

# Innovating Slicing Methods for Systematic and Efficient Collider Phenomenology at N3LO

## Gherardo Vita



*UZH/ETH QCD Seminar - Zurich, 26 November 2024*

Based on:

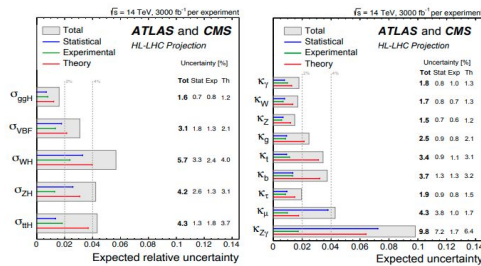
**“N3LO Power Corrections for 0-jettiness Subtractions With Fiducial Cuts”** **GV** [2401.03017]

**“Projection-to-Born-improved Subtractions at NNLO”** Campbell, Neumann, **GV** [2408.05265]

# Outline

## Intro e Motivation

- Do we need N3LO cross sections?

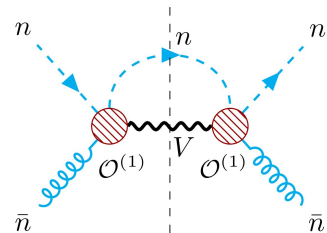


	$Q$ [GeV]	$\delta\sigma^{\text{NNLO}}$	$\delta\sigma^{\text{N}^3\text{LO}}$
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- Slicing methods

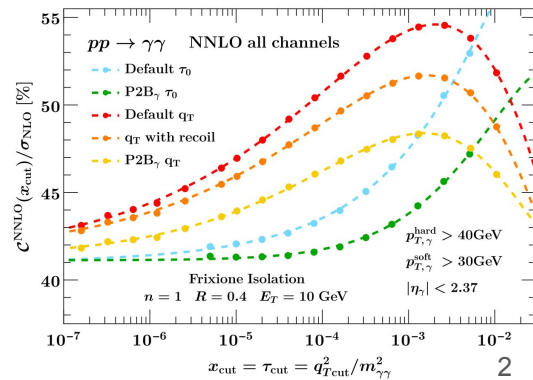
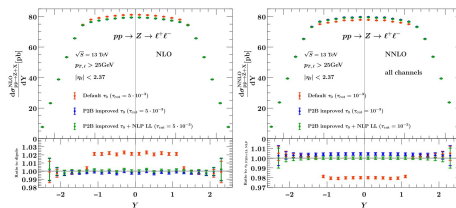
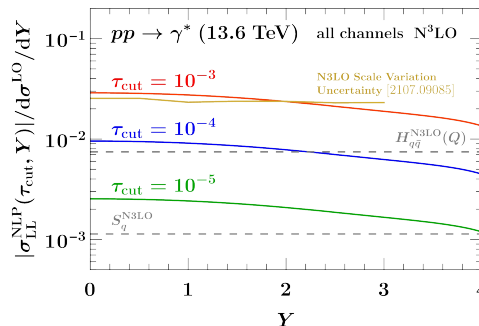
$$\sigma(X) = \int_0^{q_{T\text{cut}}} dq_T \frac{d\sigma^{\text{sing}}(X)}{dq_T} + \int_{q_{T\text{cut}}} dq_T \frac{d\sigma(X)}{dq_T} + \Delta\sigma(X, q_{T\text{cut}})$$

Below the cut region
Above the cut region
Residual



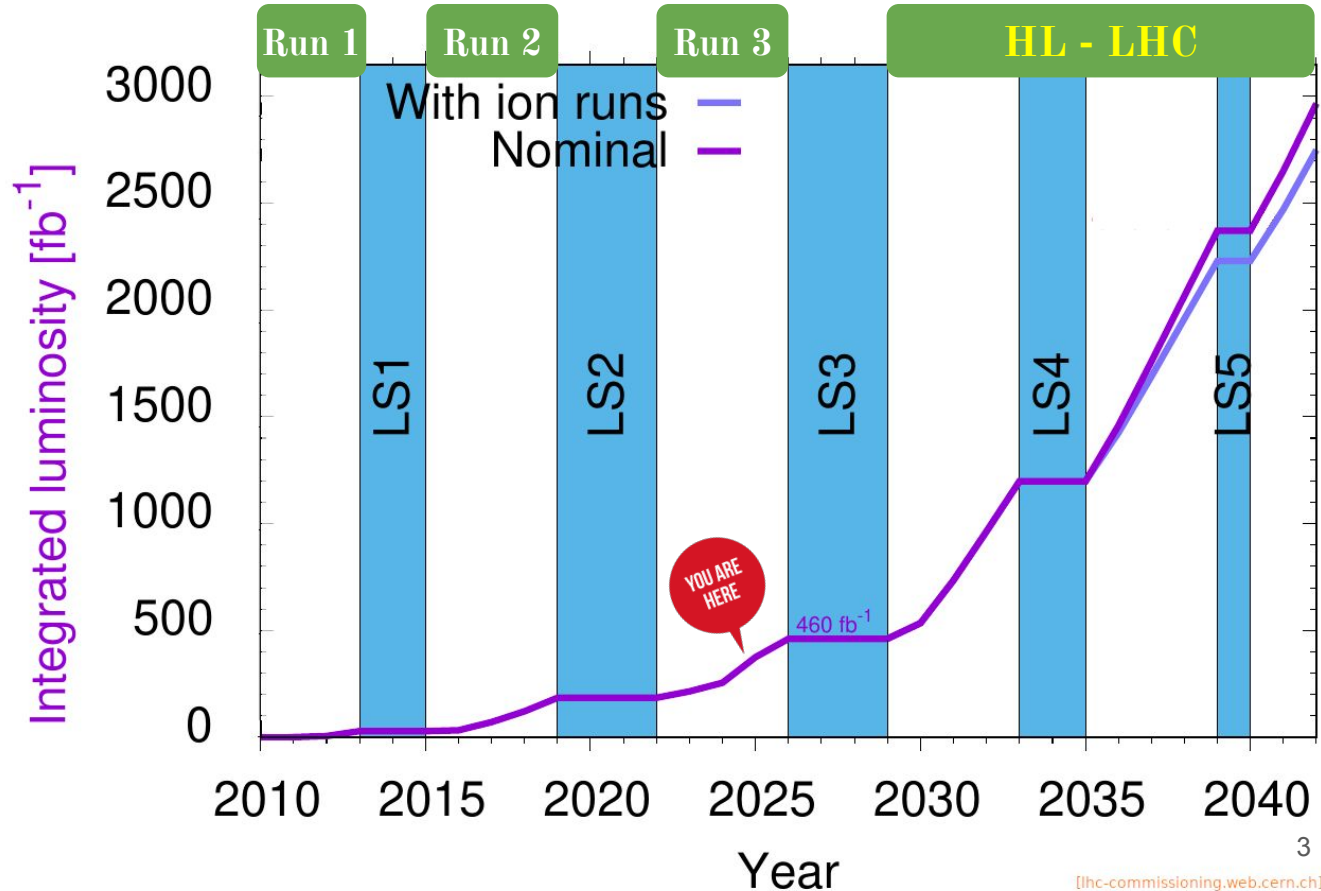
## Improving Slicing

- N3LO Power corrections for 0-jettiness subtraction
- Projection-to-Born-improved Subtractions



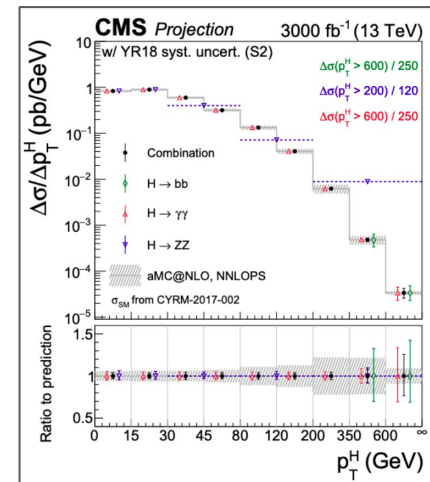
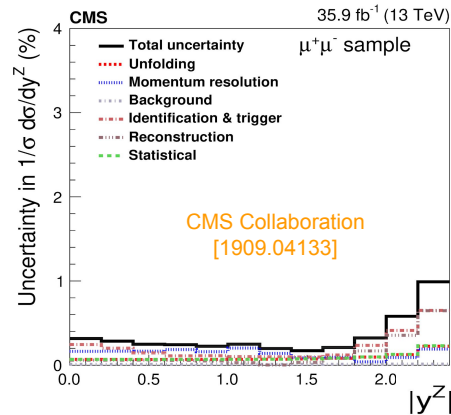
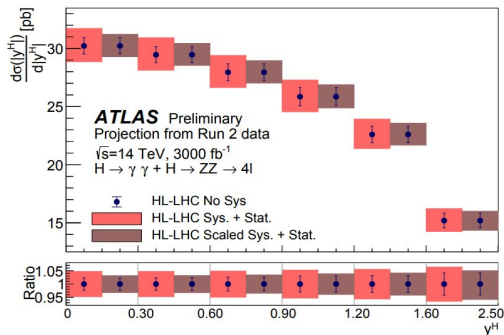
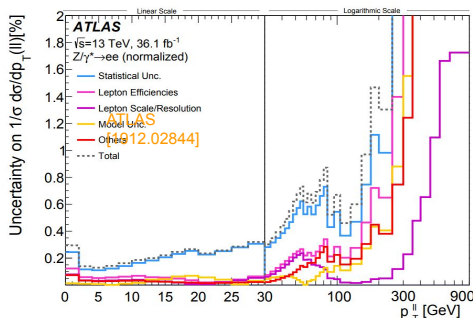
# LHC Timeline

- The LHC physics program is just at the beginning!
- LHC analysis **today** are based on less than **150 fb<sup>-1</sup>**
- **460 fb<sup>-1</sup>** will be collected by the end of **2025**
- **3000 fb<sup>-1</sup>** will be collected after **HL - LHC**



# Testing the Standard Model at Colliders

Ability to test the SM at (sub)-percent accuracy!



