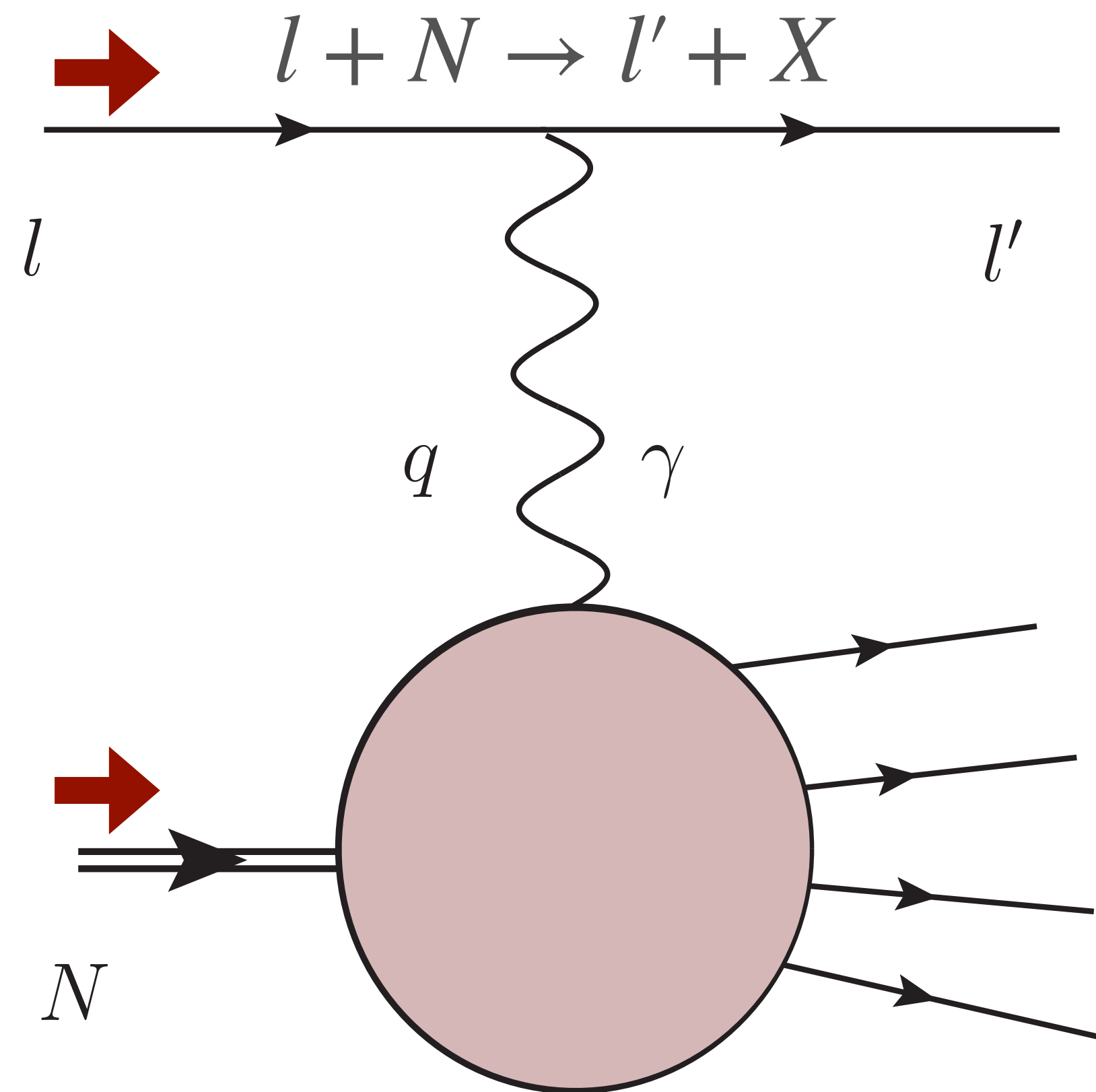
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Quark's spin    Gluon's spin    OAM

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# Introduction - The proton's spin structure

How is the proton's spin distributed among its constituents?



$$\Delta\sigma \equiv \frac{1}{2} [\sigma^{\uparrow\uparrow} - \sigma^{\uparrow\downarrow}]$$

$$\Delta\sigma = \sum_a \int dz \Delta f_a(z, \mu_F^2) \Delta\hat{\sigma}_i(\alpha_S(\mu_R), \mu_F^2, \mu_R^2)$$

Polarized PDFs
Polarized Partonic cross-section

$$\Delta f_a \equiv f_a^{\uparrow} - f_a^{\downarrow}$$

$$\Delta\hat{\sigma} \equiv \frac{1}{2} [\hat{\sigma}^{\uparrow\uparrow} - \hat{\sigma}^{\uparrow\downarrow}]$$

Contribution of parton  $a$  to the proton's spin

$$\Delta f_a(\mu_F^2) = \int_0^1 \Delta f_a(x, \mu_F^2) dx$$

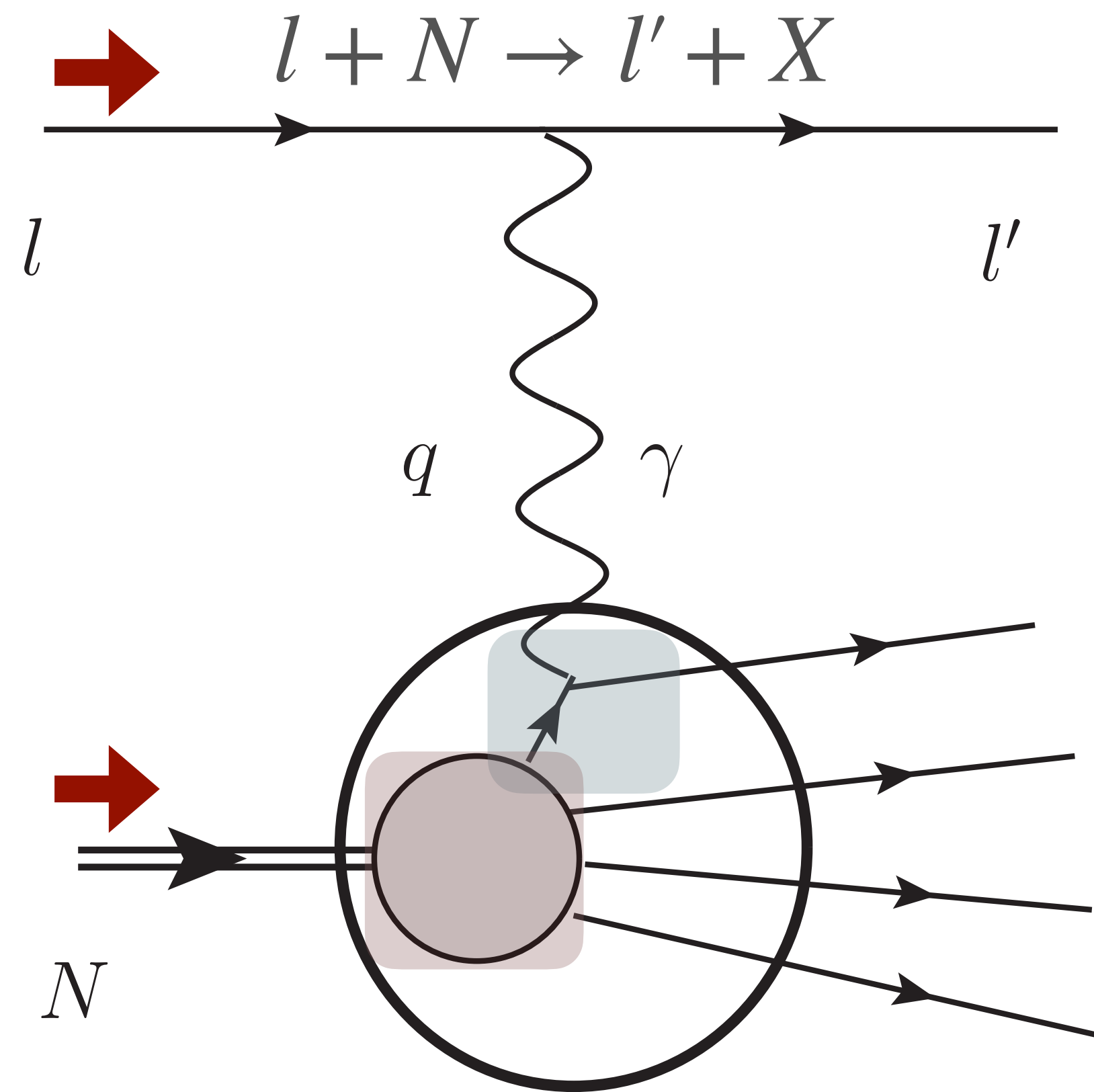
PDFs' scale dependence can be calculated perturbatively in QCD

$$\frac{\partial}{\partial \ln \mu^2} \Delta f_i(z, \mu^2) = \sum_j \int_z^1 \frac{dy}{y} \Delta P_{ji}(y, \alpha_S(\mu^2)) \Delta f_j\left(\frac{z}{y}, \mu^2\right)$$

They can be determined at some input scale from a set of experimental measurements  $\rightarrow$  QCD global analyses

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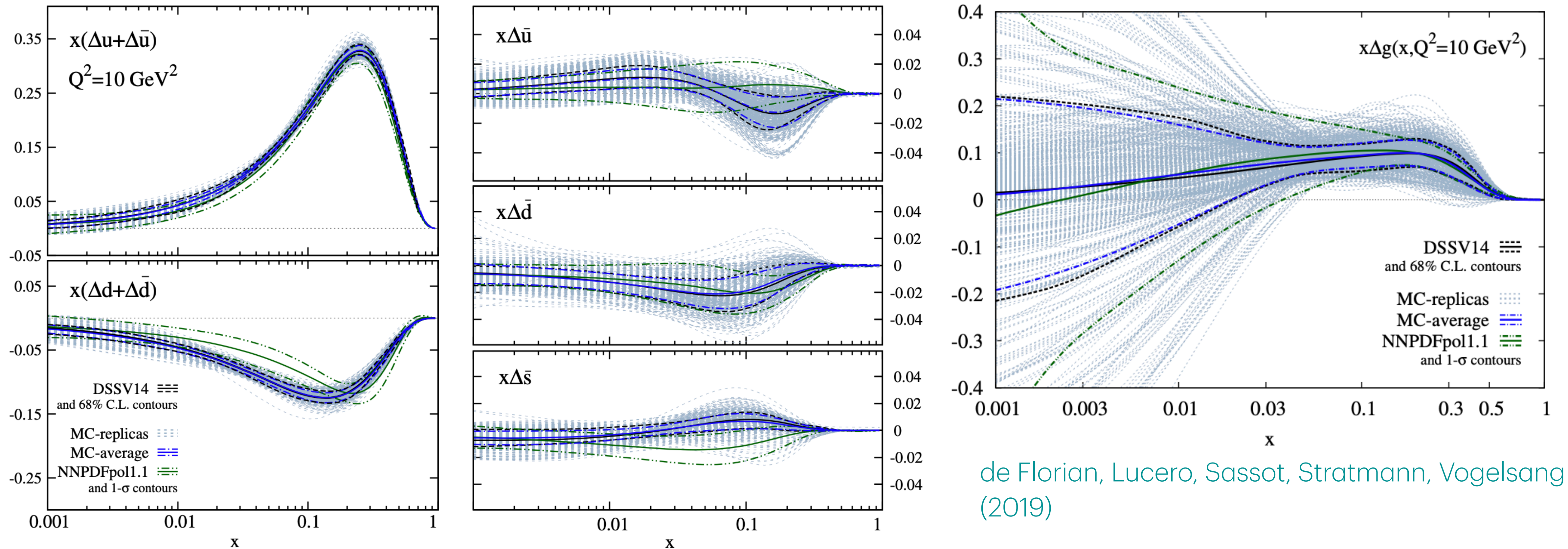
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# Introduction - The proton's spin structure

## How well do we know polarized PDFs?



▶ Well constrained singlet combination

▶ Still incomplete picture in terms of flavor separation and contribution from gluons

# Introduction - The proton's spin structure

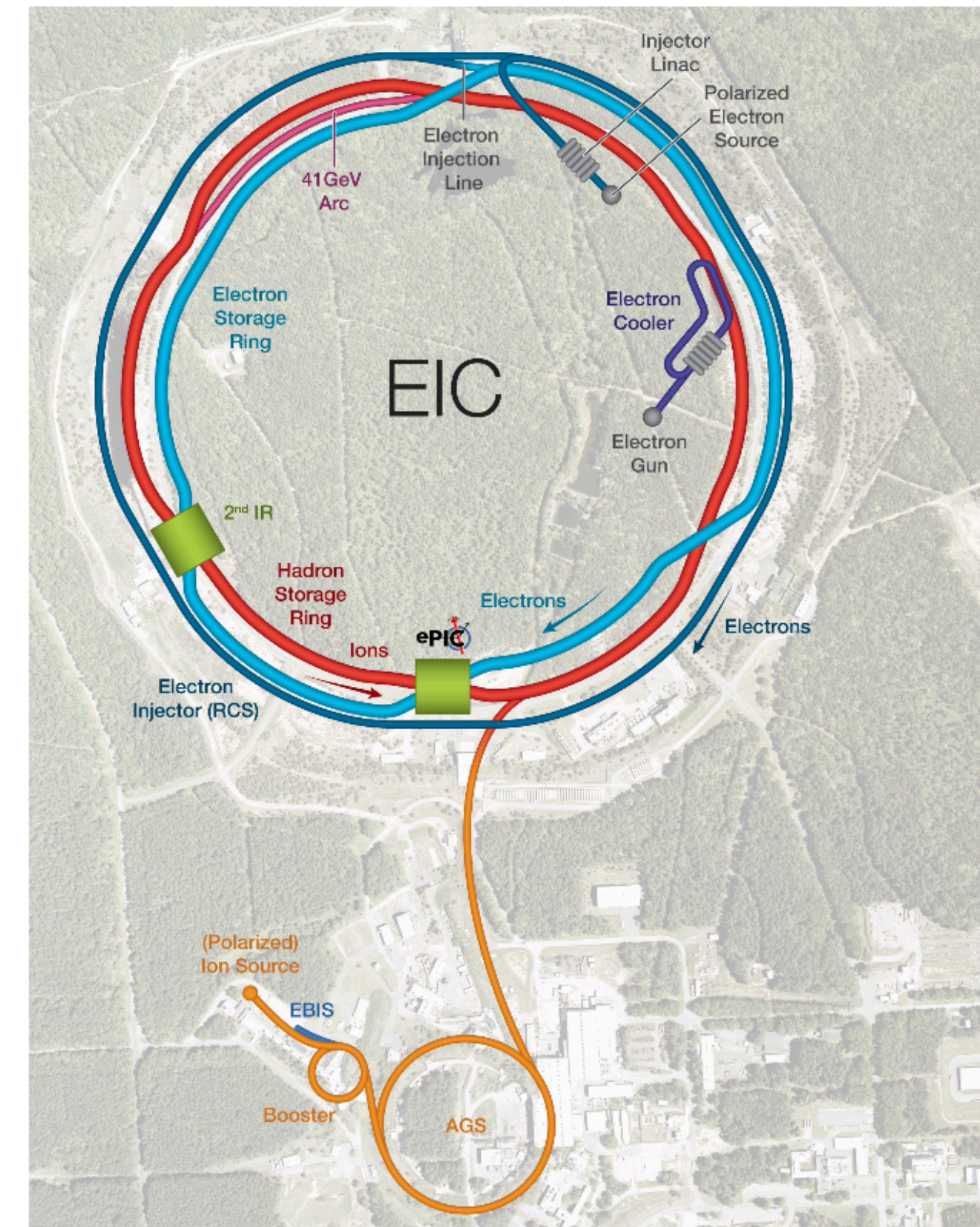
## Why NNLO? - Precision physics at the Electron-Ion Collider

BNL-based EIC on its path towards construction

- ▶ High Luminosity:  $\mathcal{L} = 10^{33} - 10^{34} \text{ cm}^{-2} \text{ sec}^{-1}$
- ▶ Center-of-mass energy range: 20 – 140 GeV
- ▶ Highly polarized electron & light hadron beams

Unique access to the proton's spin structure in terms of helicity parton distributions!

