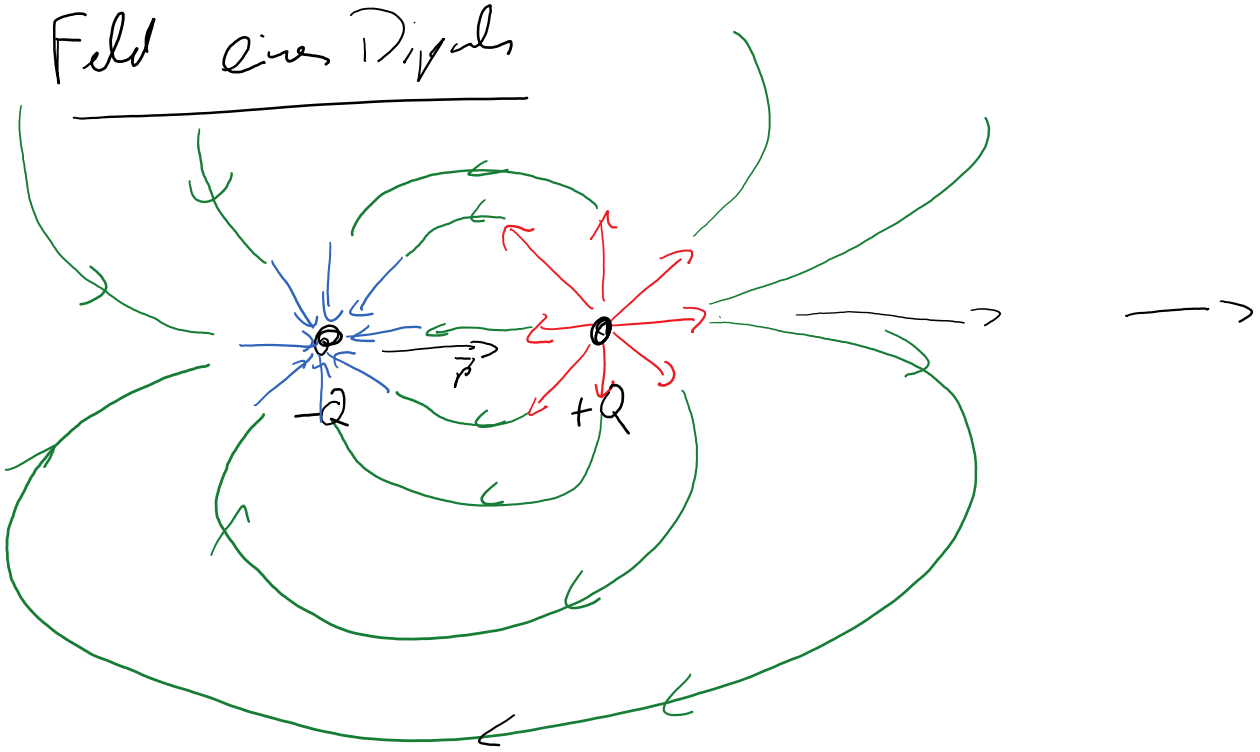
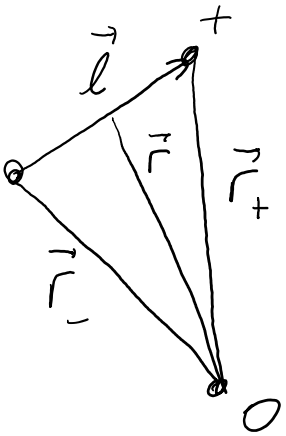
Feld eines Dipols



$$V_{dipol} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_+} - \frac{1}{r_-}\right) = \frac{Q}{4\pi\varepsilon_0}\left(\frac{r_- - r_+}{r_+ r_-}\right)$$



$$|\vec{r}_{\mp}| = |\vec{r} \mp \vec{l}/2| = r_{\mp}$$

Einschränkung wenn $r \gg l$

$$r_+ r_- = r^2 - (l/2)^2 = r^2\left(1 - \left(\frac{l}{2r}\right)^2\right) \approx r^2$$

↳ quadratische Term in l/r → weglassen in 1. Ordnung Taylor

$$V_{dipol} \cong \frac{Q}{4\pi\varepsilon_0} \cdot \frac{r_- - r_+}{r^2}$$

$$r_- - r_+ = |\vec{r}_-| - |\vec{r}_+| = \left|\vec{r} - \vec{l}/2\right| - \left|\vec{r} + \vec{l}/2\right|$$

$$= \sqrt{\left(\vec{r} - \vec{l}/2\right)^2} - \sqrt{\left(\vec{r} + \vec{l}/2\right)^2}$$

$$= \sqrt{\vec{r}^2 - \vec{r}\cdot\vec{l} + \left(\vec{l}/2\right)^2} - \sqrt{\vec{r}^2 + \vec{r}\cdot\vec{l} + \left(\vec{l}/2\right)^2}$$

$$= r\left(\sqrt{1 - \frac{\vec{r}\cdot\vec{l}}{\vec{r}^2}} - \sqrt{1 + \frac{\vec{r}\cdot\vec{l}}{\vec{r}^2}}\right)$$

$$\overset{\text{1. Ordnung Taylor}}{\cong} r\left(\left(1 - \frac{\vec{r}\cdot\vec{l}}{2\vec{r}^2}\right) - \left(1 + \frac{\vec{r}\cdot\vec{l}}{2\vec{r}^2}\right)\right) = -\frac{\vec{r}\cdot\vec{l}}{r}$$

$\frac{\vec{r}\cdot\vec{l}}{\vec{r}^2}$

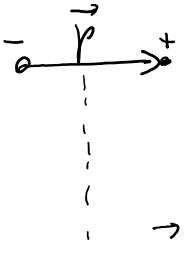
$$\Rightarrow \boxed{V_{dipol} \cong \frac{\vec{p}\cdot\vec{r}}{4\pi\varepsilon_0 r^3}} \qquad Q\cdot\vec{l} = \vec{p}$$

$\vec{r} \gg \vec{l}$

$$\vec{E}_{dipol} = -\vec{\nabla} V_{dipol} = \frac{3(\vec{p}\cdot\vec{r})\vec{r} - r^2\vec{p}}{4\pi\varepsilon_0 r^5} \leftarrow$$

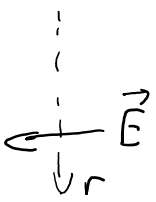
Spezialfälle

$\vec{r} \perp \vec{p} \Rightarrow (\vec{r}\cdot\vec{p}) = 0$



$\vec{r} \perp \vec{p} \Rightarrow (\vec{r} \cdot \vec{p}) = 0$

$$\Rightarrow \vec{E} = -\frac{\vec{p}}{4\pi\varepsilon_0 r^3}$$



$\vec{r} \parallel \vec{p} \Rightarrow \vec{r} \cdot \vec{p} = rp$

$\vec{r} = r\vec{e}_r$

$\vec{p} = p\vec{e}_r$

$$\Rightarrow \vec{E} = \frac{2pr\vec{e}_r}{4\pi\varepsilon_0 r^5} = \frac{\vec{p}}{2\pi\varepsilon_0 r^3}$$

$3(\vec{p}\cdot\vec{r})\vec{r} - r^2\vec{p}$

$= 3\vec{r}\cdot pr - r^2\vec{p}$

$= 3pr^2\vec{e}_r - r^2 p\vec{e}_r$

$= 2pr^2\vec{e}_r$