High experimental accuracy for processes sensitive to some of the most interesting aspects of contemporary particle physics:

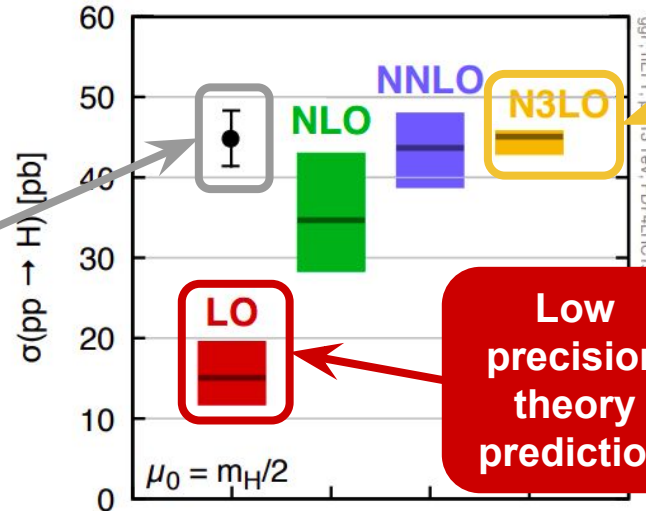
- Probing Electroweak Symmetry Breaking
- Unveiling the Nature of Yukawa Interactions
- Testing the Boundaries of the Standard Model
- Measuring the Strength of the Strong Interaction
- Deepening our Understanding of the Proton Structure

# Precision in Theoretical Predictions

To answer these fundamental questions we need comparable precision from the theory side!

Example: Higgs Production at the LHC

Experimental Measurement



Accurate theoretical prediction

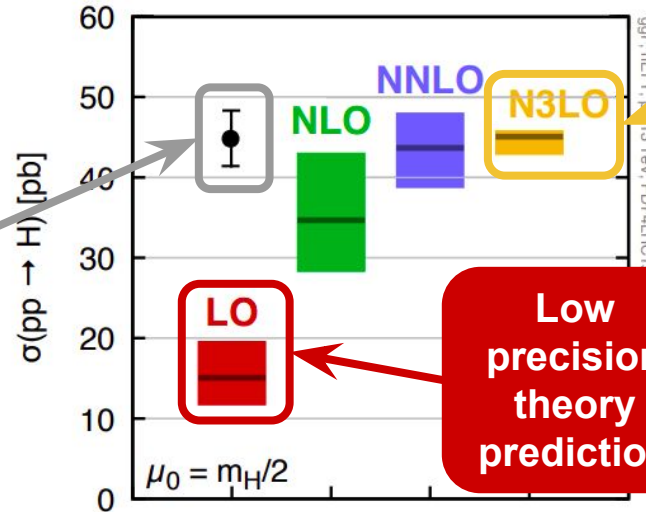
Low precision theory prediction

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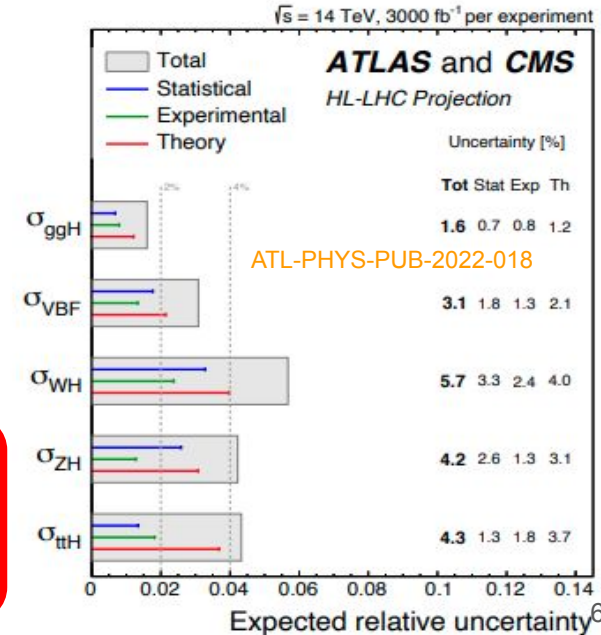
Experimental Measurement



Accurate theoretical prediction

Low precision theory prediction

Recurring theme for HL - LHC Projections: Accuracy of Theory Predictions will be limiting factor in many analysis!



# Standard Model Phenomenology at percent level

One way to achieve more accurate predictions is by advancing

$$\hat{\sigma}_{ab \rightarrow X} = \underbrace{\sigma_0}_{\text{LO}} + \underbrace{\alpha_s \sigma_1}_{\text{NLO}} + \underbrace{\alpha_s^2 \sigma_2}_{\text{NNLO}} + \underbrace{\alpha_s^3 \sigma_3}_{\text{N}^3\text{LO}} + \dots$$

**QCD Perturbation Theory**

# And many more things...

- **Accessibility and User Friendliness:** Creating frameworks that make N<sup>3</sup>LO (and NNLO) predictions easily accessible for comparison to experimental data.
- **Corrections beyond massless QCD:** EWK and masses.
- **Factorisation Violation at N<sup>3</sup>LO:** tops, PDFs.
- **Parton Showers:** Consistent combination of parton showers with fixed order perturbative computations at N<sup>3</sup>LO.
- **Resummation:** Complementing N<sup>3</sup>LO computations and resummation techniques for infrared sensitive observables.
- **Uncertainties:** Deriving/defining reliable uncertainty estimates for theoretical computations at the percent level.
- **Beyond Leading Power Factorisation:** Exploring the limitations of leading power perturbative descriptions of hadron collision cross sections.

# Do we really need N3LO cross sections?

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**Not so fast...**

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- Let's look at some explicit example...

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Inclusive  
Cross  
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**N3loxs**  
[Baglio, Duhr,  
Mistlberger, Szafron '22]

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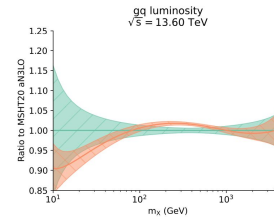
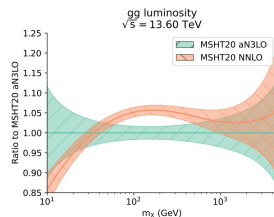
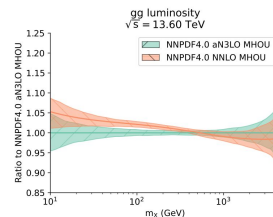
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Parton  
Distribution  
Functions

[2411.05373] NNPDF &  
MSHT Collaborations

PDF set	$\sigma(gg \rightarrow h)$	$\sigma(h \text{ VBF})$
$\Delta_{\text{NNLO}}^{\text{exact}}$ (NNPDF4.0)	2.2%	1.3%
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- Let's look at some explicit example...

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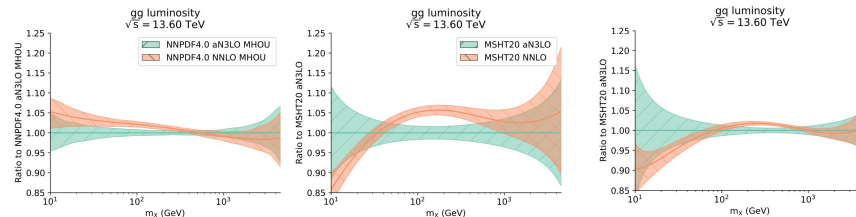
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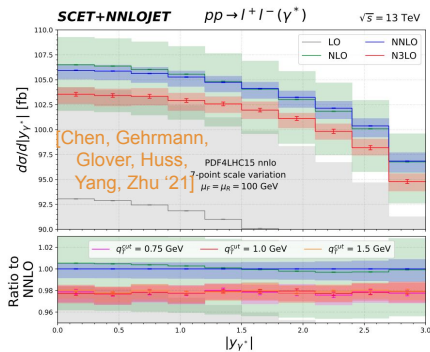
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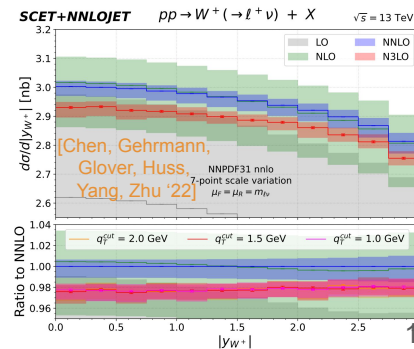


## Differential / Fiducial Cross Sections



Order	$\sigma$ [pb] Symmetric cuts		$\sigma$ [pb] Product cuts	
	$N^k\text{LO}$	$N^k\text{LO}+N^k\text{LL}$	$N^k\text{LO}$	$N^k\text{LO}+N^k\text{LL}$
0	$721.16^{+12.2\%}_{-13.2\%}$	—	$721.16^{+12.2\%}_{-13.2\%}$	—
1	$742.80(1)^{+2.7\%}_{-3.9\%}$	$748.58(3)^{+3.1\%}_{-10.2\%}$	$832.22(1)^{+2.7\%}_{-4.5\%}$	$831.91(2)^{+2.7\%}_{-10.4\%}$
2	$741.59(8)^{+0.42\%}_{-0.71\%}$	$740.75(5)^{+1.15\%}_{-2.66\%}$	$831.32(3)^{+0.59\%}_{-0.96\%}$	$830.98(4)^{+0.74\%}_{-2.73\%}$
3	$722.9(1.1)^{+0.68\%}_{-1.09\%} \pm 0.9$	$726.2(1.1)^{+1.07\%}_{-0.77\%}$	$816.8(1.1)^{+0.45\%}_{-0.73\%} \pm 0.8$	$816.6(1.1)^{+0.87\%}_{-0.69\%}$

[Chen, Gehrmann, Glover, Huss, Monni, Re, Rottoli, Torrielli '22]



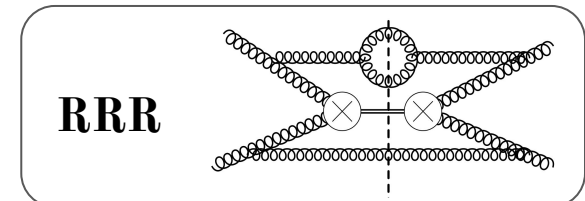
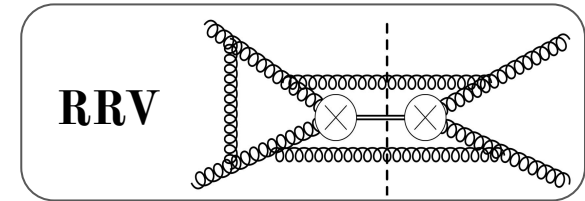
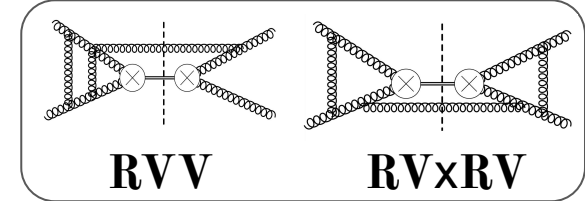
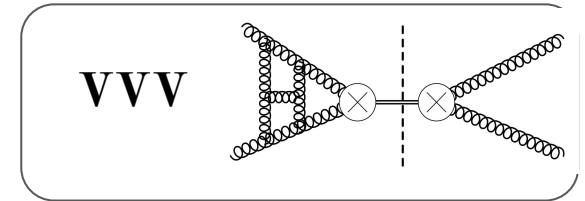
# We really need N3LO cross sections for 1% accuracy

- Numerous cases now show that **N3LO corrections can easily exceed 1%**.
- This, at the very least, indicates that **to claim percent level accuracy** in QCD sensitive observables **N3LO corrections**, or a very good quantitative estimate of them, **must be included**.
- Therefore, a key aspect of the precision program at the HL-LHC will be the ability to **systematically incorporate N3LO contributions**.