Physical interpretation of EIC data will require an increased precision of theory predictions



EIC Yellow Report. Nucl.Phys.A 1026 (2022)  
Aschenauer, IB, Lucero, Nunes, Sassot (2020)

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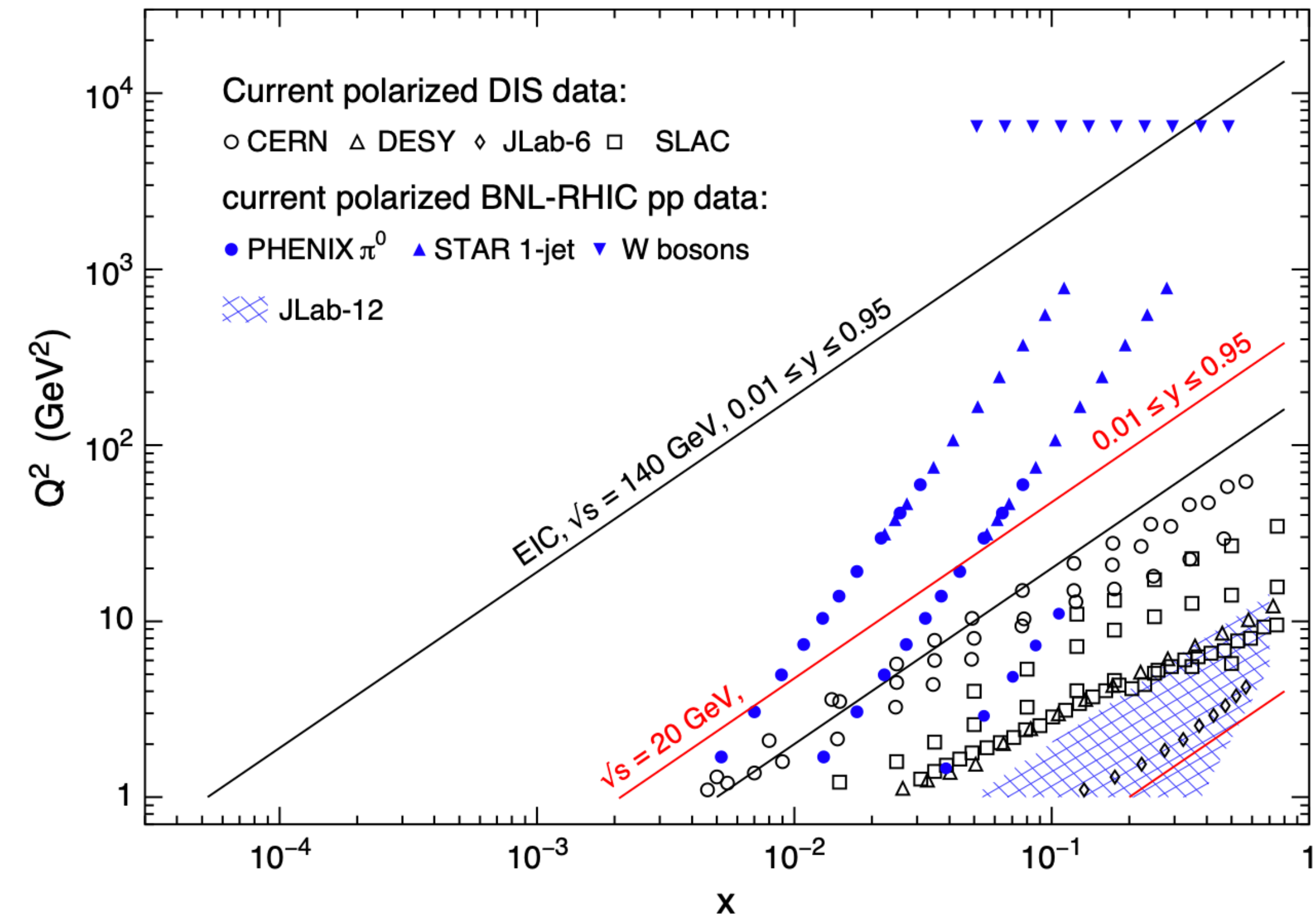
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Calculations for polarized eP observables beyond NLO:

- NNLO structure functions  $g_1$  (photon exchange)  
van Neerven, Zijlstra (1994)
- NNLO NC & CC structure functions  $g_1, g_4, g_5$   
IB, de Florian, Pedron (2022)
- Approx. NNLO and N3LO Semi-Inclusive DIS  
Abele, de Florian, Vogelsang (2022)
- NNLO Single-Jet production  
NC and CC- IB, de Florian, Pedron (2023)
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Bonino, Gehrmann, Löchner, Schönwald, Stagnitto (2024)  
Goyal, Moch, Pathak, Rana, Ravindran (2024)
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Blümlein, Marquard, Schneider, Schönwald (2023)

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This talk

### Calculations for polarized eP observables beyond NLO:

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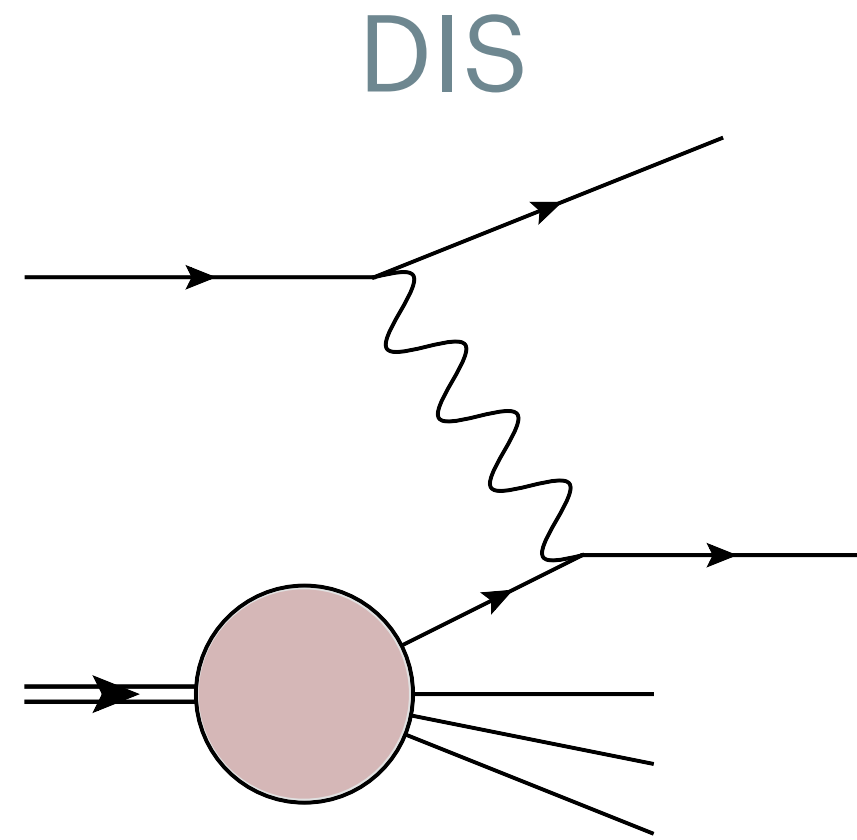
### Parton distribution functions:

- NNLO polarized PDFs  
Taghavi-Shahri, Khanpour, Atashbar Tehrani, Alizadeh Yazdi (2016)  
Bertone, Chiefa, Nocera (2024)  
IB, de Florian, Sassot, Stratmann, Vogelsang (2024)

# Global analysis of pPDFS

# Global analysis of helicity PDFs

## Accessing pPDFs in polarized high-energy scattering processes



► Mainly constrains

$$\Delta\Sigma \sim (\Delta q + \Delta\bar{q})$$

► Only indirect constraints on

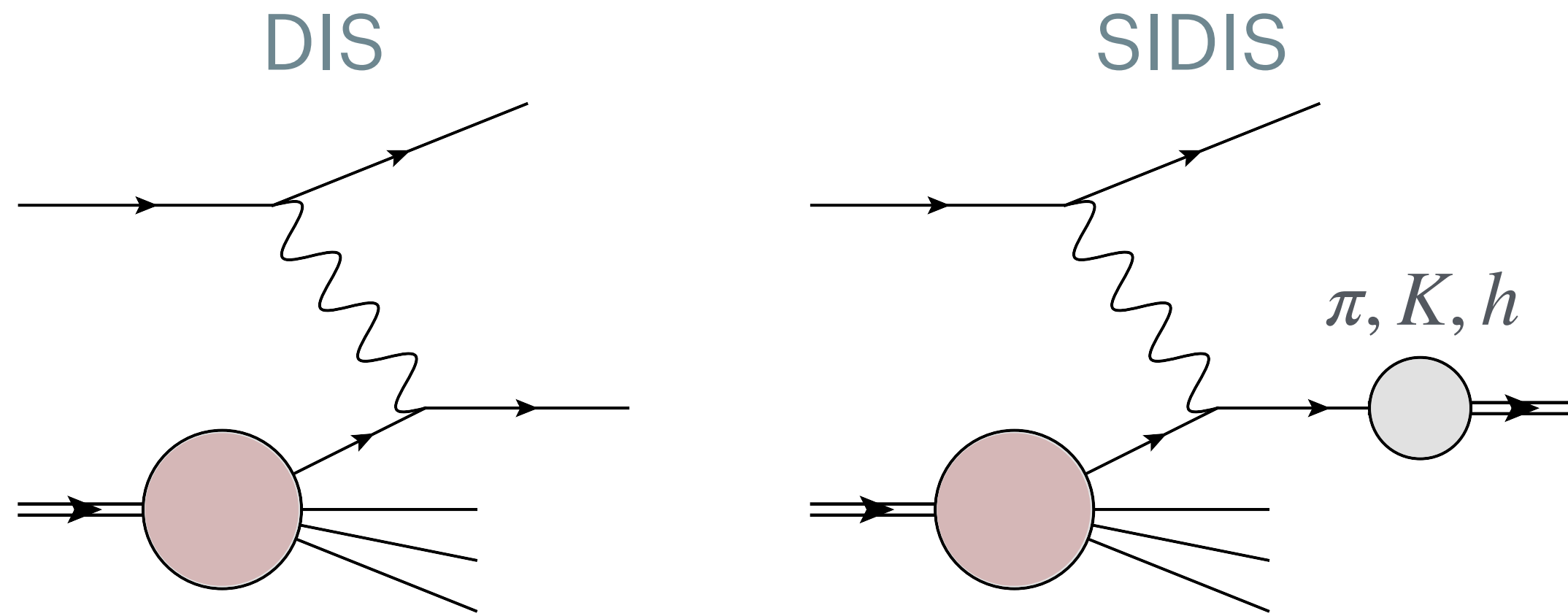
$$\Delta g, \Delta s$$

First NLO QCD analyses based on DIS data:

[Gehrmann, Stirling; Glück, Reya, Stratmann,  
Vogelsang; Blümlein, Böttcher; Leader,  
Sidorov, Stamenov; Hirai, Kumano, Saito;  
Bourely, Soffer, Bucella]

# Global analysis of helicity PDFs

## Accessing pPDFs in polarized high-energy scattering processes



► Mainly constrains

$$\Delta\Sigma \sim (\Delta q + \Delta\bar{q})$$

► Only indirect constraints on

$$\Delta g, \Delta s$$

► Improved flavor

discrimination

► Only indirect constraints

on  $\Delta g$

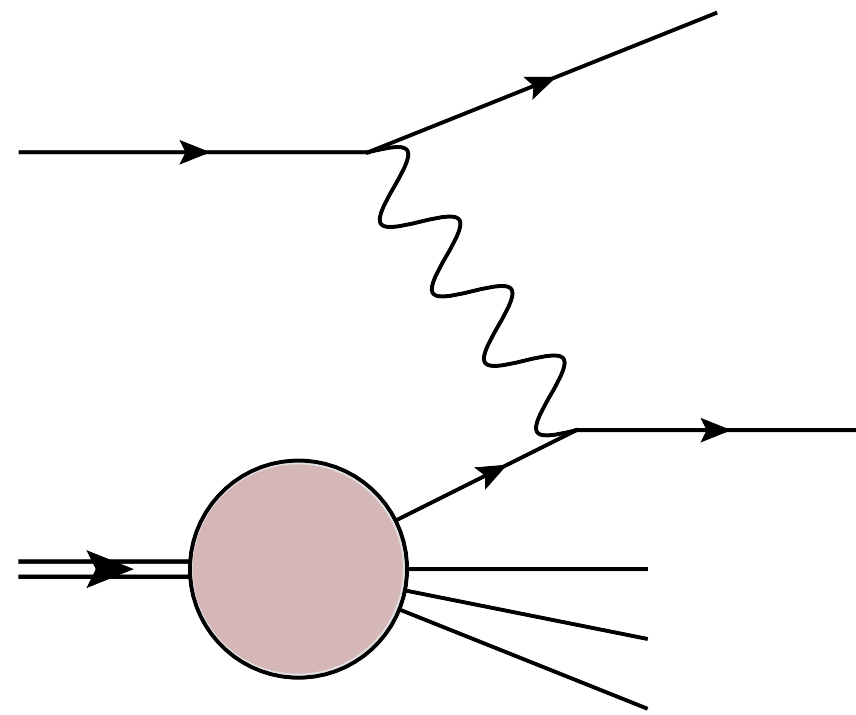
NLO QCD analyses based on combined  
DIS+SIDIS: data:

[de Florian, Sampayo, Sassot; de Florian,  
Navarro, Sassot]

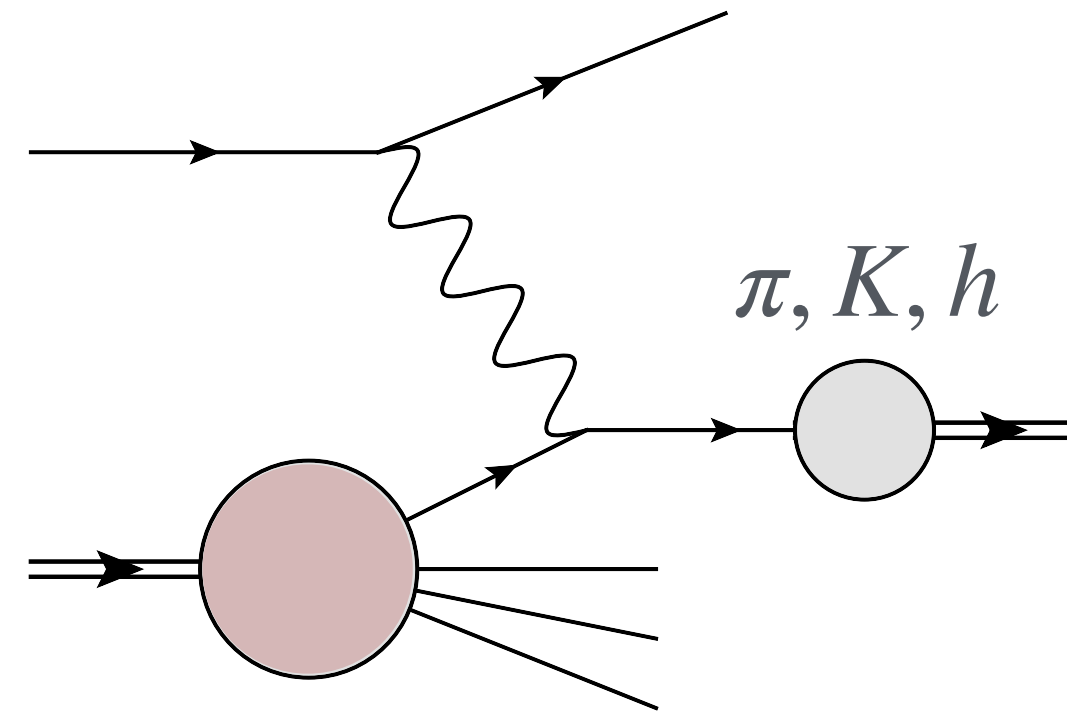
# Global analysis of helicity PDFs

## Accessing pPDFs in polarized high-energy scattering processes

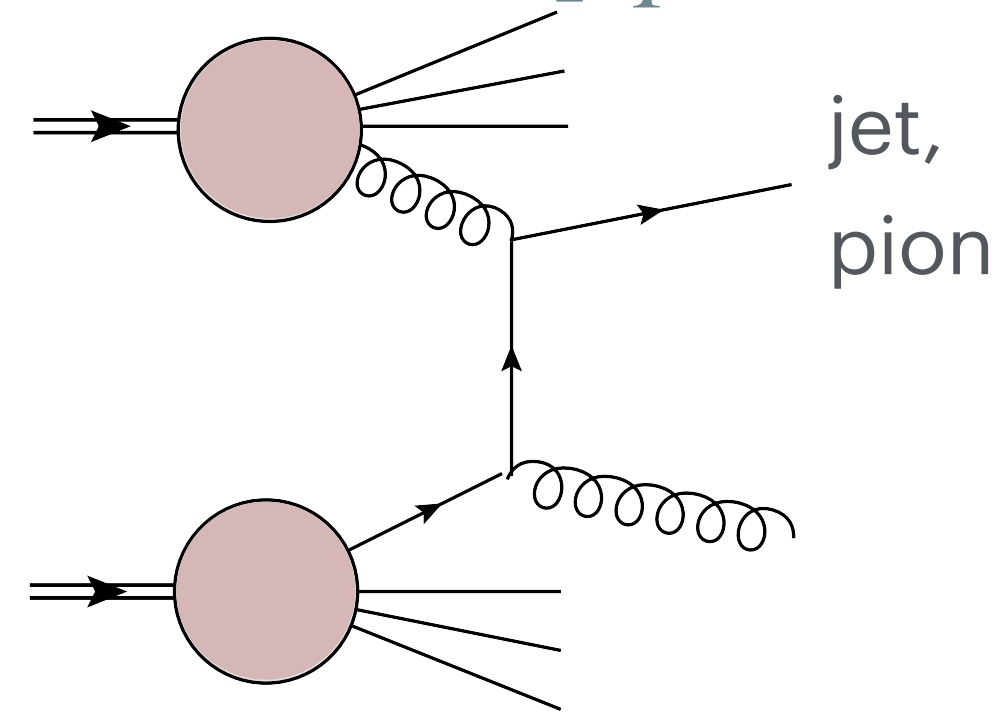
DIS



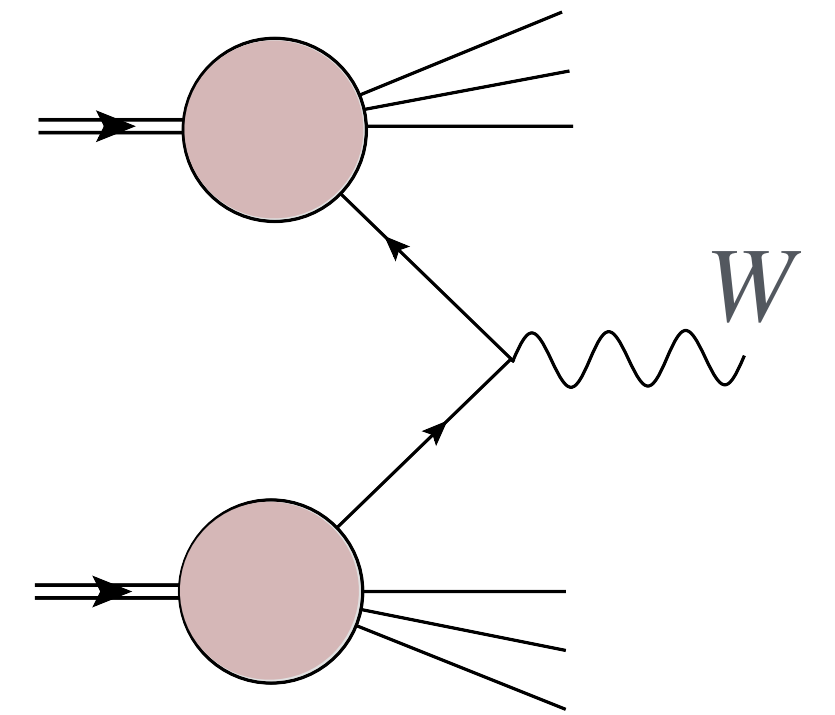
SIDIS



pp high- $p_T$



pp W boson



- ▶ Mainly constrains  $\Delta\Sigma \sim (\Delta q + \Delta\bar{q})$
- ▶ Only indirect constraints on  $\Delta g, \Delta s$

- ▶ Improved flavor discrimination
- ▶ Only indirect constraints on  $\Delta g$

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Fully global NLO analyses:

[de Florian, Sassot, Stratmann, Vogelsang (2008-);  
Nocera, Ball Forte, Ridolfi, Rojo (2013-); Cocuzza,  
Ethier, Melnitchouk, Sato (2013-)]  $\Rightarrow$  Mature analysis  
frameworks with robust estimation of uncertainties

# Global analysis of helicity PDFs

## Workflow

Parametrization for  $\Delta f(x, Q_0)$  at input scale  $Q_0$ , e.g:

$$\Delta q(z, Q_0) = N_q x^{\alpha_q} (1-x)^{\beta_q} (1 + \gamma_q x^{\delta_q})$$

Change parameters

Evolve to the relevant scale using DGLAP Equations

Evaluate cross section and calculate  $\chi^2$

$\chi^2$  minimum?