# Cross Sections in Perturbative QCD

$$\sigma = f_1 \circ f_2 \int d\Phi |M|^2$$

- Cross sections for LHC processes are obtained via **phase space integrals** over **amplitudes** (squared) convoluted with **Parton Distribution Functions (PDFs)**
- **IR divergences** at intermediate steps of the calculation **cancel only after summing** over all **real** and **virtual** contributions
- The complexity of cancellations grows dramatically with higher orders, making **systematization** of cross section calculations at NNLO very challenging and at **N3LO a monumental undertaking**



# Slicing methods

- One way to deal with IR singularities for cross sections are **slicing methods**
- The idea behind is quite simple. Take the production of color singlet  $q$  at N<sup>3</sup>LO as example.
  - Find an observable  $x$  that isolates the Born configuration in a the region where the observable vanishes (think for example at the transverse momentum of  $q$ )
  - Reorganize the cross section separating out the region around the Born configurations

$$\sigma_q^{\text{N}^3\text{LO}}(\mathcal{O}) = \int_0^{x_{\text{cut}}} dx \frac{d\sigma_q^{\text{N}^3\text{LO}}}{dx}(\mathcal{O}) + \int_{x_{\text{cut}}}^{x_{\text{max}}} dx \frac{d\sigma_{q+\text{jet}}^{\text{NNLO}}}{dx}(\mathcal{O})$$

## Below the cut region:

Only region where **genuine N<sup>3</sup>LO cancellation of IR divergences** is necessary

## Above the cut region:

- Resolved extra radiation => no events in Born configuration
- From a IR point of view this is an NNLO problem, so no N<sup>3</sup>LO subtraction needed to get this term

# Slicing methods

- One way to deal with IR singularities for N<sup>3</sup>LO is to slice the cross section
- The idea behind is quite simple. Take the cross section and slice it into two regions
  - Find an observable  $x$  that isolates the singularities and vanishes (think for example at the threshold)
  - Reorganize the cross section separating the singularities from the rest

Approximate the full distribution in  $x$  below the cut with its **Leading Power term** (obtained via resummation / factorization theorem in SCET)

$$\int_0^{x_{\text{cut}}} dx \frac{d\sigma_q^{\text{N}^3\text{LO}}}{dx}(\mathcal{O}) = \int_0^{x_{\text{cut}}} dx \frac{d\sigma_{q,(\text{LP})}^{\text{N}^3\text{LO}}}{dx}(\mathcal{O}) + \dots$$

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- Resolved extra radiation  $\Rightarrow$  no events in Born configuration
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# Slicing methods

$$\sigma_q(\mathcal{O}) = \int_0^{x_{\text{cut}}} dx \frac{d\sigma_q^{\text{sub}}}{dx}(\mathcal{O}) + \int_{x_{\text{cut}}}^{x_{\text{max}}} dx \frac{d\sigma_{q+\text{jet}}}{dx}(\mathcal{O}) + \int_0^{x_{\text{cut}}} dx \left[ \frac{d\sigma_q}{dx}(\mathcal{O}) - \frac{d\sigma_q^{\text{sub}}}{dx}(\mathcal{O}) \right]$$

$$\equiv \sigma^{\text{sub}}(x_{\text{cut}}, \mathcal{O}) + \sigma^{\text{above}}(x_{\text{cut}}, \mathcal{O}) + \Delta\sigma(x_{\text{cut}}, \mathcal{O})$$

## Below the cut region:

- Singular distribution
- Contains most complicated cancellation of IR divergences
- Control it analytically via factorization theorems

## Above the cut region:

- Resolved extra radiation
- Calculate with lower order subtraction schemes for process with jet (e.g. NNLOjet)

## Slicing Residual/Error:

Neglected non singular terms from below the cut (*power correction*).  
Minimized by going to very small values of cut parameter

**$q_T$  Subtraction:** [Catani, Grazzini '07]

**N-Jettiness Subtraction:** [Boughezal, Focke, Liu, Petriello '15]  
[Gaunt, Stahlhofen, Tackmann, Walsh '15]

- Extremely successful program for many color singlet (and top) processes at **NNLO** [MATRIX Collaboration] [DYTurbo]
- With **N-Jettiness** (or  $k_T$ -ness) ability to tackle also processes with **jets in the final state**

# Slicing, non-local subtractions, and local subtractions

- **In principle**, slicing methods are different from standard subtraction methods since the residual power corrections constitute an intrinsic error that is always present, which is not the case for a subtraction method.
- However, still in principle, with the exact same theoretical ingredients, one can very easily write a **(non-local) subtraction scheme** with **no residual power corrections**

$$\sigma_q(\mathcal{O}) = \int_0^{x_{\max}} dx \left[ \underbrace{\frac{d\sigma_q^{\text{full}}}{dx}(\mathcal{O})}_{\text{real contribution}} - \underbrace{\frac{d\sigma_q^{\text{sub}}}{dx}(\mathcal{O})}_{\text{counterterm}} \right] + \underbrace{\int_0^{x_{\text{cut}}} dx \frac{d\sigma_q^{\text{sub}}}{dx}(\mathcal{O})}_{\text{Integrated counterterm}}$$

- In practice, **every implementation of a subtraction scheme** (local, non-local, slicing) has technical cut-offs that lead to the **neglect of subleading power terms**
- For standard NLO calculations, technical cutoffs of  $\sim 10^{-6}$  in local subtractions are more than enough. For complicated NNLO final states and at N3LO these aspects are not as clear cut.

# Extending Slicing to N3LO

- **Singular region** (i.e. below the cut) can be understood at all orders via *Leading power factorization theorems* in Soft and Collinear Effective Theory (SCET). For example  $\mathbf{q}_T$

$$\frac{d\sigma}{dQ^2 dY d^2\vec{q}_T} = \sigma_0 \sum_{i,j} \boxed{H_{ij}(Q^2, \mu)} \int d^2\vec{b}_T e^{i\vec{q}_T \cdot \vec{b}_T} \underbrace{\tilde{B}_i\left(x_1^B, b_T, \mu, \frac{\nu}{\omega_a}\right) \tilde{B}_j\left(x_2^B, b_T, \mu, \frac{\nu}{\omega_b}\right)}_{\mathbf{q}_T \text{ Beam Functions}} \boxed{\tilde{S}(b_T, \mu, \nu)}$$

Hard Function
Soft Function

**N3LO ingredients  $\mathbf{q}_T$ :**

[Gehrmann, Glover, Huber, Iikizlerli, Studerus '10]

**“TMDPDFs at N3LO”**

M.Ebert, B.Mistlberger,  
GV [2006.05329]

**“Quark Transverse Parton Distribution at N3LO”**

[Luo, Yang, Zhu, Zhu '19]

[Li, Zhu '16]

**N3LO ingredients  
0-jettiness:**

[Gehrmann, Glover, Huber, Iikizlerli, Studerus '10]

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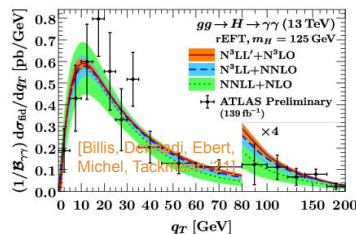
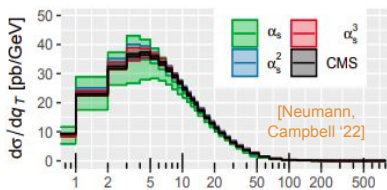
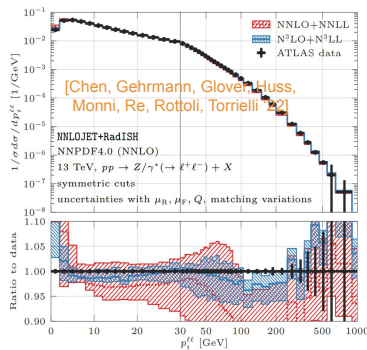
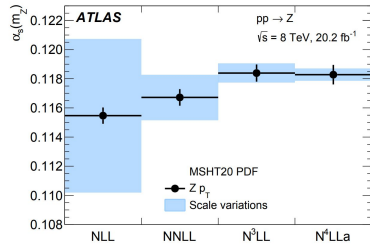
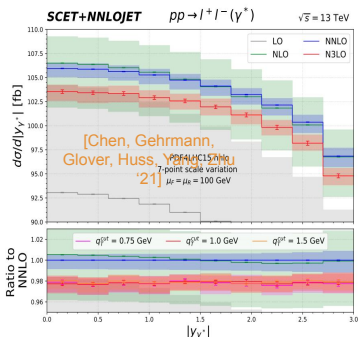
**“Zero-jettiness soft function to third order in perturbative QCD”**

[Baranowski, Delto, Melnikov, Pikelner, Wang '24]

# Precision Standard Model Phenomenology at N3LO

- **N3LO TMDPDF** were last missing ingredient for  $q_T$  slicing at N3LO
- Enabled N3LO predictions for differential and fiducial Drell-Yan and Higgs production

- Marked the advent of a new level of accuracy for the precision program at the LHC



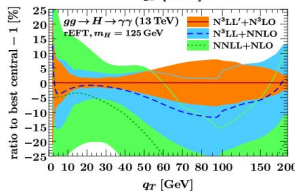
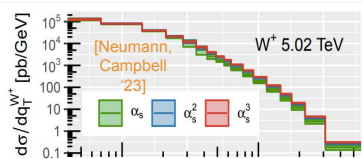
And many more:

[Ju, Schönherr '21]

[Camarda, Cieri, Ferrera '21]

[Re, Rottoli, Torrielli '21]

...

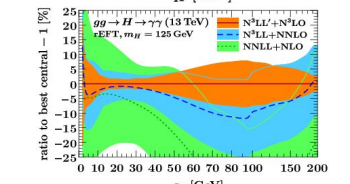
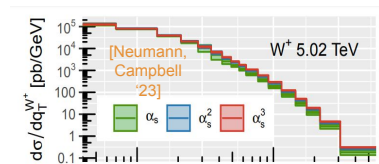
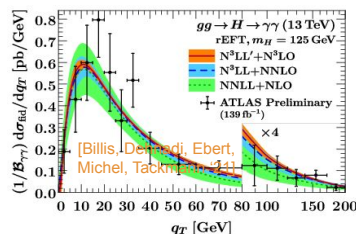
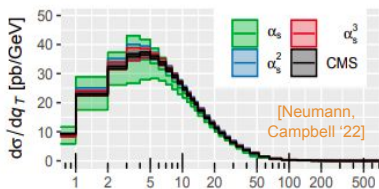
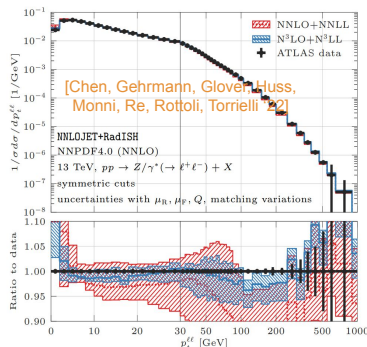
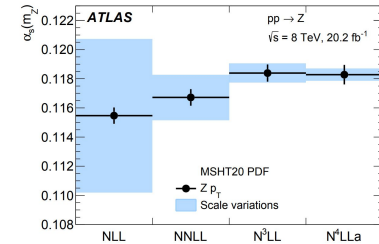
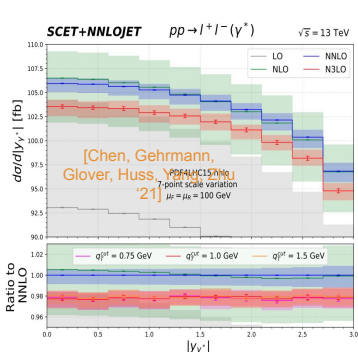


# Precision Standard Model Phenomenology at N3LO

- **N3LO TMDPDF** were last missing ingredient for  $q_T$  slicing at N3LO
- Enabled N3LO predictions for differential and fiducial Drell-Yan and Higgs production
- Marked the advent of a new level of accuracy for the precision program at the LHC

## However...

- Numerical (slicing) error of these methods very difficult to control at this order
- Extreme push of NNLO+j predictions well into the IR needed (NNLOjet pushed to  $q_T = 0.5$  GeV)
- Calculations take **O(10 million) CPU hours**
- Almost any change will require to run everything from scratch
- Other results use O(100k) CPU hours and stop at 5 GeV... this requires very delicate extrapolation to 0 to obtain finite results.
- Going forward, these facts pose issues for the practical usability of these predictions



And many more:

[Ju, Schönherr '21]

[Camarda, Cieri, Ferrera '21]

[Re, Rottoli, Torrielli '21]

...

In short, starting to think about how to move from  
making fully differential N3LO predictions **possible**,

to

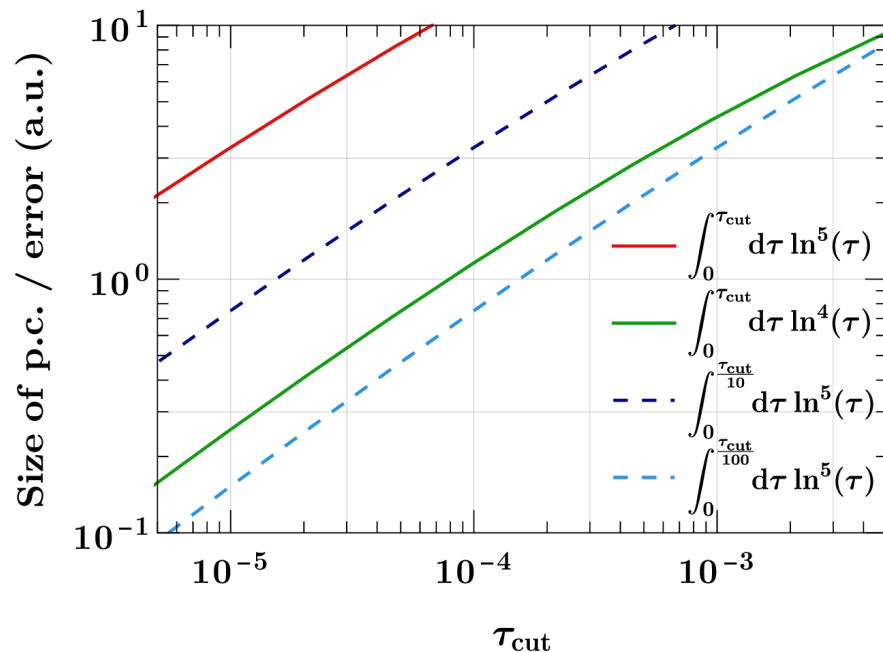
making N3LO predictions (more) **efficient, stable, and usable**

(at least for some color singlet processes...which may also turn out to be a necessary stepping  
stone to make other processes possible at N3LO)

# Improving non-local subtraction methods: Power corrections

$$\Delta\sigma(\tau_{\text{cut}}) = \int_0^{\tau_{\text{cut}}} d\tau \left[ \frac{d\sigma}{d\tau} - \frac{d\sigma^{\text{sub}}}{d\tau} \right]$$

- At N3LO power corrections start with **5th power of log**
- Taking  $\tau_{\text{cut}}$  small reduces single power, but increases size of log  $\Rightarrow$  very slow convergence
- Each order in the log equivalent to  $\sim$  a 10 fold reduction in  $\tau_{\text{cut}}$



$$\Delta\sigma^{N3LO}(\tau_{\text{cut}}) \sim \left(\frac{\alpha_s}{4\pi}\right)^3 \int_0^{\tau_{\text{cut}}} d\tau \left( c_{3,5}^{\text{NLP}} \ln^5 \tau + c_{3,4}^{\text{NLP}} \ln^4 \tau + c_{3,3}^{\text{NLP}} \ln^3 \tau + \dots \right)$$

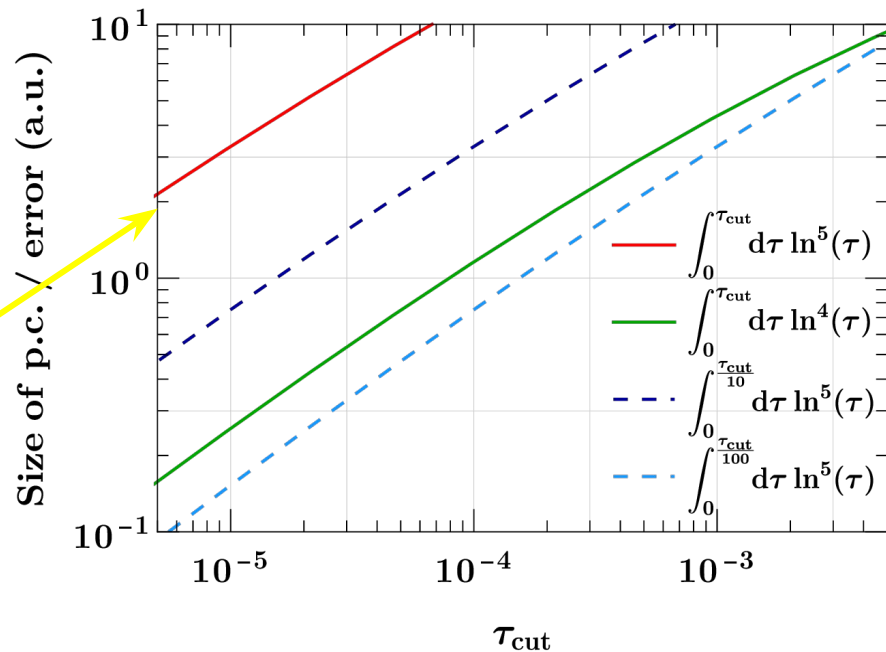
# Improving non-local subtraction methods: Power corrections

$$\Delta\sigma(\tau_{\text{cut}}) = \int_0^{\tau_{\text{cut}}} d\tau \left[ \frac{d\sigma}{d\tau} - \frac{d\sigma^{\text{sub}}}{d\tau} \right]$$

- At N<sup>3</sup>LO, the size of the non-local subtraction error is a 10 fold reduction
- Taking into account the power corrections in the subtraction method
- Each power correction term is a 10 fold reduction in error

Very straightforward way of improving slicing:

Obtain the leading logarithmic term at NLP analytically



$$\Delta\sigma^{N3LO}(\tau_{\text{cut}}) \sim \left(\frac{\alpha_s}{4\pi}\right)^3 \int_0^{\tau_{\text{cut}}} d\tau \left( c_{3,5}^{\text{NLP}} \ln^5 \tau + c_{3,4}^{\text{NLP}} \ln^4 \tau + c_{3,3}^{\text{NLP}} \ln^3 \tau + \dots \right)$$

# 0-Jettiness Power Corrections at N3LO [GV 2401.03017]

$$\Delta\sigma^{N3LO}(\tau_{\text{cut}}) \sim \left(\frac{\alpha_s}{4\pi}\right)^3 \int_0^{\tau_{\text{cut}}} d\tau \left( c_{3,5}^{\text{NLP}} \ln^5 \tau + c_{3,4}^{\text{NLP}} \ln^4 \tau + c_{3,3}^{\text{NLP}} \ln^3 \tau + \dots \right)$$

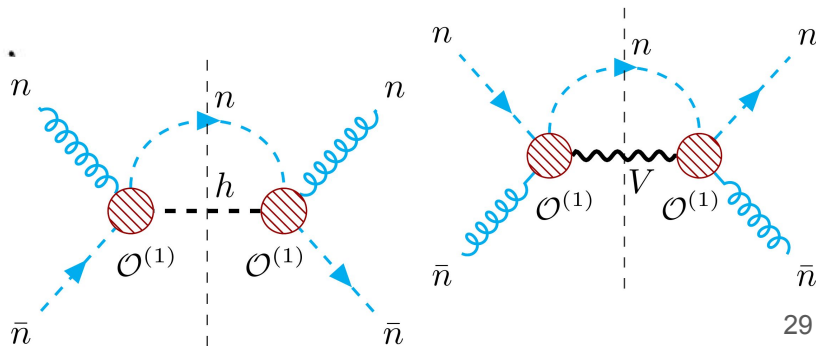
- For 0-jettiness, use consistency relations to relate full LL to RVV correction in collinear limit.