No

Yes

Optimum set of parameters

+ Prescription for uncertainties

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“Time-consuming” convolution integrals:

$$\frac{\partial}{\partial \ln \mu^2} \Delta f_i(x, \mu^2) = \Delta P_{ij} \otimes \Delta f_j(x, \mu^2)$$

Eg:

$$\Delta \sigma_{pp} = \sum_{a,b} \Delta f_a(x_a, \mu) \otimes \Delta f_b(x_b, \mu) \otimes \Delta \hat{\sigma}_{ab}$$

$$a(x) \otimes b(x) \equiv \int_0^1 dy \int_0^1 dz \delta(x - yz) a(y) b(z)$$

⇒ Switch to Mellin space

# Global analysis of helicity PDFs

## Mellin Technique

- ▶ Fast evaluation in Mellin N-space  $\Rightarrow$  Convolutions become simple products

$$\tilde{a}(N) = \int_0^1 dx x^{N-1} a(x) \quad [a \otimes b](N) = \tilde{a}(N) \cdot \tilde{b}(N)$$

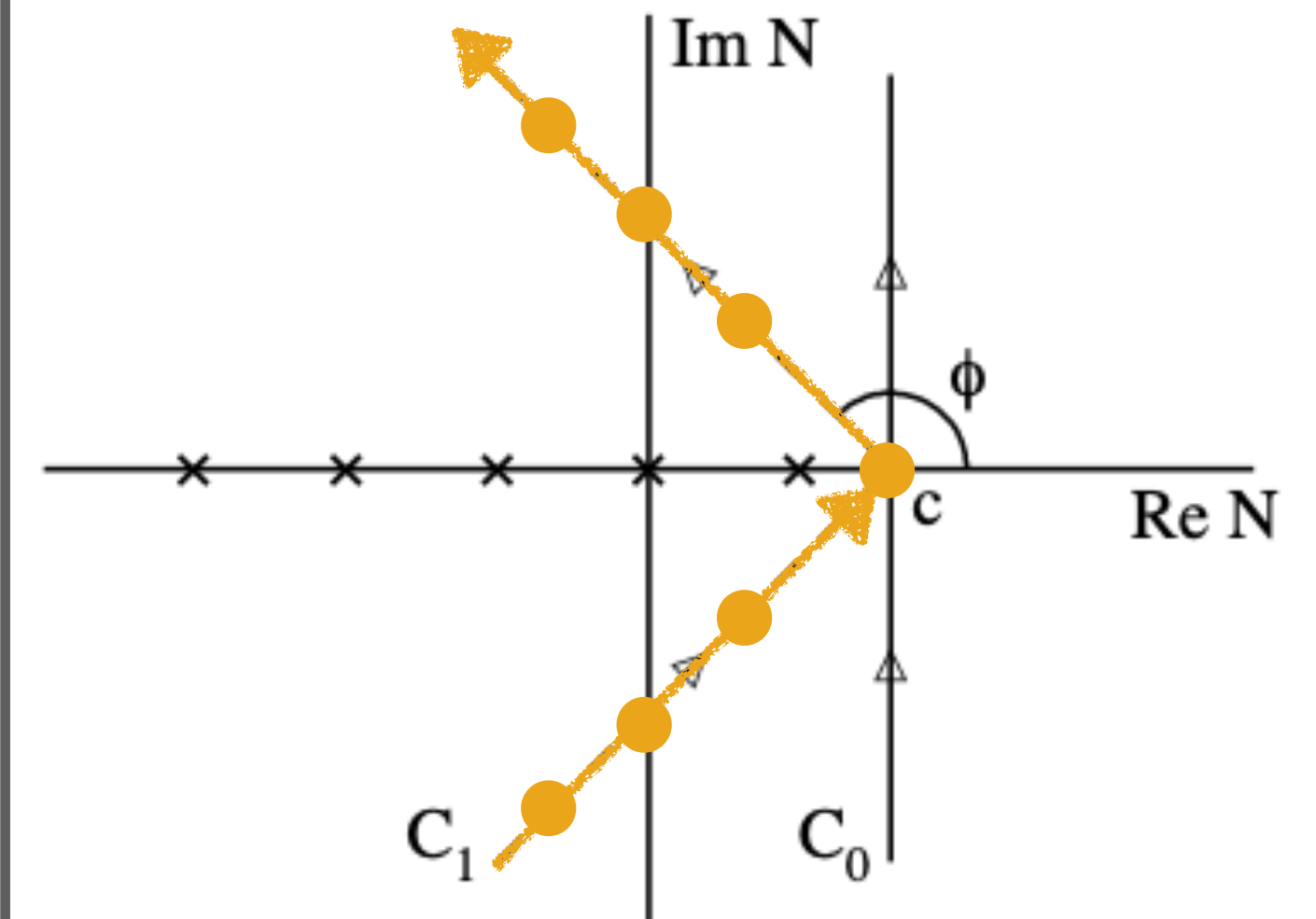
- ▶ Simple Gaussian integral for each convolution in the cross-section
- ▶ If analytical coefficients are known in Mellin space (e.g: DIS)  $\Rightarrow$  Direct application
- ▶ "Trick" if coefficients not known in N-space (e.g:  $pp \rightarrow \pi X$ )  $\Rightarrow$  write PDFs in

terms of their Mellin inverse  $\Delta f_i(x) = \frac{1}{2\pi i} \int_{\mathcal{C}_N} dN x^{-N} \Delta \tilde{f}_i(N)$

$$\frac{d\Delta\sigma}{dO} = \sum_{ab} \int_{\mathcal{C}_N} \int_{\mathcal{C}_M} \Delta \tilde{f}_a(N) \Delta \tilde{f}_b(M) \Delta \tilde{\sigma}_{ab}^h(N, M, O)$$

Mellin inversion
Fit
Precomputed grid

$$a(x) = \frac{1}{2\pi i} \int_{\mathcal{C}_N} dN x^{-N} \tilde{a}(N)$$



From NLO to NNLO

# Global analysis of helicity PDFs

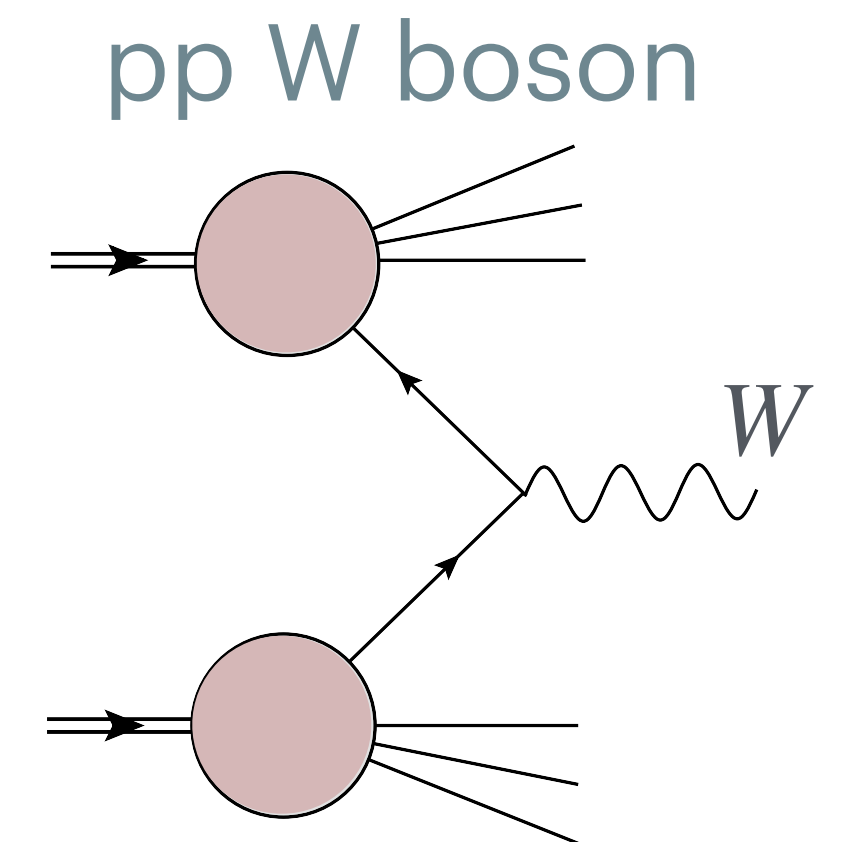
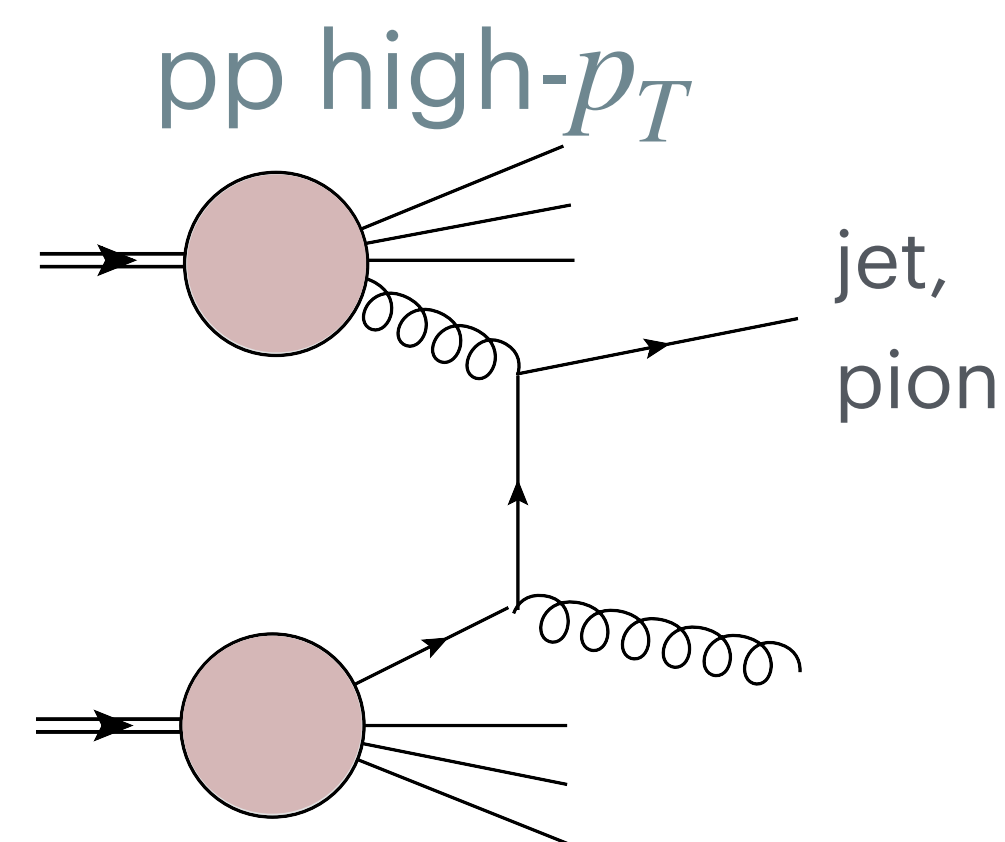
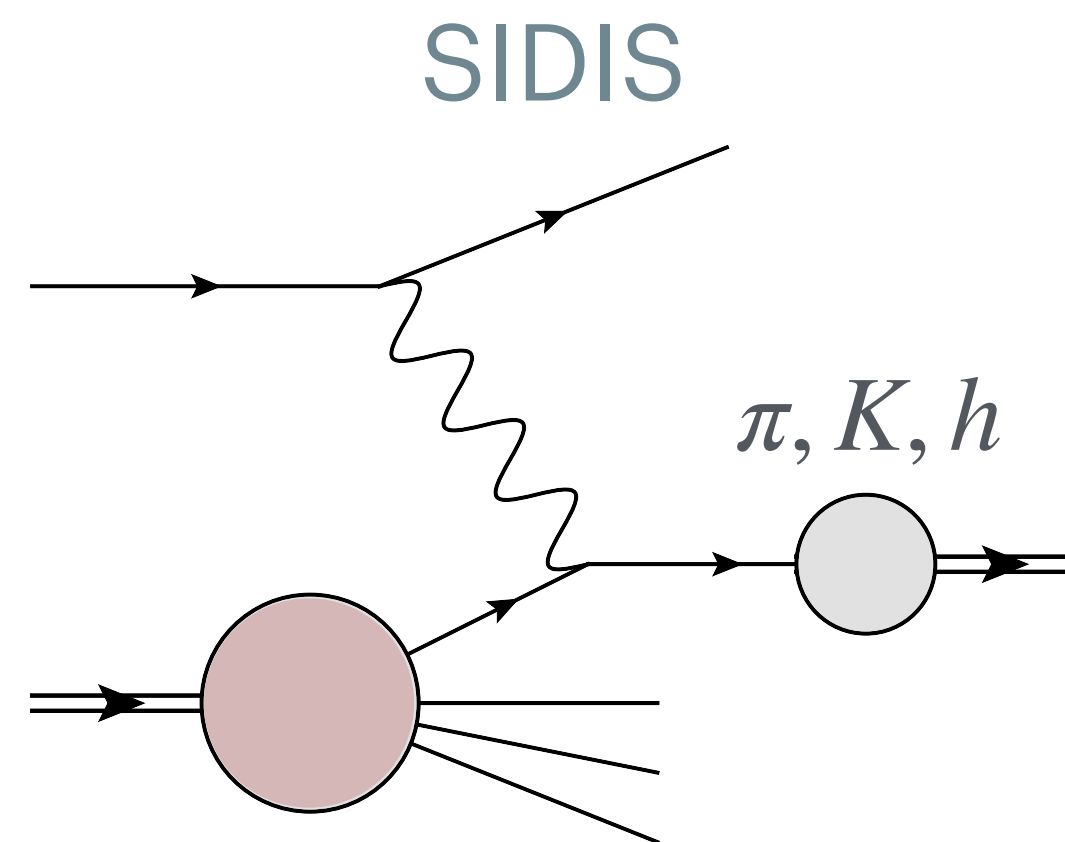
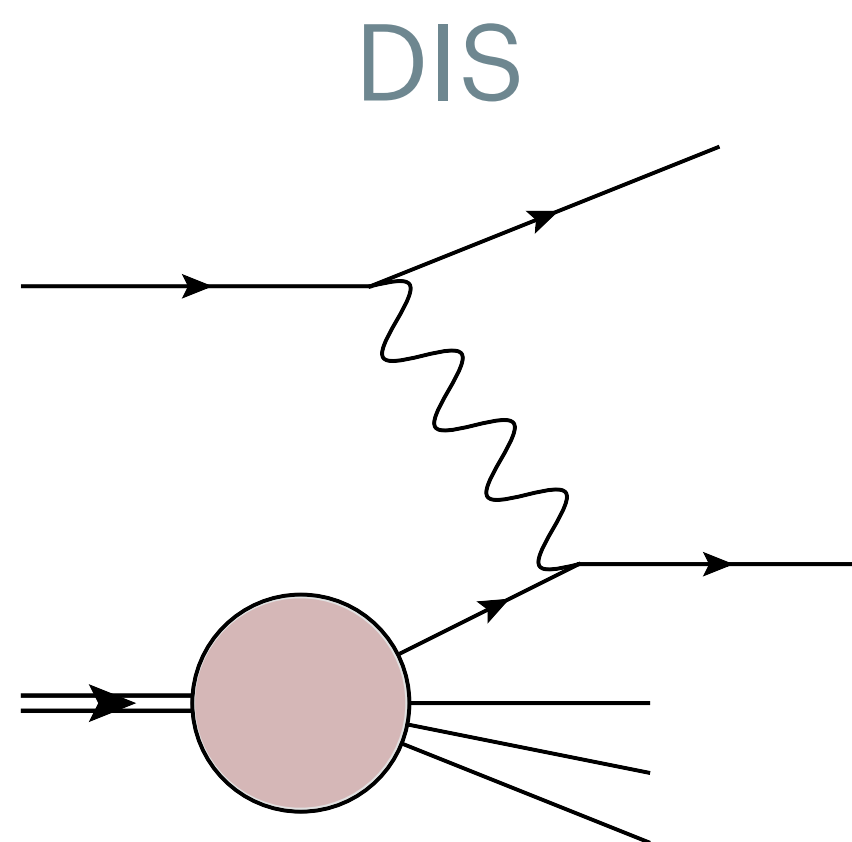
## Ingredients for NNLO