[Moult, Rothen, Stewart, Tackmann, Zhu '16] [Moult, Stewart, GV, Zhu '19]

- Focus on Drell-Yan and Higgs production. Single collinear emission fully differential in rapidity:

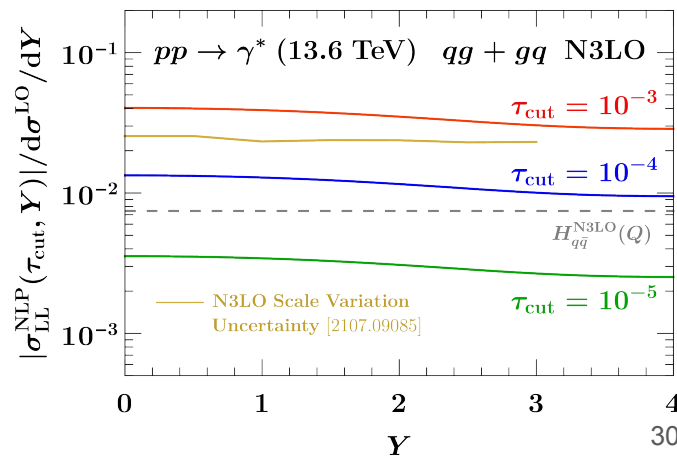
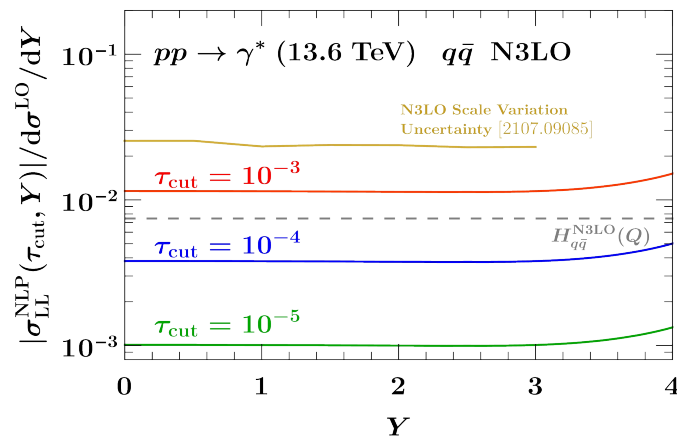
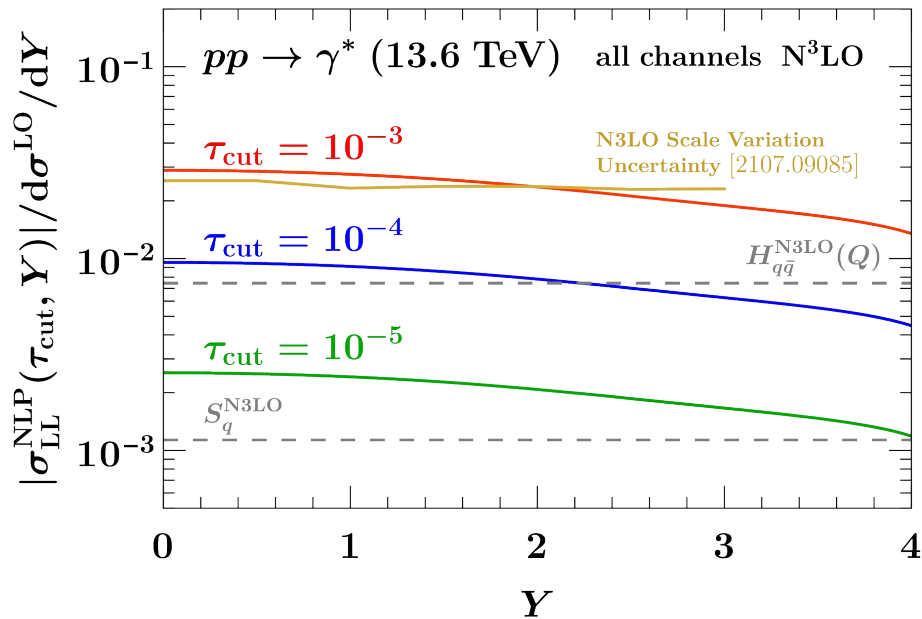
$$\frac{d\sigma_n^{\text{NLP}}}{dQ^2 dY d\mathcal{T}} \sim \int_{x_a}^1 \frac{dz_a}{z_a} \frac{(Q^2 \tau)^{-\epsilon}}{(1-z_a)^\epsilon} \left\{ \underbrace{\tau A^{(0)}(\tau, z_a, \epsilon)}_{\text{LP Matrix Element}} \left[ \underbrace{-f_a\left(\frac{x_a}{z_a}\right) f_b(x_b) + f_a\left(\frac{x_a}{z_a}\right) x_b f_b'(x_b)}_{\text{NLP Phase Space}} \right] \right. \\ \left. + \underbrace{f_a\left(\frac{x_a}{z_a}\right) f_b(x_b)}_{\text{LP Phase Space}} \underbrace{A^{(2)}(\tau, z_a, \epsilon)}_{\text{NLP Matrix Element}} \right\}.$$

[Ebert, Moult, Stewart, Tackmann, GV, Zhu '18]



- LL contributions also from off-diagonal  $gg + gg$  channels via subleading power hard scattering operators and Lagrangian insertions

# 0-Jettiness Power Corrections at N3LO: Results for DY



- By the size of LL NLP: 0-jettiness with standard setup (only LP in subtraction term) would require  $\tau_{\text{cut}} \sim 10^{-5}$  or even smaller.
- Off-diagonal channel has large power corrections (in line with empirical observation in  $q_T$  slicing at N3LO)

Ok, but what about fiducial power corrections?

# Fiducial Power Corrections

- These are **purely kinematic effects**, but have very **large impact** on non-local subtractions due to non canonical scaling in the cut parameter.

- In short: (Ebert, Tackmann) [1911.08486]
 
$$\frac{d\sigma^{(\text{cuts})}(X)}{dQ^2 dY dq_T^2} \sim \frac{1}{q_T^2} \frac{q_T}{Q}, \quad \frac{d\sigma^{(\text{cuts})}(X)}{dQ^2 dY d\mathcal{T}_0} \sim \frac{1}{\mathcal{T}_0} \sqrt{\frac{\mathcal{T}_0}{Q}}.$$
  - **Cuts on leptons** induce *linear* terms

For  $q_T$  subtraction they can be captured analytically by a boost, but not for 0-jettiness.

- **Photon Isolations** induce p.c. with wild and complicated scaling
 
$$\frac{d\sigma^{(\text{smooth})}(X)}{dQ^2 dY dq_T^2} \sim \frac{R^2}{q_T^2} \left(\frac{q_T}{Q}\right)^{1/n} \left(\frac{Q}{E_T^{\text{iso}}}\right)^{1/n}$$

No simple boost trick to account for them.

$$\frac{d\sigma^{(\text{smooth})}(X)}{dQ^2 dY d\mathcal{T}_0} \sim \begin{cases} \frac{R^2}{\mathcal{T}_0} \left(\frac{\mathcal{T}_0}{Q}\right)^{1+1/(2n)} \left(\frac{Q}{E_T^{\text{iso}}}\right)^{1/n} \\ \frac{R^2}{\mathcal{T}_0} \left(\frac{\mathcal{T}_0}{Q}\right)^{1/n} \left(\frac{Q}{E_T^{\text{iso}}}\right)^{1/n} \end{cases}$$

- So, although fiducial power corrections are more trivial conceptually, account for them comes first numerically compared to dynamical power corrections.

# Projection to Born Improved Slicing

[Cacciari et al. '15]  
 [Ebert, Tackmann '19]  
 [GV '24]  
 [Campbell, Neumann, GV '24]

Cut-induced power corrections can be numerically accounted for by using  
**“Projection-to-Born Improved Slicing”**

$$\sigma_{h, \text{N}^3\text{LO}}(\mathcal{O}) = \sigma_{h, \text{N}^3\text{LO}}(\tilde{\mathcal{O}}) + \sigma_{h+j, \text{NNLO}}(\mathcal{O} - \tilde{\mathcal{O}}) \quad \text{P2B correction factor}$$

Slicing calculation for  
 Born projected  
 observable

$$= \int_0^{\tau_{\text{cut}}} d\tau \frac{d\sigma_{h, \text{N}^3\text{LO}}^{\text{sub}}}{d\tau}(\tilde{\mathcal{O}}) + \int_{\tau > \tau_{\text{cut}}} d\sigma_{h+j, \text{NNLO}}^{\text{full}}(\tilde{\mathcal{O}})$$

Below the cut term Above the cut term

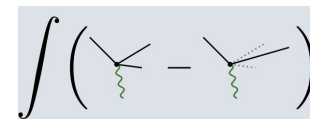
## Note:

Because of local cancellation using exact matrix elements, P2B is very efficient numerically.

Sometimes referred as the “perfect” subtraction scheme

$$+ \int_0^{\tau_{\text{cut}}} d\tau \left[ \frac{d\sigma_{h, \text{N}^3\text{LO}}^{\text{full}}}{d\tau} - \frac{d\sigma_{h, \text{N}^3\text{LO}}^{\text{sub}}}{d\tau} \right](\tilde{\mathcal{O}}) \quad \text{Residual Error}$$

$$+ \int d\sigma_{h+j, \text{NNLO}}^{\text{full}}(\mathcal{O} - \tilde{\mathcal{O}}) \quad \text{P2B correction factor}$$



# Projection to Born Improved Slicing

[Cacciari et al. '15]  
[Ebert, Tackmann '19]  
[GV '24]  
[Campbell, Neumann, GV '24]

Equivalently, perform standard slicing and correct with P2B only below the cut

$$\begin{aligned}\sigma_{h, N^n \text{LO}}(\mathcal{O}) &= \int_0^{x_{\text{cut}}} dx \frac{d\sigma_{h, N^n \text{LO}}^{\text{sub}}(\mathcal{O})}{dx} + \int_{x > x_{\text{cut}}} d\sigma_{h+j, N^{n-1} \text{LO}}^{\text{full}}(\mathcal{O}) \\ &\quad \text{Below the cut term} \qquad \qquad \qquad \text{Above the cut term} \\ &+ \int_0^{x_{\text{cut}}} dx \left[ \frac{d\sigma_{h, N^n \text{LO}}^{\text{full}}}{dx} - \frac{d\sigma_{h, N^n \text{LO}}^{\text{sub}}}{dx} \right] (\tilde{\mathcal{O}}) \quad \text{Residual Error} \\ &+ \int_0^{x_{\text{cut}}} d\sigma_{h+j, N^{n-1} \text{LO}}^{\text{full}}(\mathcal{O} - \tilde{\mathcal{O}}). \quad \text{P2B correction factor below the cut}\end{aligned}$$

# Projection to Born Improved Slicing

[Cacciari et al. '15]  
[Ebert, Tackmann '19]  
[GV '24]

This enable us to

- Focus on *analytic* calculation of *dynamical* power corrections
- *Numerically* treat *fiducial* power corrections efficiently with P2B method

Slicing calculation  
Born improved  
observables

correction factor

$\sigma(\tilde{\mathcal{O}})$   
sub term

## Note:

Because of local cancellation using exact matrix elements, P2B is very efficient numerically.

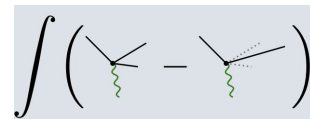
Sometimes referred as the “*perfect*” subtraction scheme

$$+ \int_0^{\tau_{\text{cut}}} d\tau \left[ \frac{d\sigma_{h, \text{N}^3\text{LO}}^{\text{full}}}{d\tau} - \frac{d\sigma_{h, \text{N}^3\text{LO}}^{\text{sub}}}{d\tau} \right] (\tilde{\mathcal{O}})$$

Residual Error

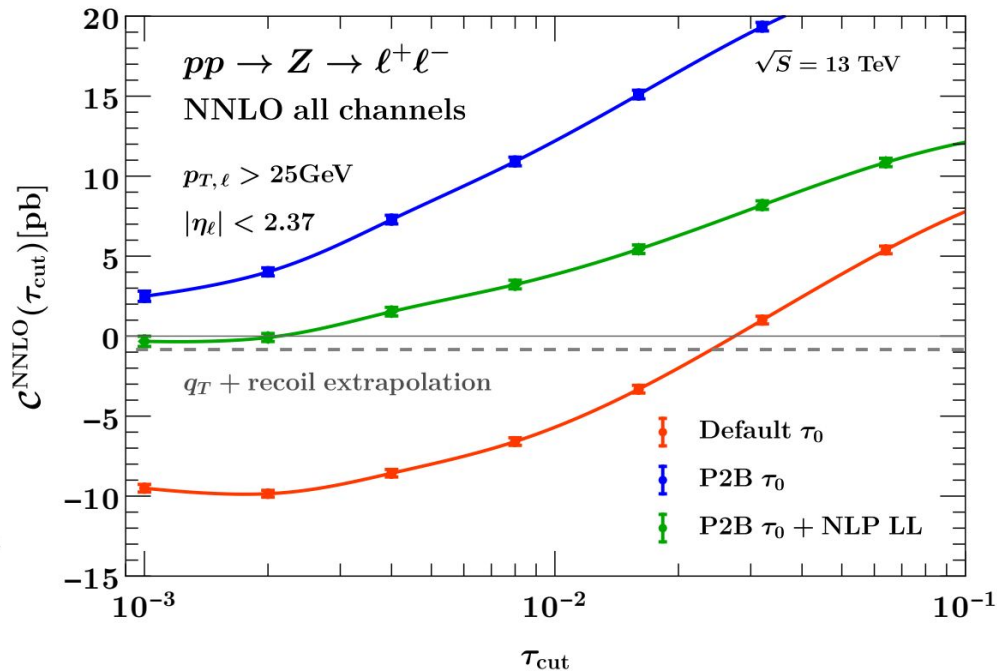
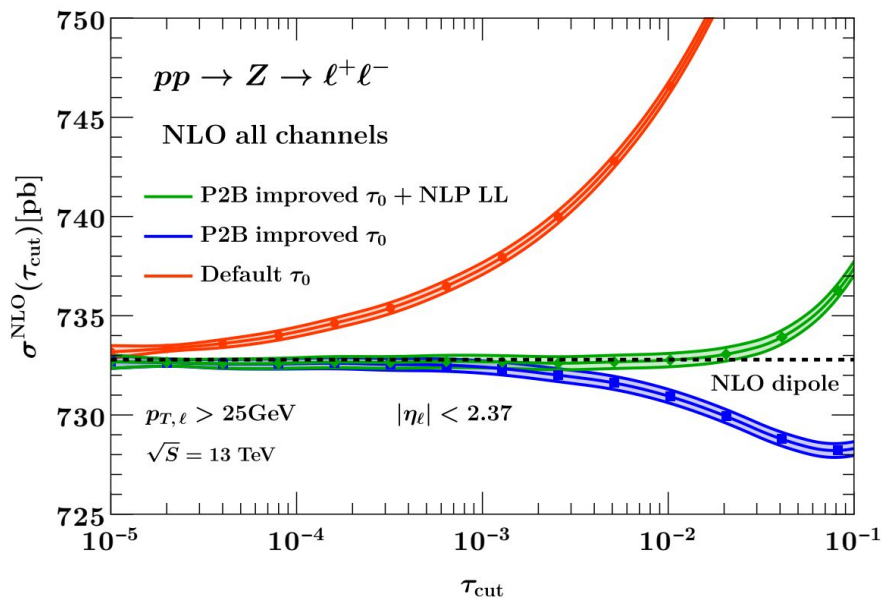
$$+ \int d\sigma_{h+j, \text{NNLO}}^{\text{full}} (\mathcal{O} - \tilde{\mathcal{O}})$$

P2B correction factor

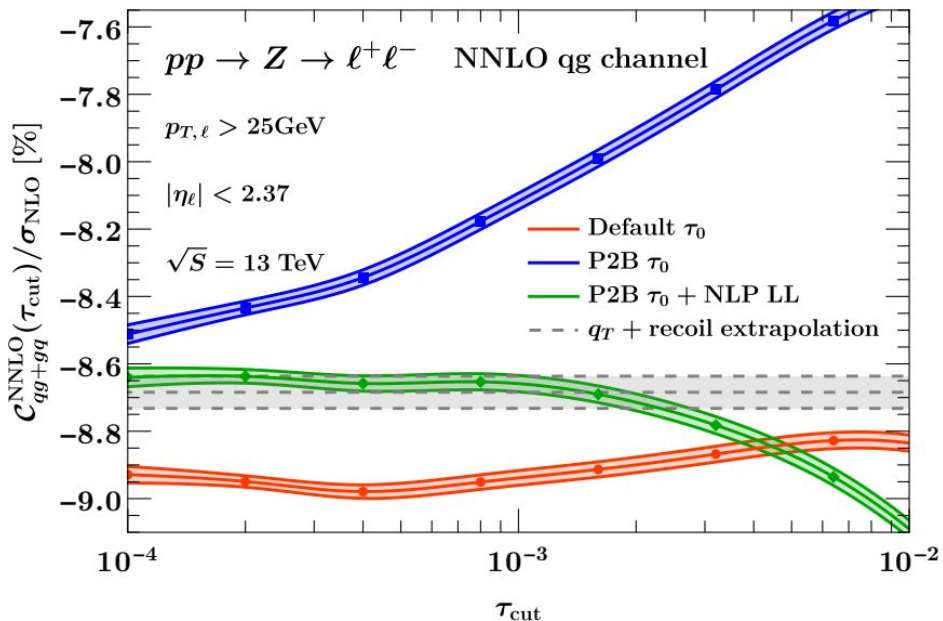
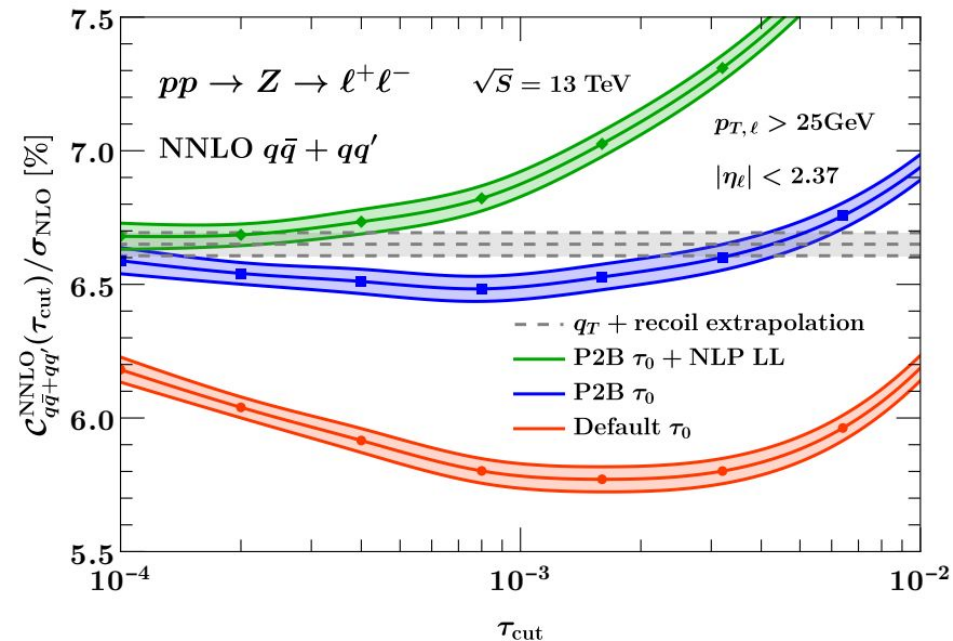