► PDF evolution kernels

$$\Delta P_{ij} = \frac{\alpha_s}{2\pi} \Delta P_{ij}^{\text{LO}} + \left( \frac{\alpha_s}{2\pi} \right)^2 \Delta P_{ij}^{\text{NLO}} + \left( \frac{\alpha_s}{2\pi} \right)^3 \Delta P_{ij}^{\text{NNLO}} + \dots$$

► Partonic hard scattering:

$$\Delta \hat{\sigma}_{ab} = \Delta \hat{\sigma}_{ab}^{\text{LO}} + \frac{\alpha_s}{\pi} \Delta \hat{\sigma}_{ab}^{\text{NLO}} + \left( \frac{\alpha_s}{\pi} \right)^2 \Delta \hat{\sigma}_{ab}^{\text{NNLO}} + \dots$$



# Global analysis of helicity PDFs

## Ingredients for NNLO

NNLO helicity PDFs:

- DIS-only analysis [Taghavi-Shahri, Khanpour, Atashbar Tehrani, Alizadeh Yazdi (2016)]
- DIS and (approximate) SIDIS [MAP: Bertone, Chiefa, Nocera (2024)]
- Fully global analysis with approximate NNLO for SIDIS and  $pp$  [BDSSV: IB, de Florian, Sassot, Stratmann, Vogelsang (2024)]

# Global analysis of helicity PDFs

## NNLO PDF evolution

$$\frac{\partial}{\partial \ln \mu^2} \Delta f_i(x, \mu^2) = \Delta P_{ji} \otimes \Delta f_j(x, \mu^2) \quad \Delta P_{ij} = \frac{\alpha_s}{2\pi} \Delta P_{ij}^{\text{LO}} + \left(\frac{\alpha_s}{2\pi}\right)^2 \Delta P_{ij}^{\text{NLO}} + \left(\frac{\alpha_s}{2\pi}\right)^3 \Delta P_{ij}^{\text{NNLO}} + \dots$$

- ▶ NNLO polarized evolution kernels known [Moch, Vermaseren, Vogt (2008, 2014, 2015)]; [Blümlein, Marquard, Schneider, Schönwald (2022)].
- ▶ NNLO evolution of pPDFs implemented in extended PEGASUS library [Vogt (2004)].
  - Evolution equations solved in Mellin space  $\Rightarrow$  direct interface.
  - Included missing NNLO polarized kernels
  - Included OMEs for matching conditions [Bierenbaum, Blümlein, De Freitas, Goedicke, Klein (2022)].
- ▶ Evolution benchmarked against libraries EKO [Candido, Hekhorn, Magni (2022)] and APFEL [Bertone, Carrazza, Rojo (2013)]; [Bertone (2017)].

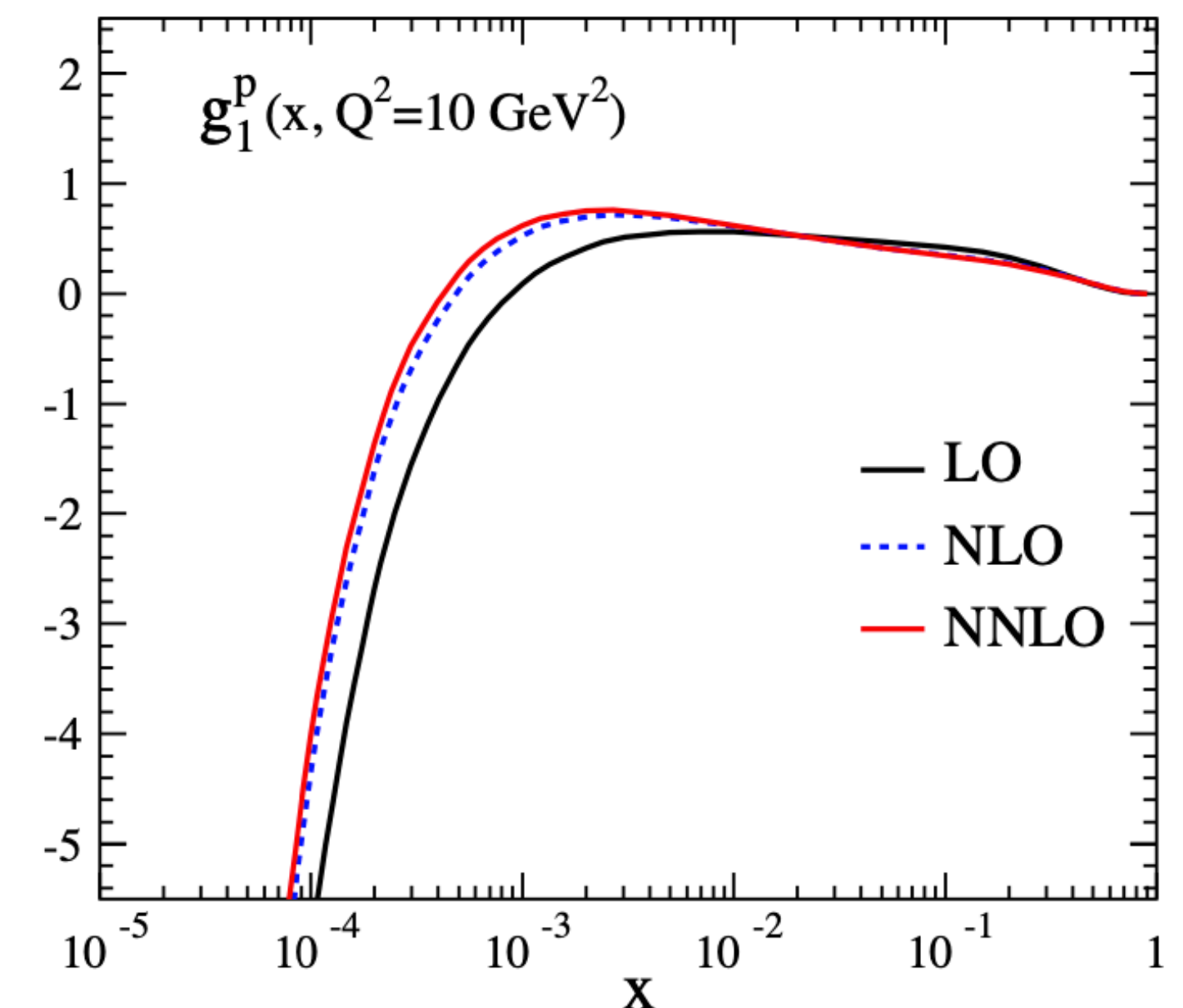
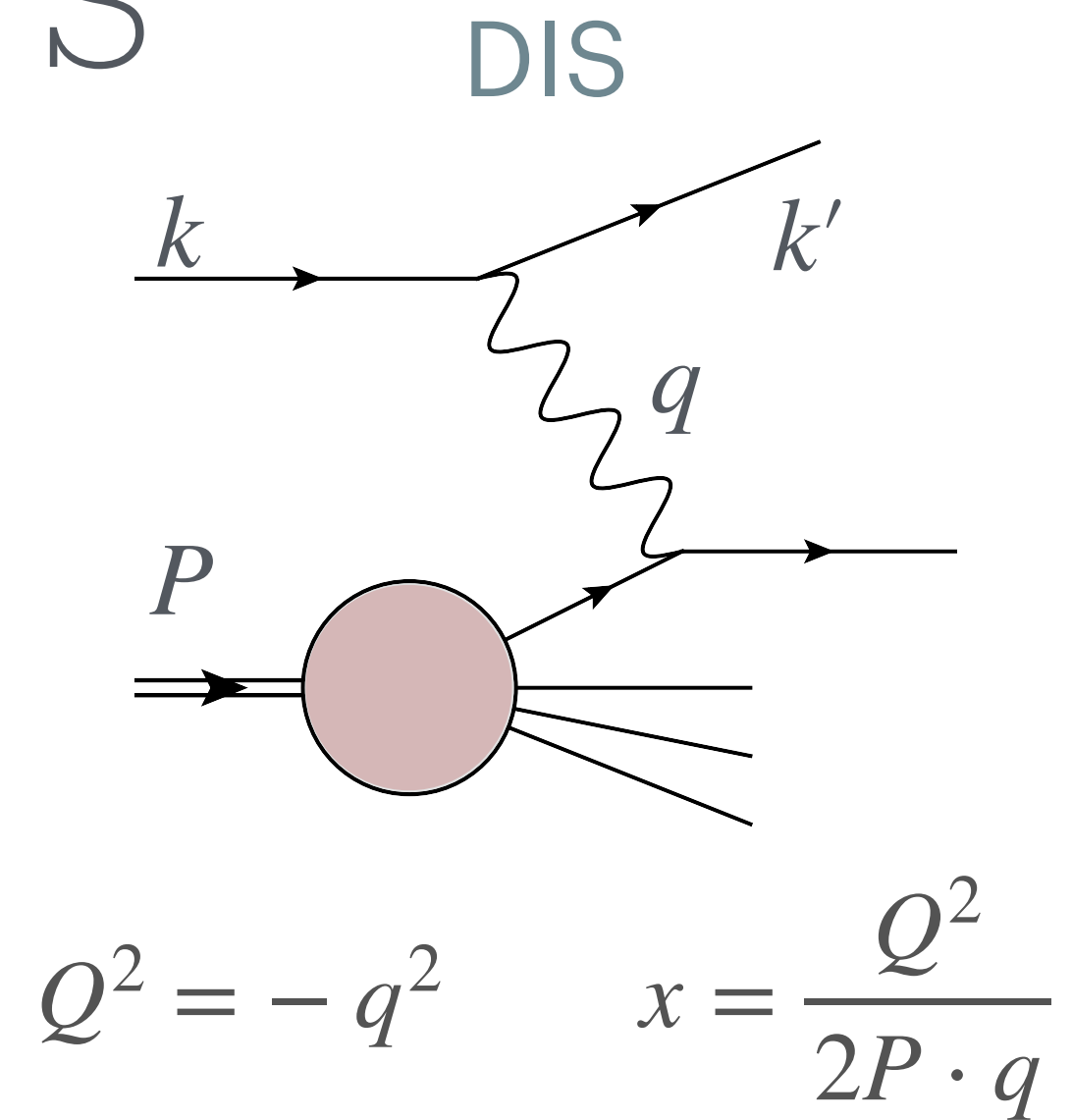
# Global analysis of helicity PDFs

## NNLO Coefficient functions for DIS

$$\frac{d^2 \Delta\sigma}{dx dy} = \frac{8\pi\alpha^2}{Q^2} [(2-y) g_1(x, Q^2)] \quad g_1(x, Q^2) = \sum_{f=q,g} \Delta C^f(x, Q^2) \otimes \Delta f(x, Q^2)$$

$$\Delta C^f = \Delta C^{f,(0)} + \frac{\alpha_s}{2\pi} \Delta C^{f,(1)} + \left(\frac{\alpha_s}{2\pi}\right)^2 \Delta C^{f,(2)} + \dots$$

- ▶ NNLO corrections to  $g_1(x, Q^2)$  known [van Neerven, Zijlstra (1994)] (N3LO corrections recently obtained for Larin scheme [Blümlein, Marquard, Schneider, Schönwald (2023)]).
- ▶ Possible to obtain analytical expression in Mellin space (MT package [Höschele, Hoff, Oak, Steinhauser, Ueda (2013)]; ANCONT program [Blümlein (2000)]).
- ▶ Already used in DIS-only analysis [Taghavi-Shahri, Khanpour, Tehran, Yazdi (2023)].



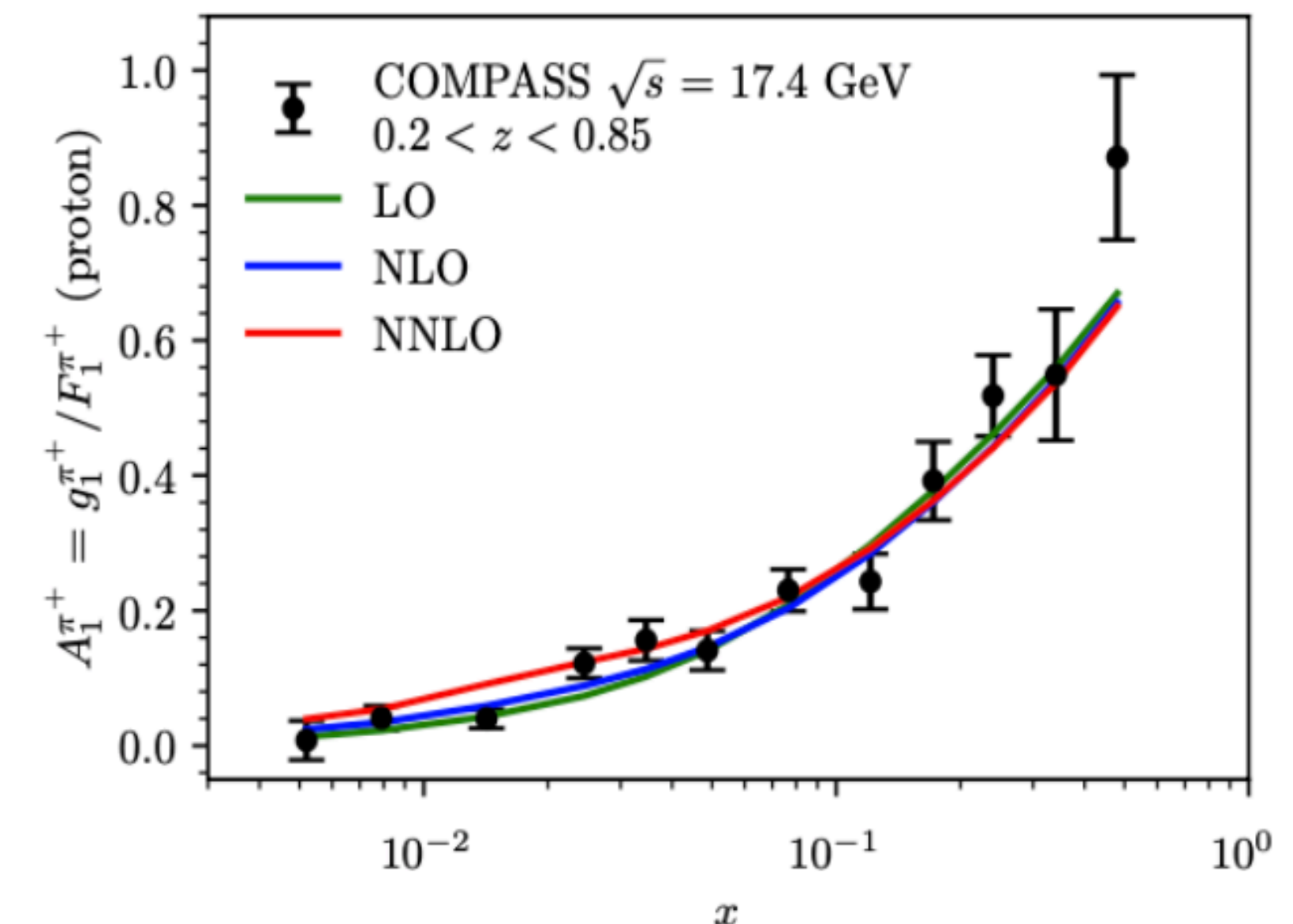
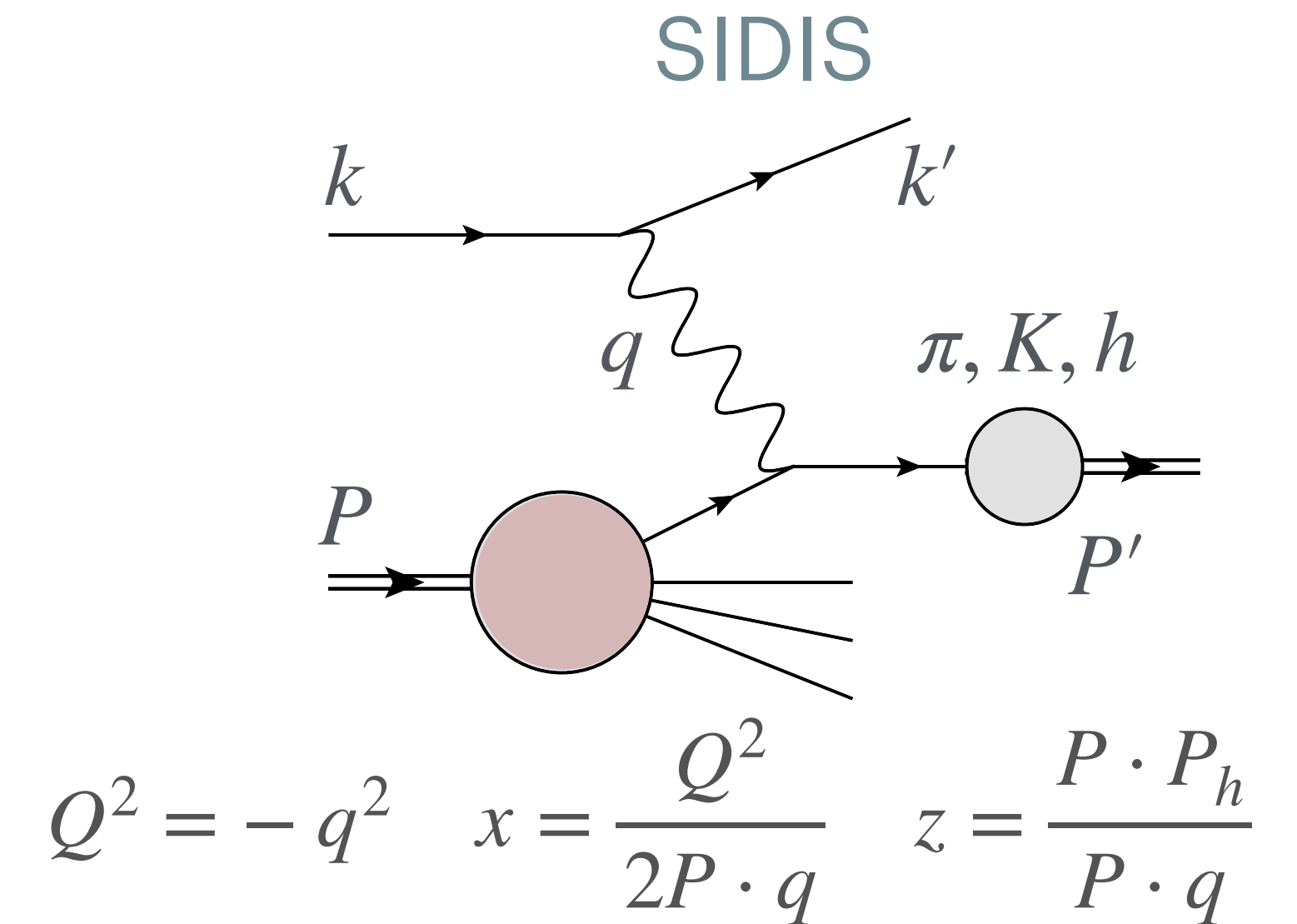
# Global analysis of helicity PDFs