# Projection-to-Born-improved Subtractions at NNLO

We studied this at NNLO in MCFM in 2408.05265



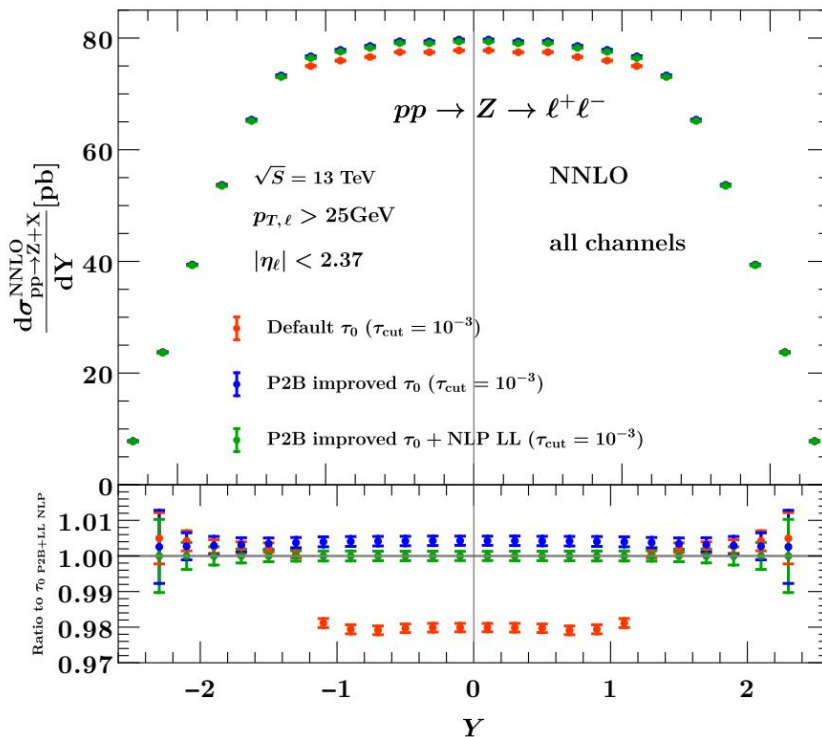
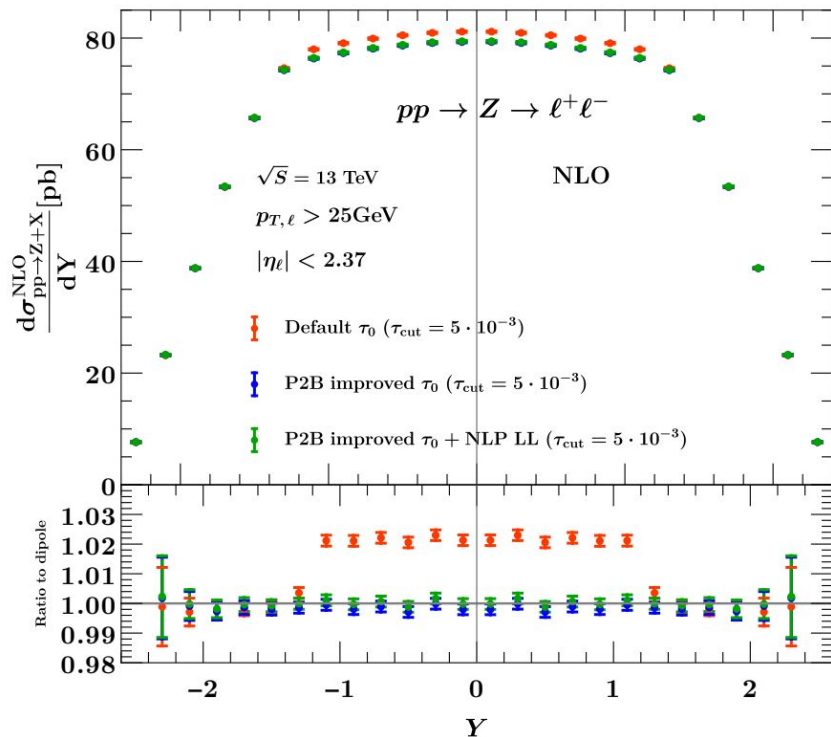
# Projection-to-Born-improved Subtractions at NNLO



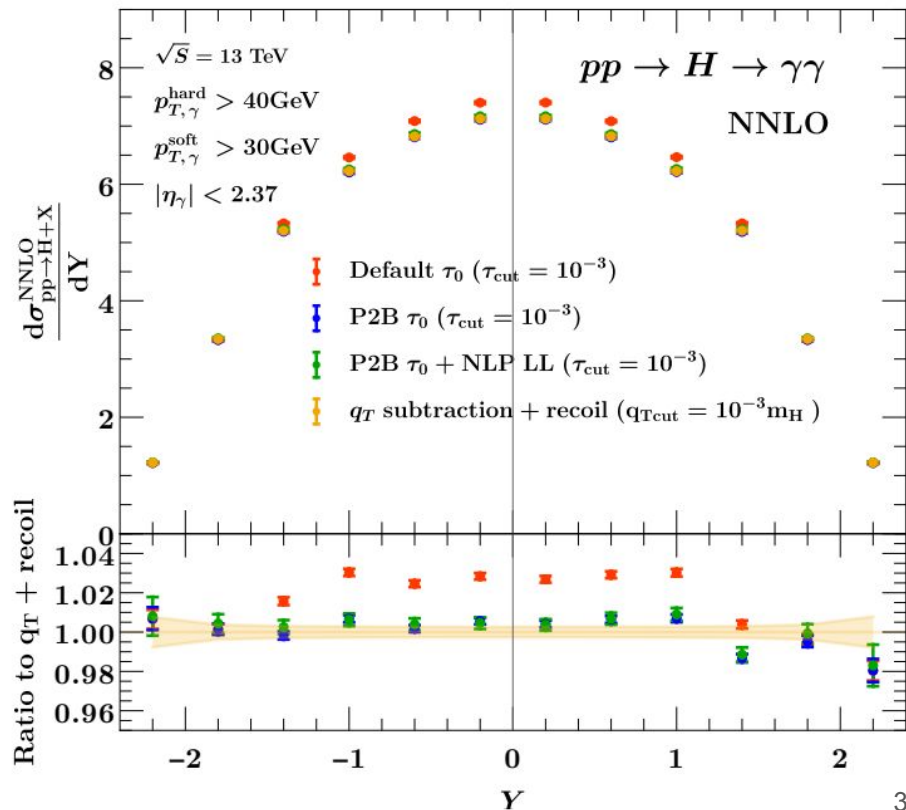
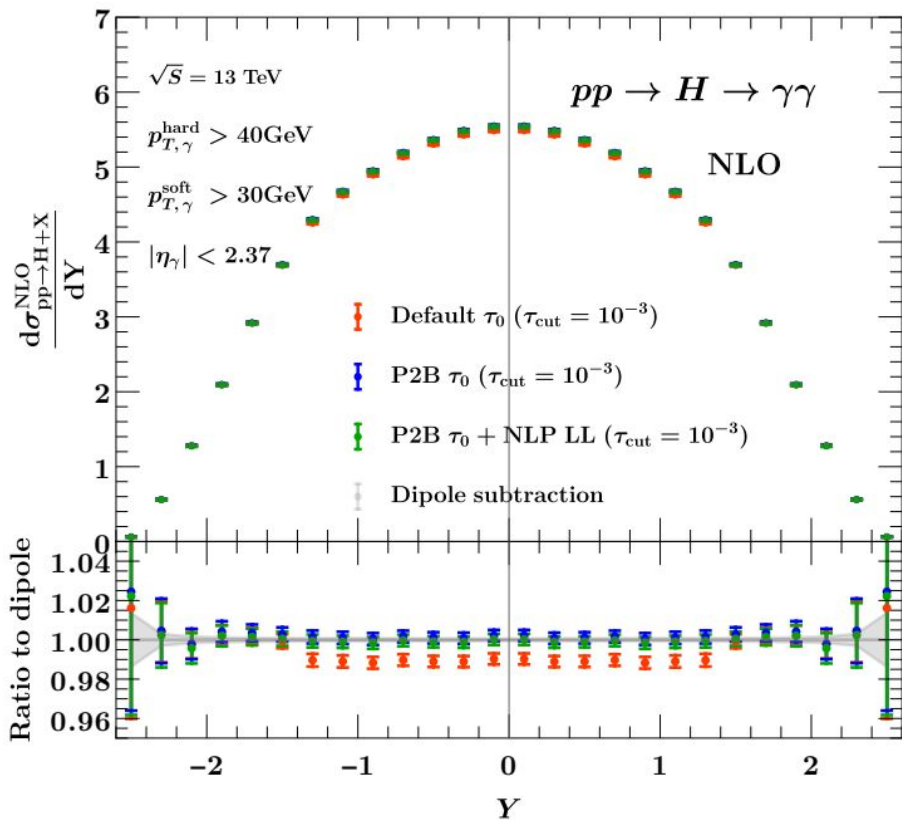
Extrapolation from  $q_T$  with recoil  
 in MCFM taken as reference  
 (excellent agreement with result  
 from MATRIX based on same  
 method)

$pp \rightarrow Z \rightarrow \ell^+ \ell^-$ NNLO coefficient	$q\bar{q} + qq'$	$qg$	$gg$
MCFM $q_T$ + recoil	$48\,732 \pm 316$ fb	$-31\,819 \pm 175$ fb	$13\,870 \pm 25$ fb
MATRIX $q_T$ + recoil	$48\,695 \pm 364$ fb	$-31\,798 \pm 131$ fb	$13\,786 \pm 205$ fb
Relative Difference	$0.08 \pm 0.99$ %	$0.07 \pm 0.69$ %	$0.61 \pm 1.53$ %

# Projection-to-Born-improved Subtractions at NNLO



# Projection-to-Born-improved Subtractions at NNLO

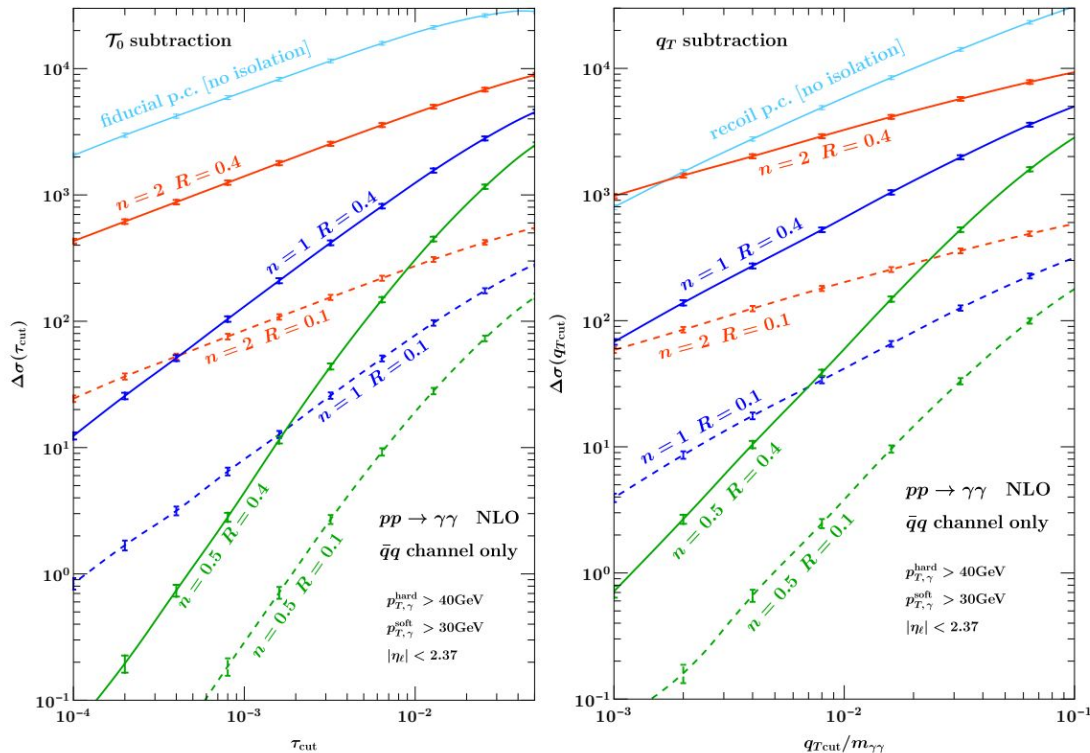


# Di-photon

- Di-photon is notoriously challenging
- Isolation prescription required to avoid QED singularity when quarks are in the final state

$$\sum_{d(i,\gamma)\leq r} E_T^i \leq E_T^{\text{iso}} \left[ \frac{1 - \cos(r)}{1 - \cos(R)} \right]^n, \quad \forall r \leq R$$