## NNLO Coefficient functions for SIDIS

$$\frac{d^3 \Delta\sigma^h}{dx dy dz} = \frac{4\pi\alpha^2}{Q^2} [(2-y) g_1^h(x, z, Q^2)] \quad g_1^h(x, z, Q^2) = \sum_{f, f'} \Delta f(x, Q^2) \otimes \Delta C_{ff'}(x, z, Q^2) \otimes D_f^h$$

$$\Delta C_{ff'} = \Delta C_{ff'}^{(0)} + \frac{\alpha_s}{2\pi} \Delta C_{ff'}^{(1)} + \left(\frac{\alpha_s}{2\pi}\right)^2 \Delta C_{ff'}^{(2)} + \dots$$

- ▶ NNLO coefficients recently obtained [Bonino, Gehrmann, Löchner, Schönwald, Stagnitto (2024)]; [Goyal, Lee, Much, Pathak, Rana, Ravindran (2024)] ⇒ Soon to be included in global analyses of FFs and pPDFs.
- ▶ Soft gluon approximate NNLO [Anderle, Ringer, Vogelsang (2012)]; [Abelde, de Florian, Vogelsang (2021)].
- ▶ NNLO FFs available, but ... based only on SIA [Anderle, Ringer, Stratmann (2015)]; [Bertone, Carrazza, Hartland, Nocera, Rojo (2017)], or SIA+approx. SIDIS [IB, de Florian, Sassot, Stratmann, Vogelsang (2021)], [Abdul-Khalek, Bertone, Khoudii, Nocera (2021)].

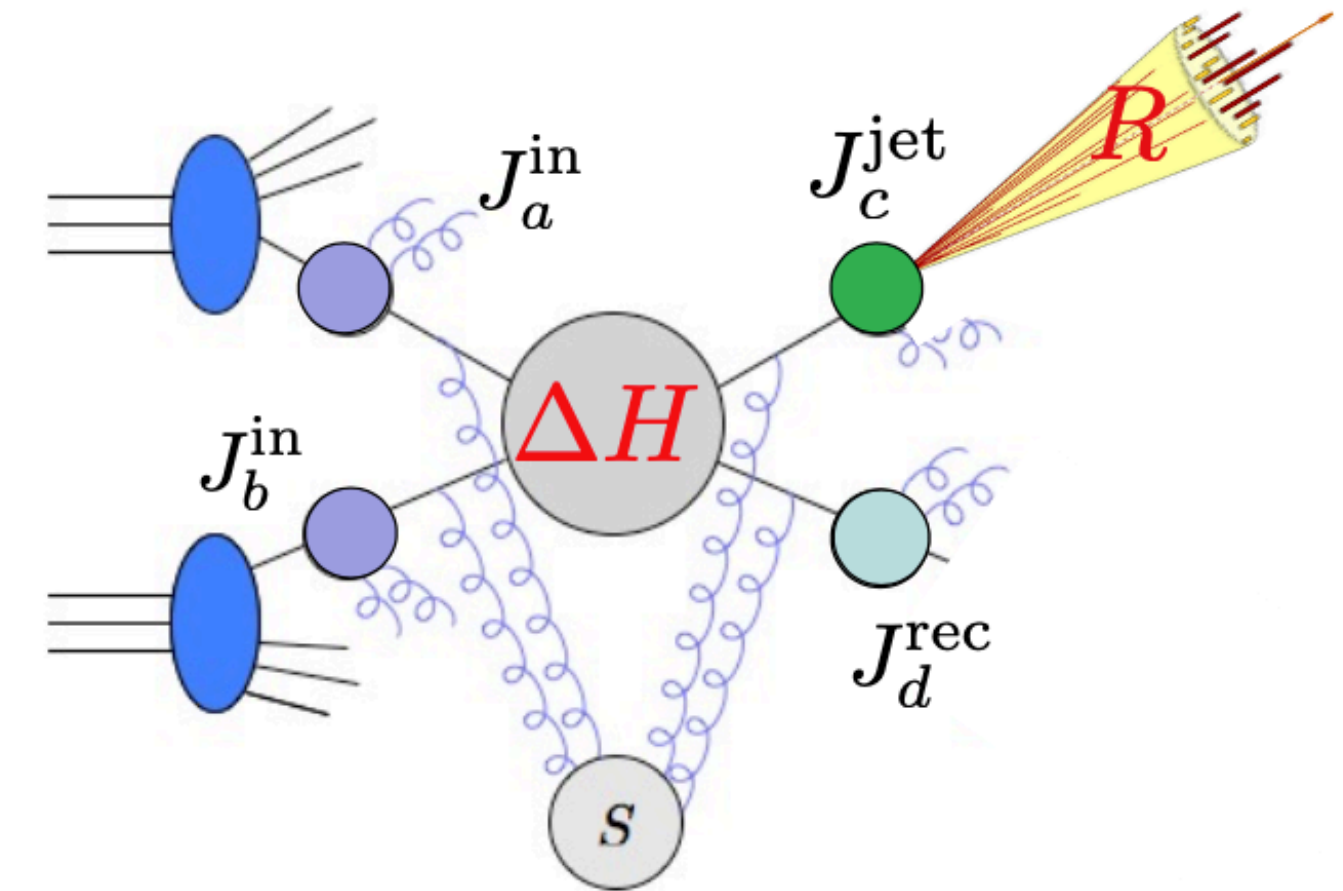


# Global analysis of helicity PDFs

## NNLO Coefficient functions for pp observables

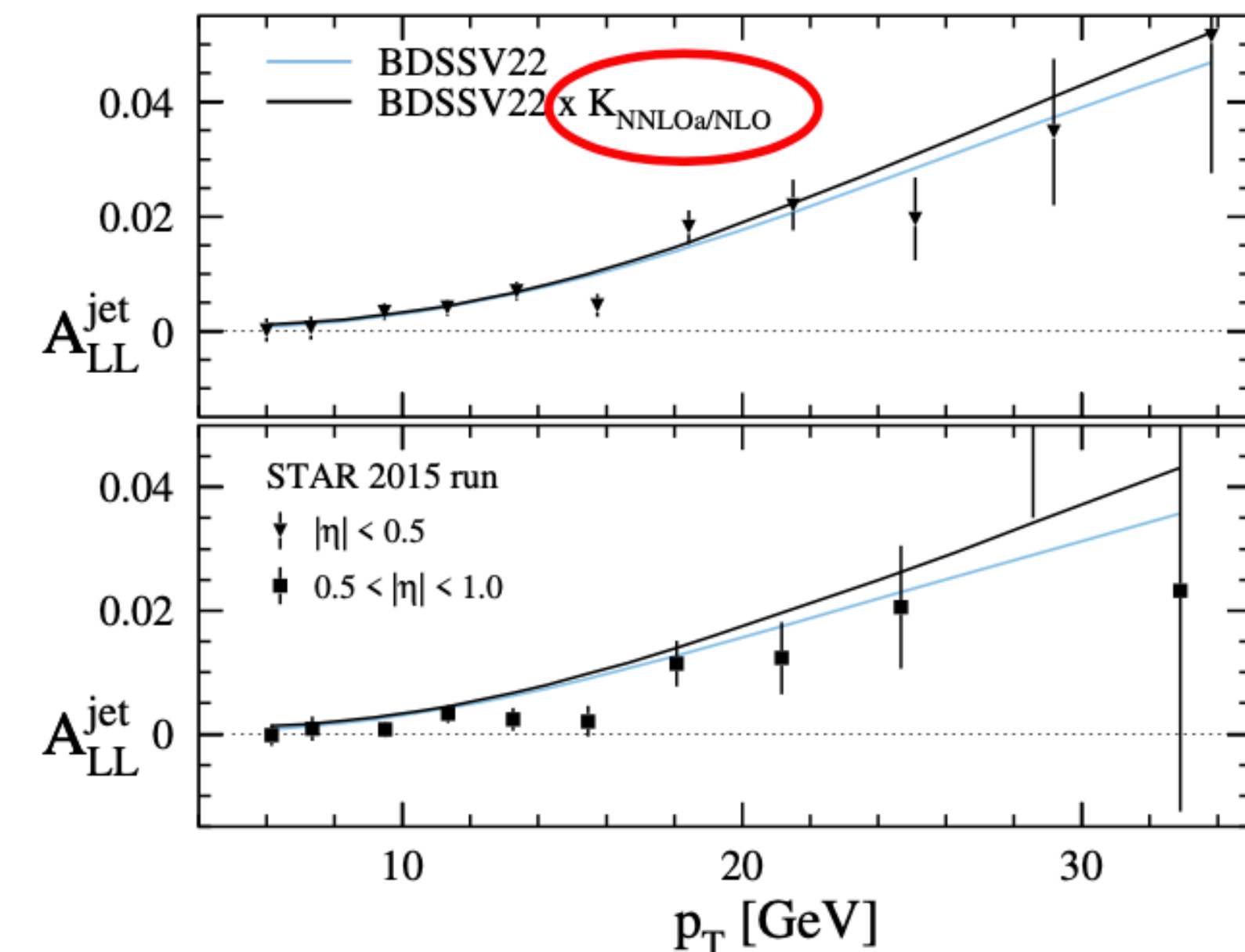
$$\Delta\sigma^{ab\rightarrow cd} \sim J_a^{\text{in}} \times J_b^{\text{in}} \times J_c^{\text{jet}} \times J_d^{\text{rec}} \times \text{Tr}[\Delta H S]_{ab\rightarrow cd}$$

Threshold logarithms  $\ln\left(1 - \frac{S_{\text{rad}}}{S}\right)$



► NNLO corrections not known for jet nor pion production.

► Still possible to derive approximate NNLO corrections based on the resummation of threshold logs [Kidonakis, Oderda, Sterman, (1998); de Florian, Vogelsang (2005); Hinderer, Ringer, Sterman, Vogelsang (2019)].



# Global analysis of helicity PDFs

## NNLO Coefficient functions for pp observables

$$\Delta\sigma = \sum_{a,b} \Delta f_a \otimes \Delta f_b \otimes \Delta\hat{\sigma}_{ab}$$

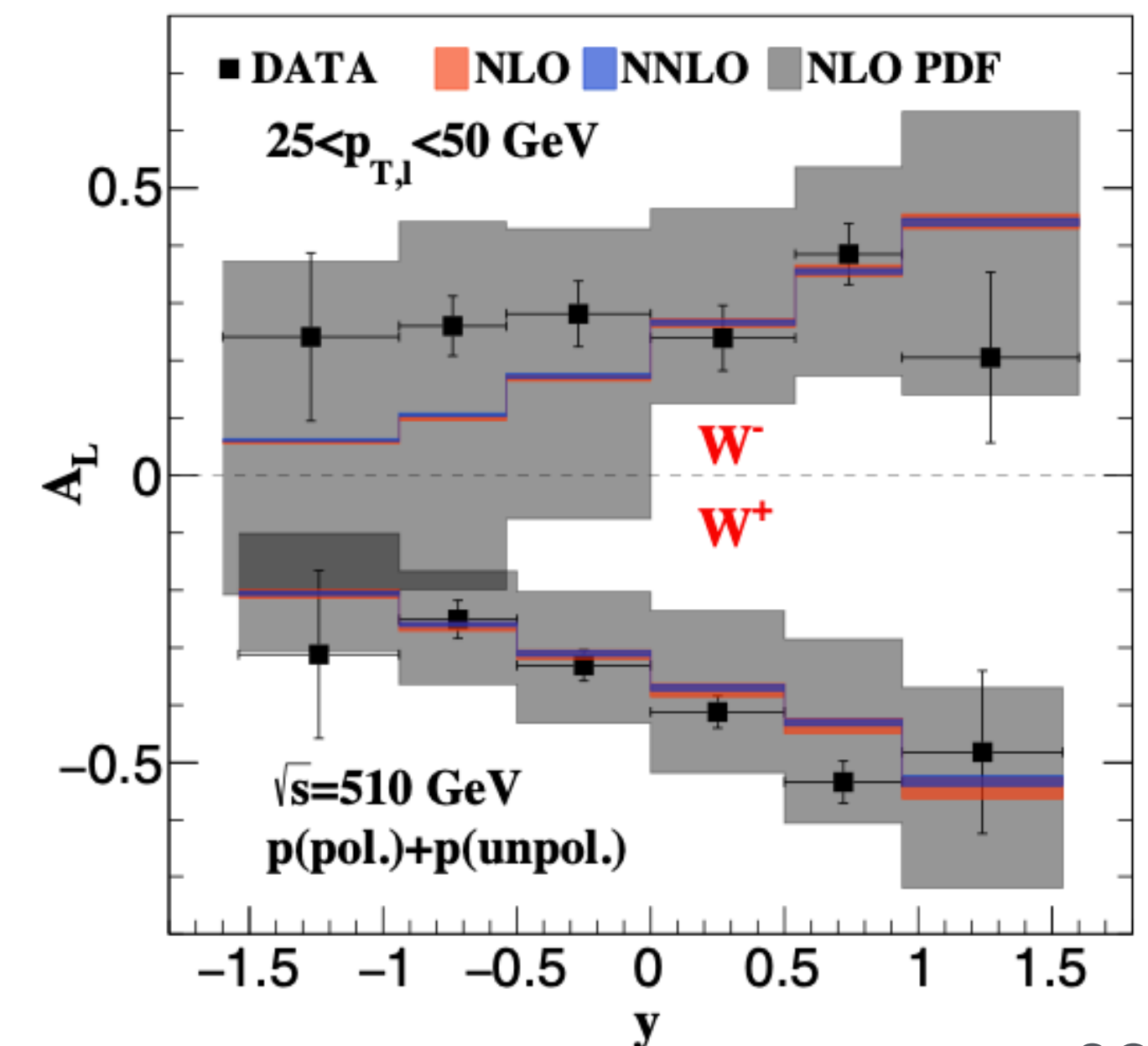
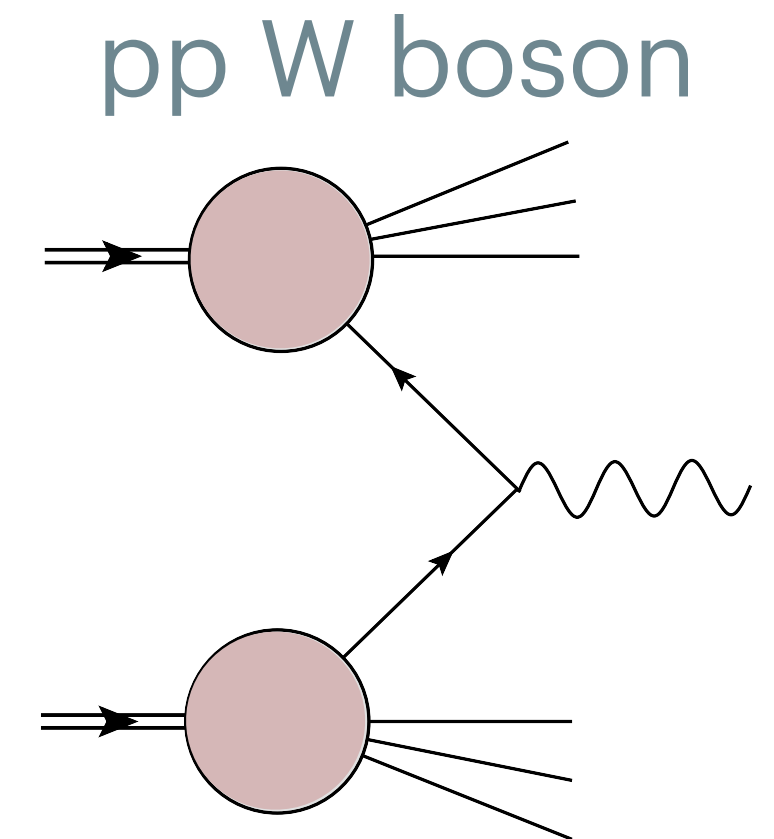
At LO :

$$A_L^{W^+} \propto \Delta\bar{d}u - \Delta u\bar{d}$$

$$A_L^{W^-} \propto \Delta\bar{u}d - \Delta d\bar{u}$$

► NNLO corrections for  $W^\pm$  known [Boughezal, Li, Petriello (2021)].

► NNLO implemented with K-factors in the analysis.



# Global analysis of helicity PDFs

## Technical specifications & data selection

► Parameterizations (at  $Q_0^2 = 1 \text{ GeV}^2$ ):

- $(\Delta q + \Delta \bar{q})(x, Q_0^2) = N_q x^{\alpha_q} (1 - x^{\beta_q}) (1 + \gamma_q x^{\delta_q} + \eta_q x)$  for  $(u, d)$
- $\Delta \bar{q}(x, Q_0^2) = N_{\bar{q}} x^{\alpha_{\bar{q}}} (1 - x^{\beta_{\bar{q}}}) (1 + \gamma_{\bar{q}} x^{\delta_{\bar{q}}})$  for  $(u, d, s)$
- $\Delta g(x, Q_0^2) = N_g x^{\alpha_g} (1 - x^{\beta_g}) (1 + \gamma_g x^{\delta_g})$

► Evolution:

- Zero-Mass Variable Flavor Number Scheme (ZMVFNS). HQ Matching coefficients from [\[Bierenbaum et al.\]](#)
- Extended QCD-PEGASUS library [\[Vogt\]](#)

► Assumptions

- No SU(2)/SU(3) constraints
- Positivity enforced (with respect to MSHT20 [\[Bailey, Cridge, Harland-Lang, Martin, Thorne \(2020\)\]](#))

► NLO FFs [\[IB, de Florian, Sassot, Stratmann \(2021, 2024\)\]](#)

► Uncertainties

- Monte Carlo Sampling [\[de Florian, Lucero, Sassot, Stratmann, Vogelsang \(2019\)\]](#)

# Global analysis of helicity PDFs

## Technical specifications & data selection

► Parameterizations (at  $Q_0^2 = 1 \text{ GeV}^2$ ):

- $(\Delta q + \Delta \bar{q})(x, Q_0^2) = N_q x^{\alpha_q} (1 - x^{\beta_q}) (1 + \gamma_q x^{\delta_q} + \eta_q x)$  for  $(u, d)$
- $\Delta \bar{q}(x, Q_0^2) = N_{\bar{q}} x^{\alpha_{\bar{q}}} (1 - x^{\beta_{\bar{q}}}) (1 + \gamma_{\bar{q}} x^{\delta_{\bar{q}}})$  for  $(u, d, s)$
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► NLO FFs [\[IB, de Florian, Sassot, Stratmann \(2021, 2024\)\]](#)

► Uncertainties

- ~~Monte Carlo Sampling~~ Montecarlo Error Sampling Systematic Implementation

# Global analysis of helicity PDFs

## Technical specifications & data selection

Data:	data-points
▶ DIS: EMC, SMC, E142, E143, E154, E155, HERMES, COMPASS, HALL-A, CLAS (p, n, d, He targets)	378
▶ SIDIS: HERMES, COMPASS (p, n, targets; identified $\pi^\pm, K^\pm, h^\pm$ )	277
▶ PP-JETS: STAR run 5, 6, 9, 12, 13, 15 ( $\sqrt{s} = 200, 510$ GeV)	91
▶ PP- $\pi^0/\pi^\pm$ : PHENIX, STAR	78
▶ PP- $W^\pm$ : PHENIX, STAR	22
▶ TOTAL:	850

# Global analysis of helicity PDFs

## Technical specifications & data selection

Data:

► DIS: EMC, SMC, E142, E143, E154, E155, HERMES, COMPASS, HALL-A, CLAS (p, n, d, He targets)

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► PP-JETS: STAR run 5, 6, 9, 12, 13, 15 ( $\sqrt{s} = 200, 510$  GeV)

► PP- $\pi^0/\pi^\pm$ : PHENIX, STAR

► PP- $W^\pm$ : PHENIX, STAR

► TOTAL:

data-points

378

But

277

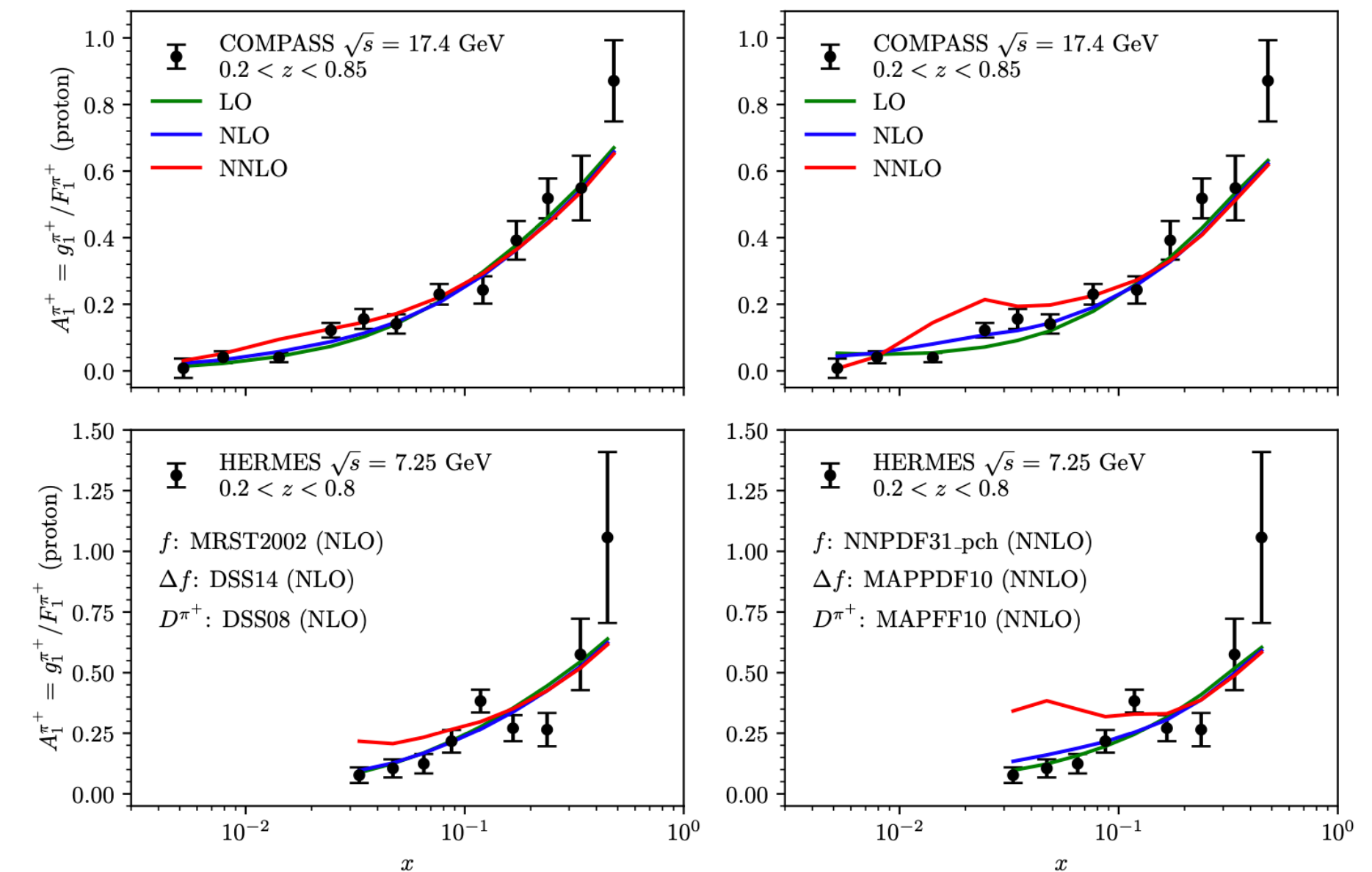
91

78

22

850

Bonino, Gehrmann, Löchner, Schönwald, Stagnitto (2024)



► Bump in the asymmetry not captured by the threshold approximation

# Global analysis of helicity PDFs

## Technical specifications & data selection

Data:

	data-points	$\chi^2$ -NLO	$\chi^2$ -NNLO
▶ DIS: EMC, SMC, E142, E143, E154, E155, HERMES, COMPASS, HALL-A, CLAS (p, n, d, He targets)	378	304.7	308.74
▶ SIDIS: HERMES, COMPASS (p, n, targets; identified $\pi^\pm, K^\pm, h^\pm$ )	277	276.1	322.5
▶ PP-JETS: STAR run 5, 6, 9, 12, 13, 15 ( $\sqrt{s} = 200, 510$ GeV)	91		
▶ PP- $\pi^0/\pi^\pm$ : PHENIX, STAR	78		
▶ PP- $W^\pm$ : PHENIX, STAR	22		
▶ TOTAL:	850		

Similar observations:

- MAP analysis [Bertone et al. (2024)]
- NNLO FFs fits [IB et al. (2022); Abdul-Khalek et al. (2022)]

⇒ Conservative cut of  $x_{\text{SIDIS}} > 0.12$   
imposed on SIDIS data

# Results

# Results

## $\chi^2$ -numerology

Data:

- ▶ DIS: EMC, SMC, E142, E143, E154, E155, HERMES, COMPASS, HALL-A, CLAS (p, n, d, He targets)
- ▶ SIDIS: HERMES, COMPASS (p, n, targets; identified  $\pi^\pm, K^\pm, h^\pm$ )
- ▶ PP-JETS: STAR run 5, 6, 9, 12, 13, 15 ( $\sqrt{s} = 200, 510$  GeV)
- ▶ PP- $\pi^0/\pi^\pm$ : PHENIX, STAR
- ▶ PP- $W^\pm$ : PHENIX, STAR
- ▶ TOTAL:

data-points

378

114 (277)

91

78

22

673

No cut on  $x_{\text{SIDIS}}$

$\chi^2$ -NLO

304.7

$\chi^2$ -NNLO

308.7

276.1

322.5

$x_{\text{SIDIS}} > 0.12$

$\chi^2$ -NLO

302.8

$\chi^2$ -NNLO

294.5

127.6

122.9

111.1

104.7

63.5

66.0

22.3

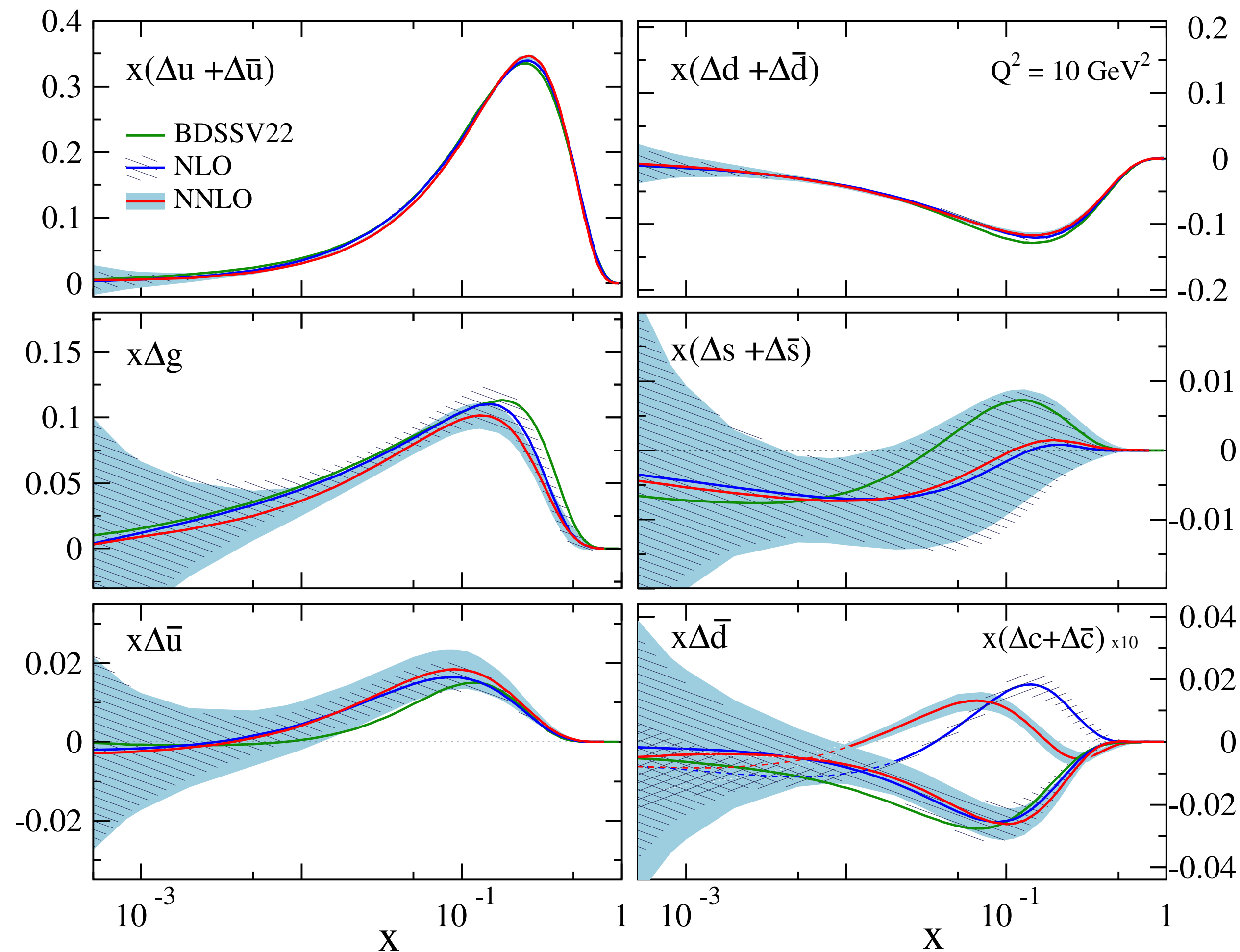
20.3

627.2

607.5

# Results

## NNLO polarized distributions

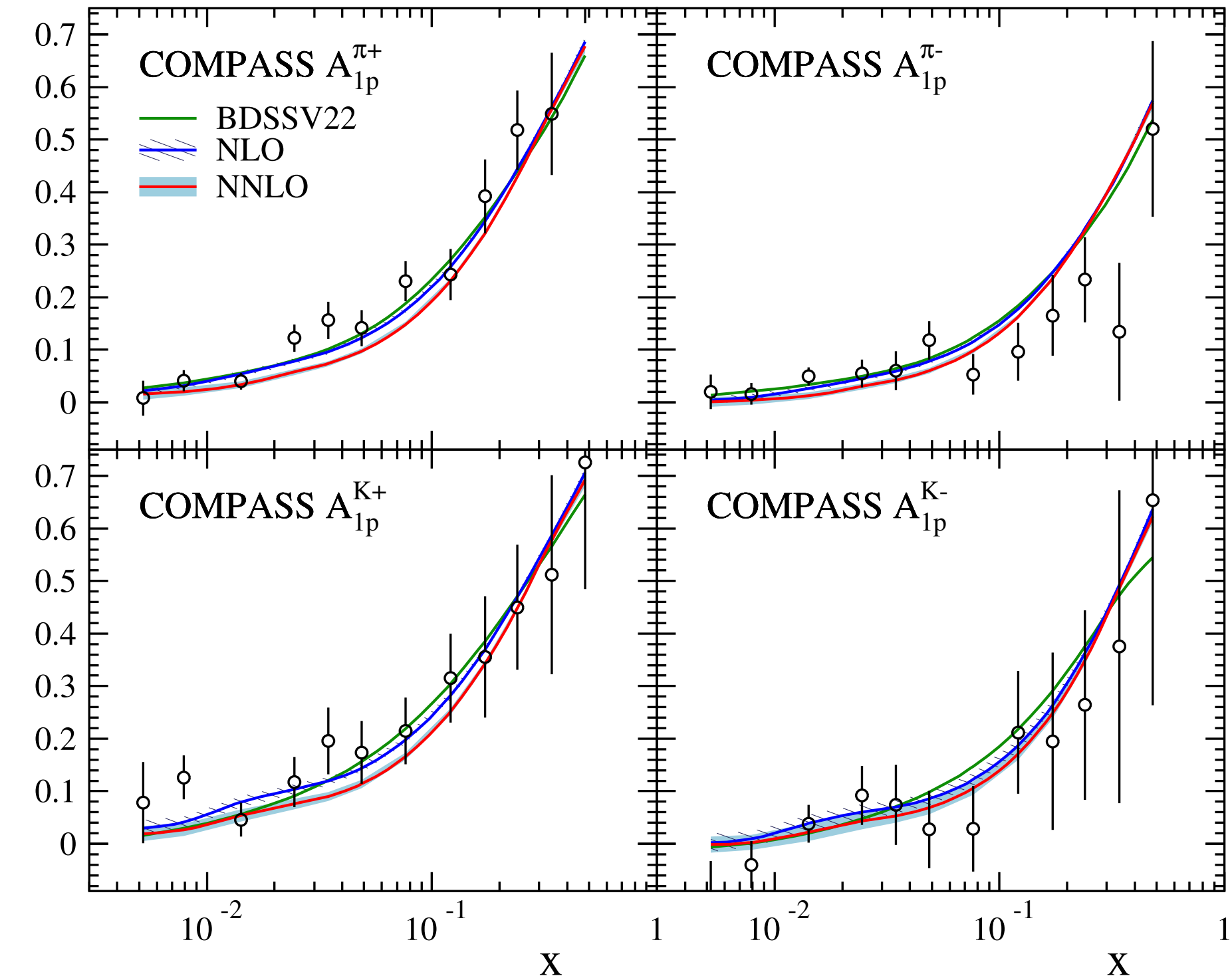
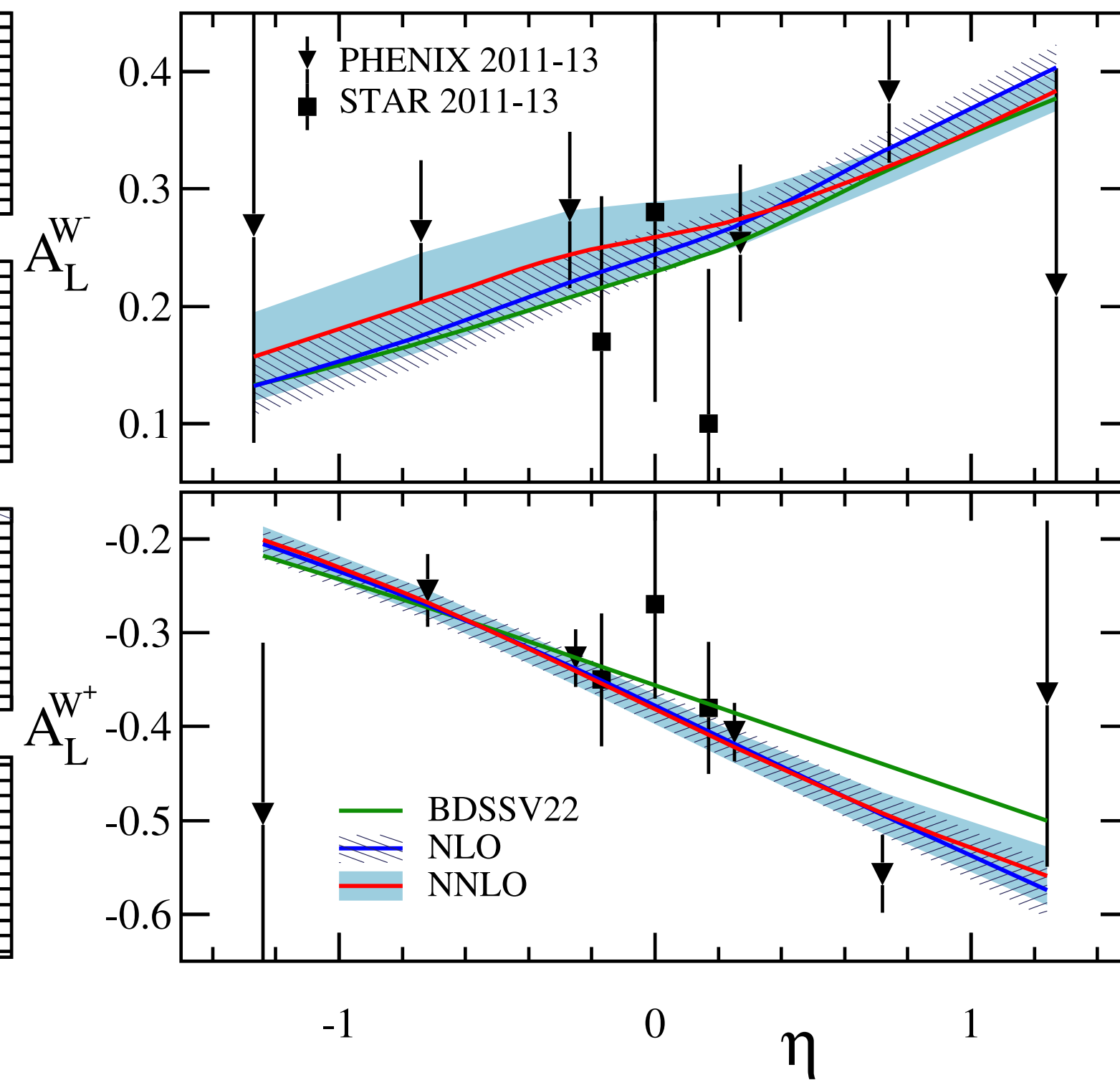
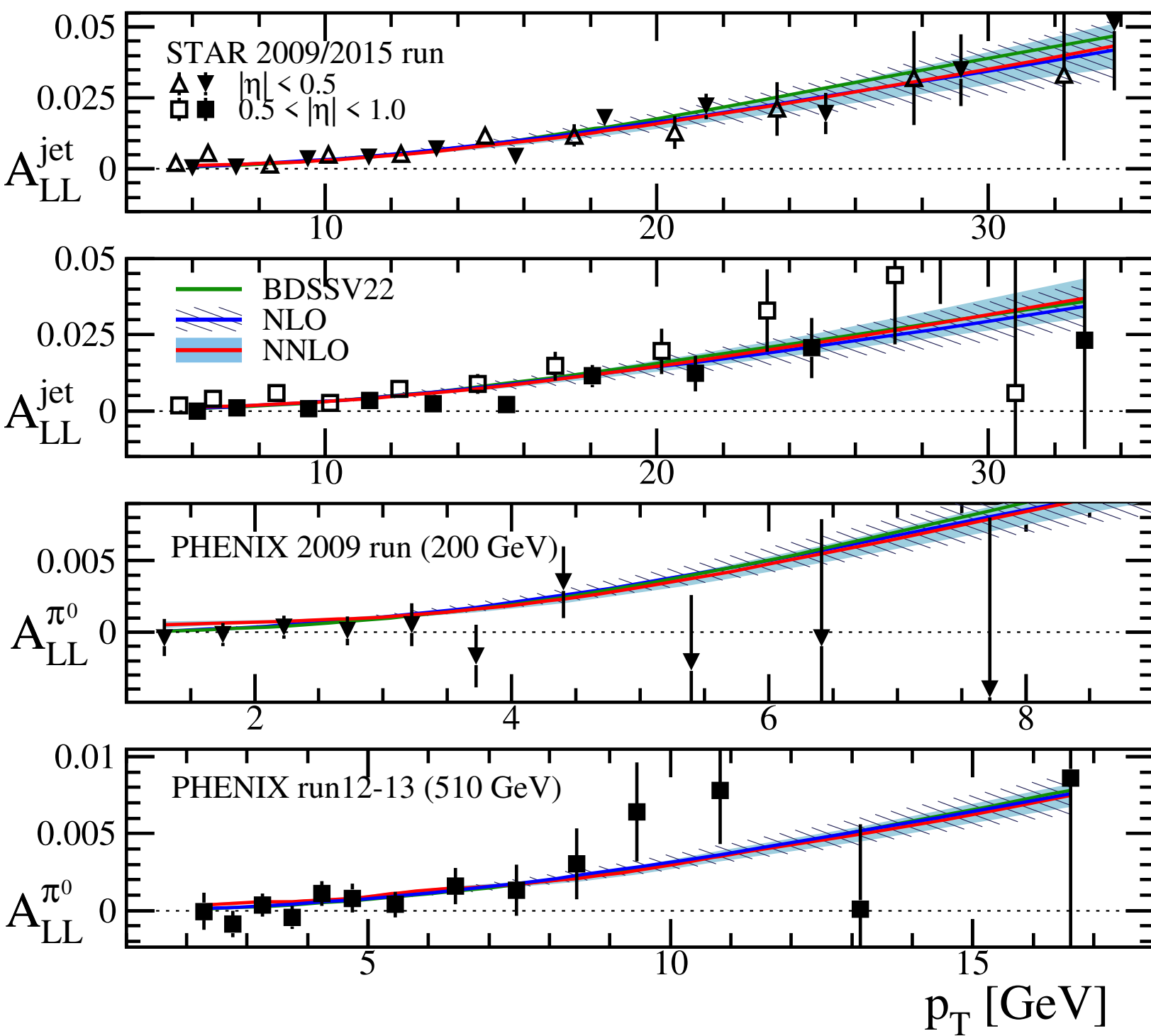


BDSSV22: NLO including dijets,  $W^\pm$  data and no cuts on SIDIS

- ▶  $(\Delta u + \Delta \bar{u})$  and  $(\Delta d + \Delta \bar{d})$  well constrained. No significant differences between NLO & NNLO.
- ▶  $\Delta g$  positive, and constrained for RHIC kinematics. NLO/NNLO differences well within uncertainties.
- ▶  $(\Delta s + \Delta \bar{s})$  consistent with zero  $\rightarrow$  Reduced number of data-points from SIDIS & lack of  $F, D$  constraints.
- ▶  $\Delta \bar{u}$  and  $\Delta \bar{d}$  constrained by  $W^\pm$  data.
- ▶  $(\Delta c + \Delta \bar{c})$  small, and strongly dependent on perturbative order (no intrinsic-charm).

# Results

## Selected data sets

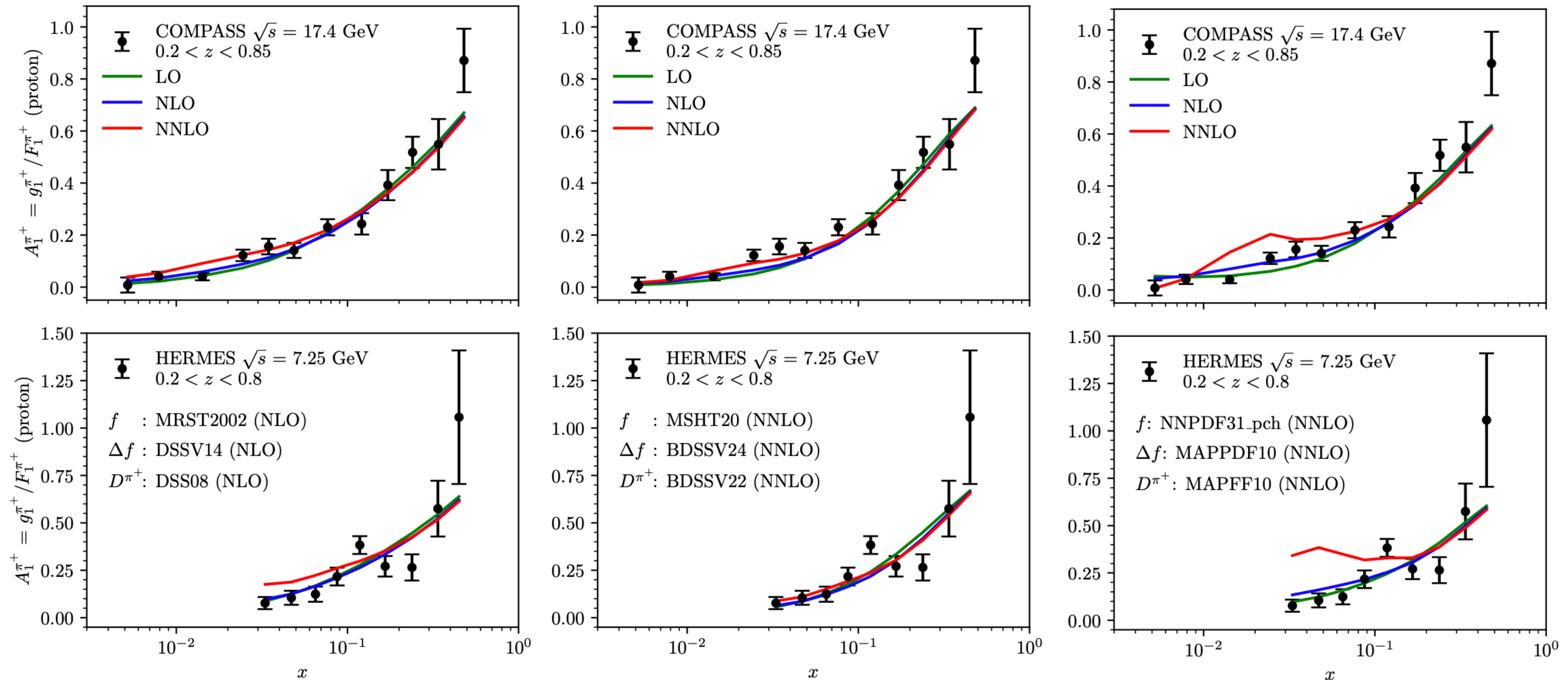


- In general, good description of data; similar results for NLO and NNLO
- For SIDIS, slight suppression of the NNLO asymmetry for low- $x$

# Results

## Revisiting SIDIS data

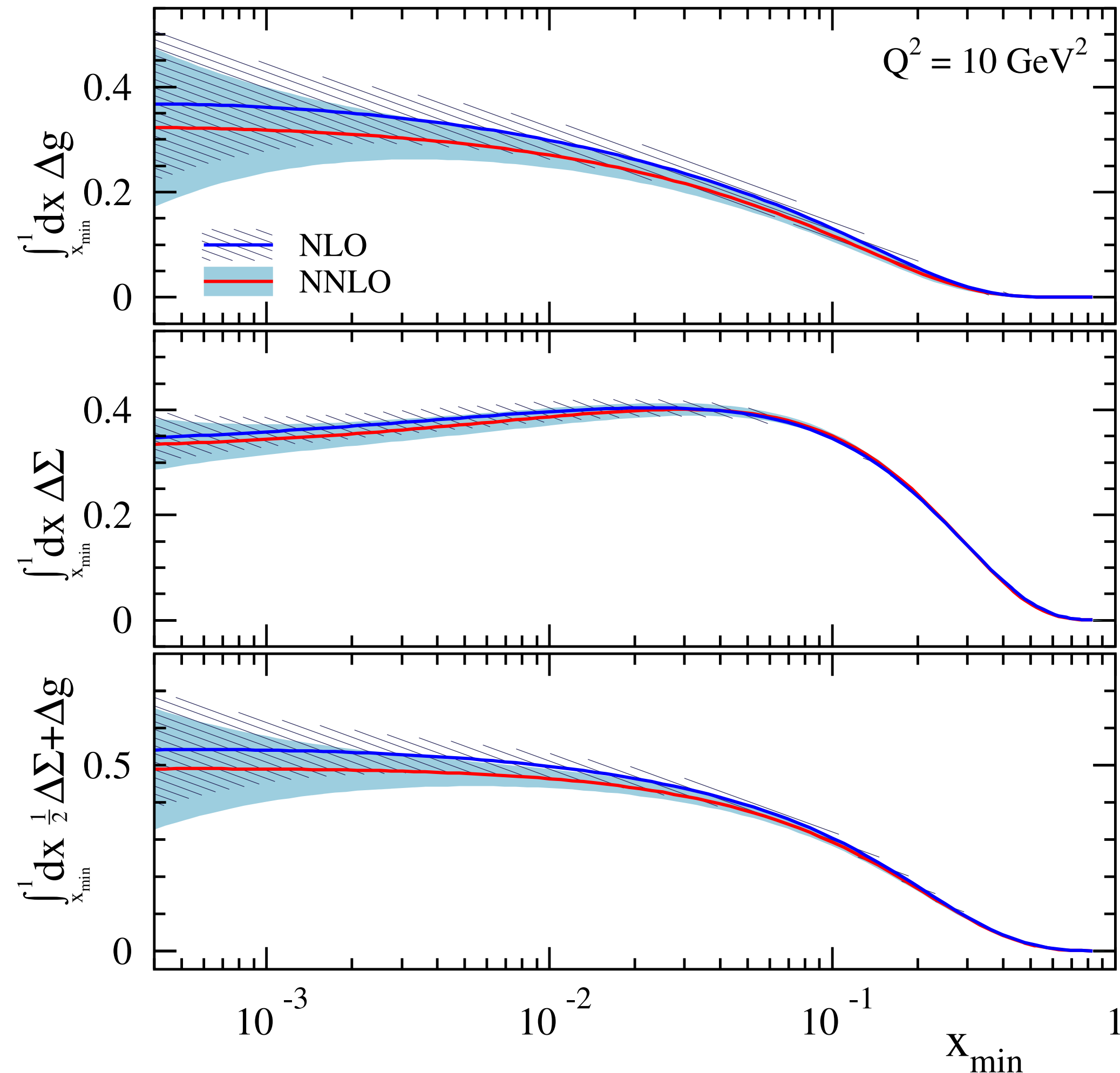
Bonino, Gehrmann, Löchner, Schönwald, Stagnitto (2024)



- ▶ Good description of the asymmetries even at low values of  $x$
- ▶ NNLO corrections seem to solve some of the tension between HERMES and COMPASS data

# Results

## Revisiting the spin sum rule



$$\frac{1}{2} = \frac{1}{2} \Delta \Sigma + \Delta g + L_q + L_g$$

► Room for OAM?

► Improved picture with EIC  
low- $x$  data

# Summary

# Summary

- ▶ Still rather incomplete picture of the proton's spin in terms of the contribution from quarks, anti-quarks and gluons.
- ▶ The spin program at the future EIC expected to give unique access to the proton's spin structure.
- ▶ First “proof-of-principle” NNLO global analysis of polarized PDFs:
  - Good perturbative stability going from NLO to NNLO
  - Slight improvement in the description of data (after imposing cut on  $x_{\text{SIDIS}}$ )
- ▶ Outlook:
  - Include full NNLO SIDIS results → First stage: new NNLO analysis of FFs.

Thank you

# Global analysis of helicity PDFs

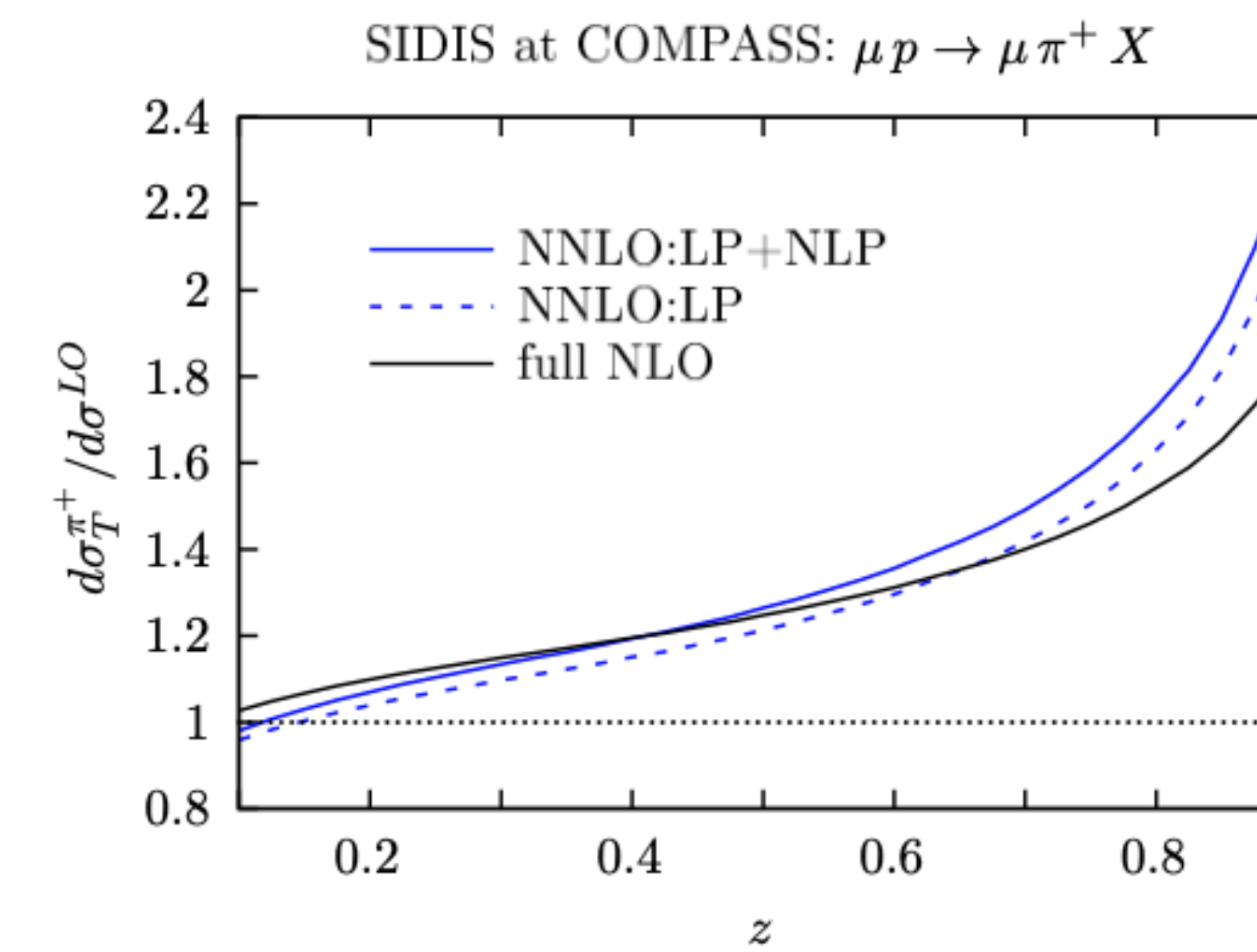
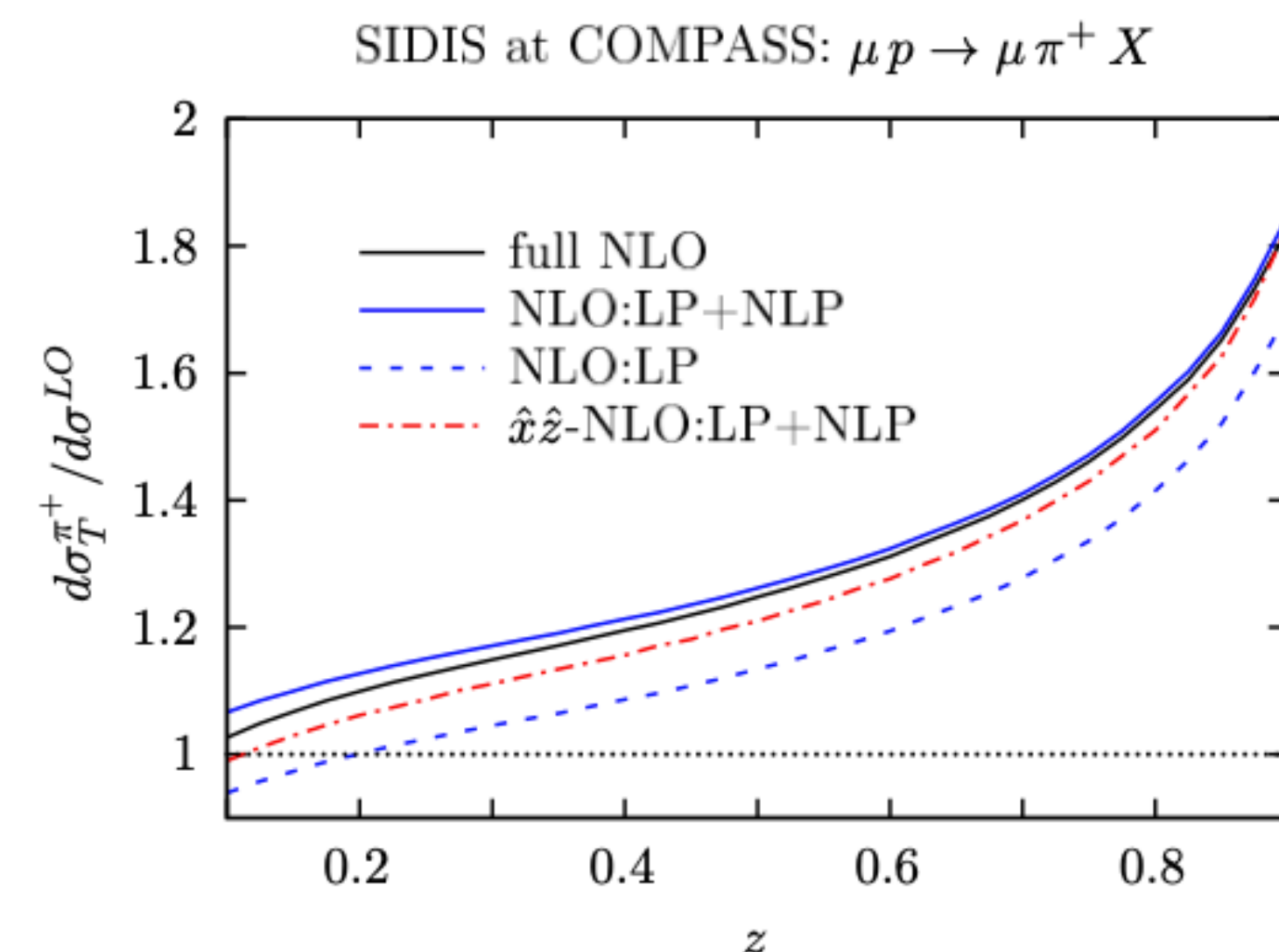
## Approximate NNLO Coefficient functions for SIDIS

$k^{\text{th}}$  order of perturbation theory:

$$\Delta \hat{\sigma}_{qq}^{\text{N}^k\text{LO}}(\hat{x}, \hat{z}) \sim \alpha_s^k \left[ \delta(1-\hat{x}) \left( \frac{\ln^{2k-1}(1-\hat{z})}{1-\hat{z}} \right)_+ + \delta(1-\hat{z}) \left( \frac{\ln^{2k-1}(1-\hat{x})}{1-\hat{x}} \right)_+ \right. \\ \left. + \frac{1}{(1-\hat{x})_+} \left( \frac{\ln^{2k-2}(1-\hat{z})}{1-\hat{z}} \right)_+ + \frac{1}{(1-\hat{z})_+} \left( \frac{\ln^{2k-2}(1-\hat{x})}{1-\hat{x}} \right)_+ + \dots \right]$$

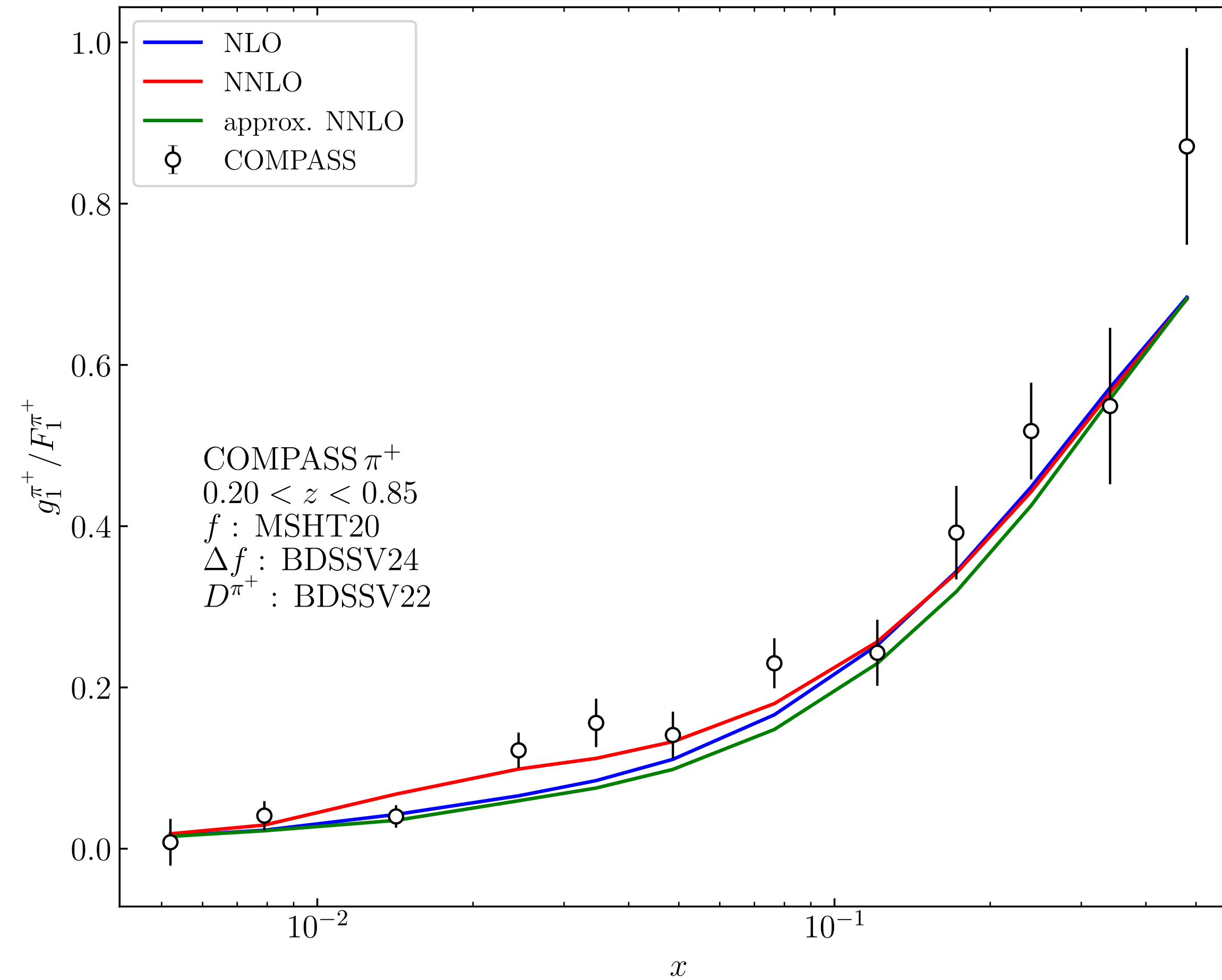
► Near the threshold for hadronic production  $x, z \rightarrow 1$ , logs can be resummed to all orders: **threshold resummation** [Anderle, Ringer, Vogelsang (2012)]; [Abelde, de Florian, Vogelsang (2021)].

► Approximate NNLO coefficients derived for the  $q \rightarrow q$  channel.



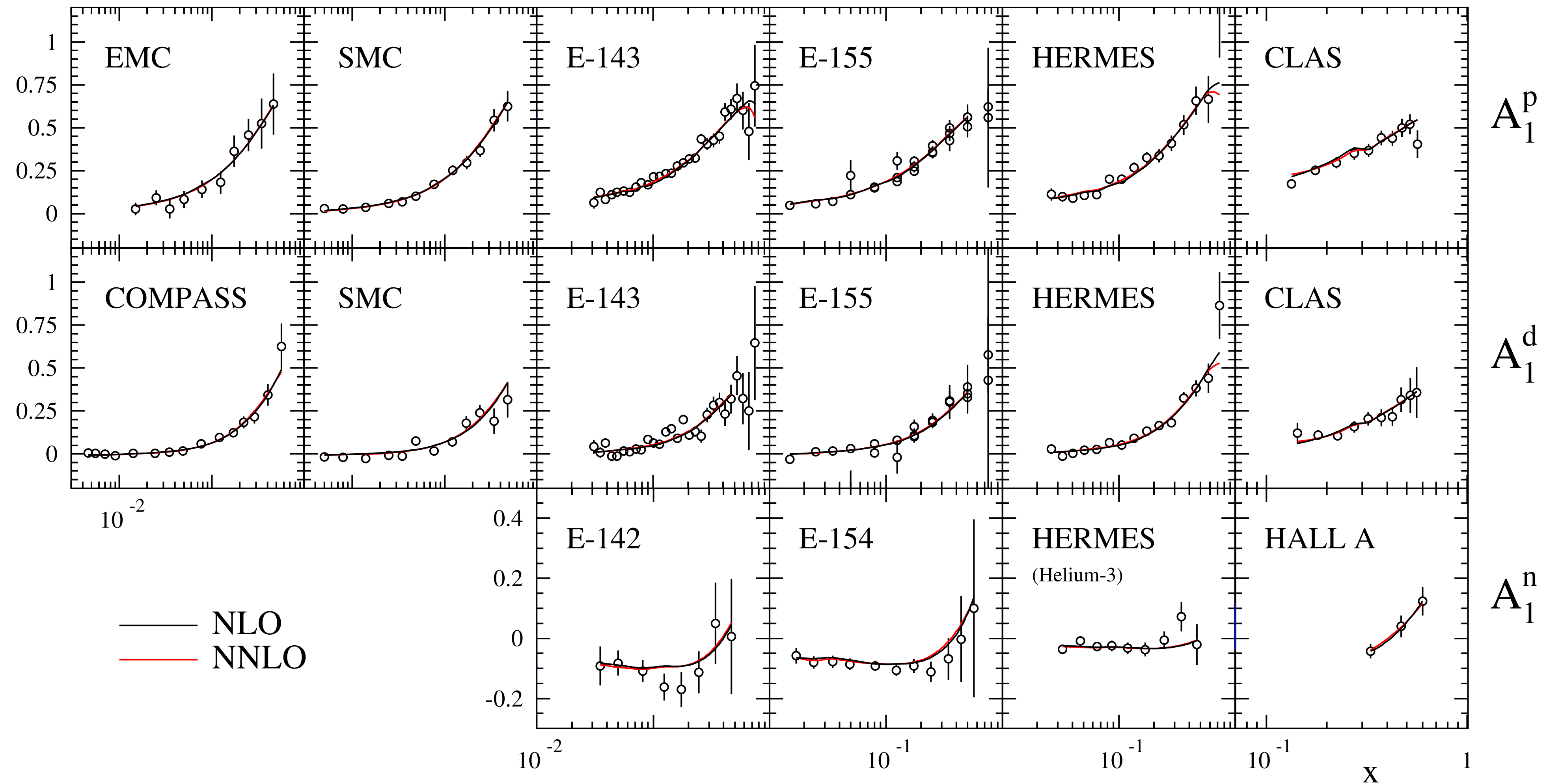
# Results

## Post validation



# Results

## Other data sets : DIS



# Results

## Other data sets : Full SIDIS