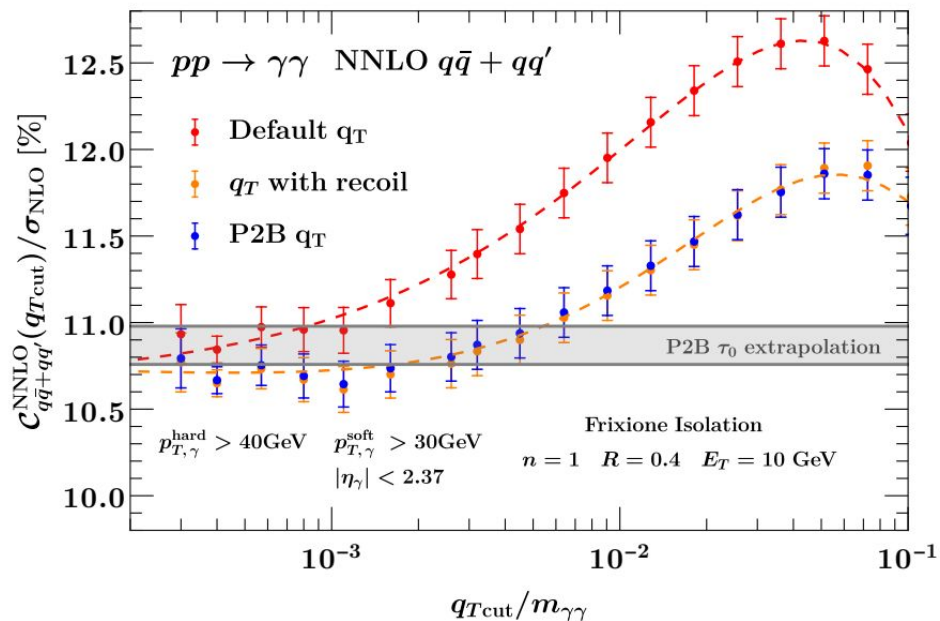
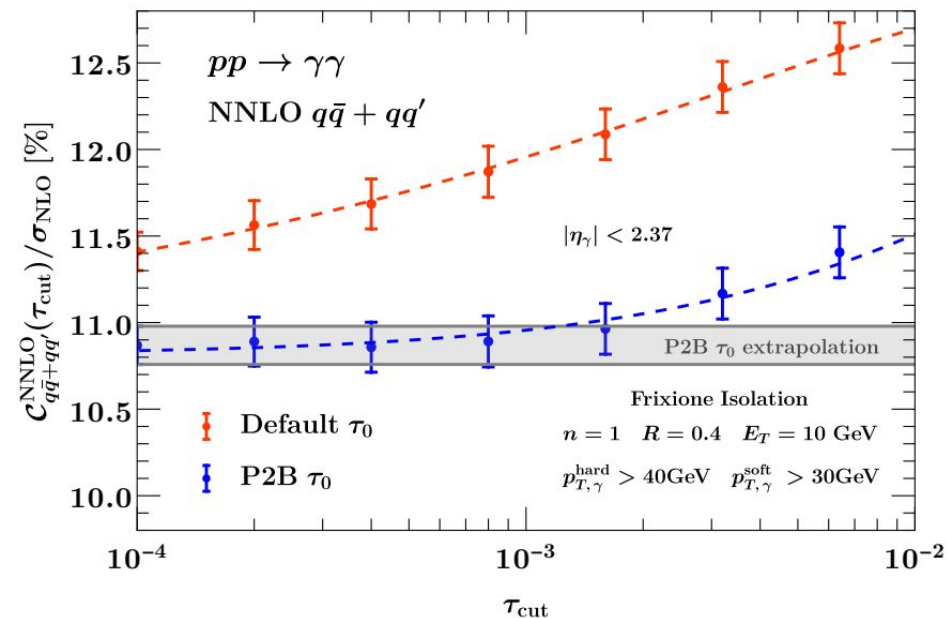
- Mix of isolation and kinematical cuts and large QCD corrections
- Large impact of the isolation parameters on the power corrections



$$\Delta\sigma_{p_T^{\gamma}+\text{iso},n=1}^{\text{fiducial+hadronic}} \sim \begin{cases} \sqrt{\frac{T_0^{\text{cut}}}{Q}} + \frac{T_0^{\text{cut}}}{Q} R^2 \frac{Q}{E_T^{\text{iso}}} + \frac{T_0^{\text{cut}}}{Q} \log(R) + \frac{T_0^{\text{cut}}}{Q}, & 0\text{-jettiness subtraction} \\ \frac{q_T^{\text{cut}}}{Q} + \frac{q_T^{\text{cut}}}{Q} R^2 \frac{Q}{E_T^{\text{iso}}} + \frac{q_T^{\text{cut}}}{Q} \log(R) + \frac{q_T^{\text{cut}}}{Q^2}, & q_T\text{-subtraction} \end{cases} \quad 40$$

# Di-photon at NNLO: quark - anti quark channels

- For this channel, the P2B and recoil capture the fiducial power corrections



# Di-photon at NNLO: fragmentation channel

- For this channel, the fiducial corrections due to the cuts on the of the photons are small, but the isolation corrections are very large

$$\sum_{d(i,\gamma)\leq r} E_T^i \leq E_T^{\text{iso}} \left[ \frac{1 - \cos(r)}{1 - \cos(R)} \right]^n, \quad \forall r \leq R$$

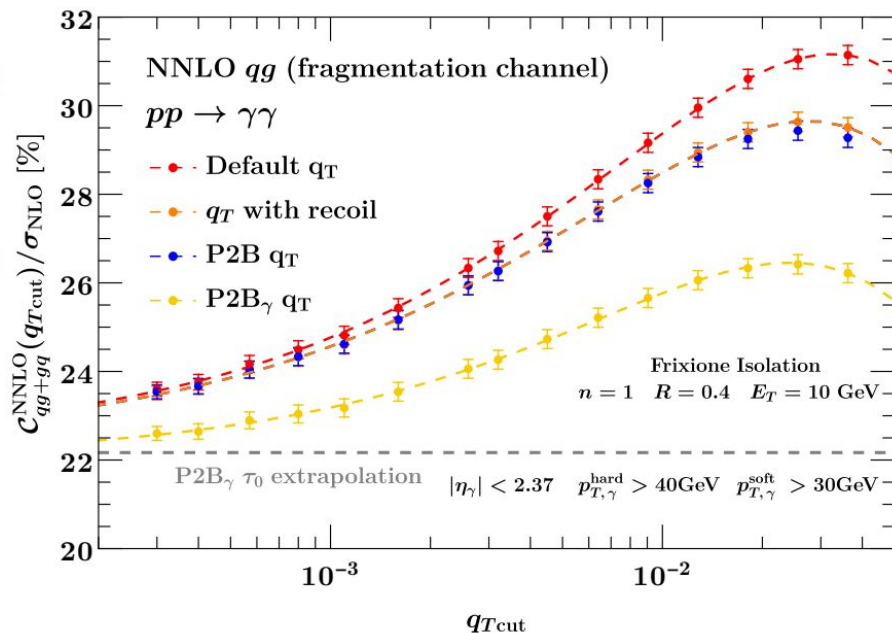
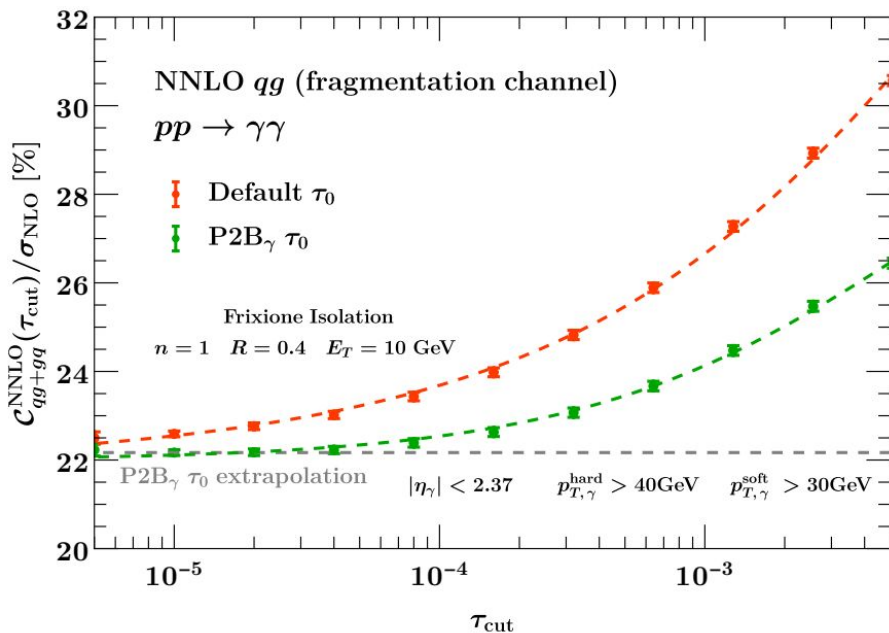
- To account for isolation p.c. we modified the P2B improved slicing by switching off the P2B counterterm. I'll refer to this prescription as **P2B<sub>Y</sub>**

$$\int_0^{x_{\text{cut}}} d\sigma_{h+j, N^{n-1}\text{LO}}^{\text{full}}(\mathcal{O} - \tilde{\mathcal{O}}) \longrightarrow \int d\sigma_{h+j, N^{n-1}\text{LO}}^{\text{full}}(\mathcal{O} - \tilde{\mathcal{O}}\theta(r > R))$$

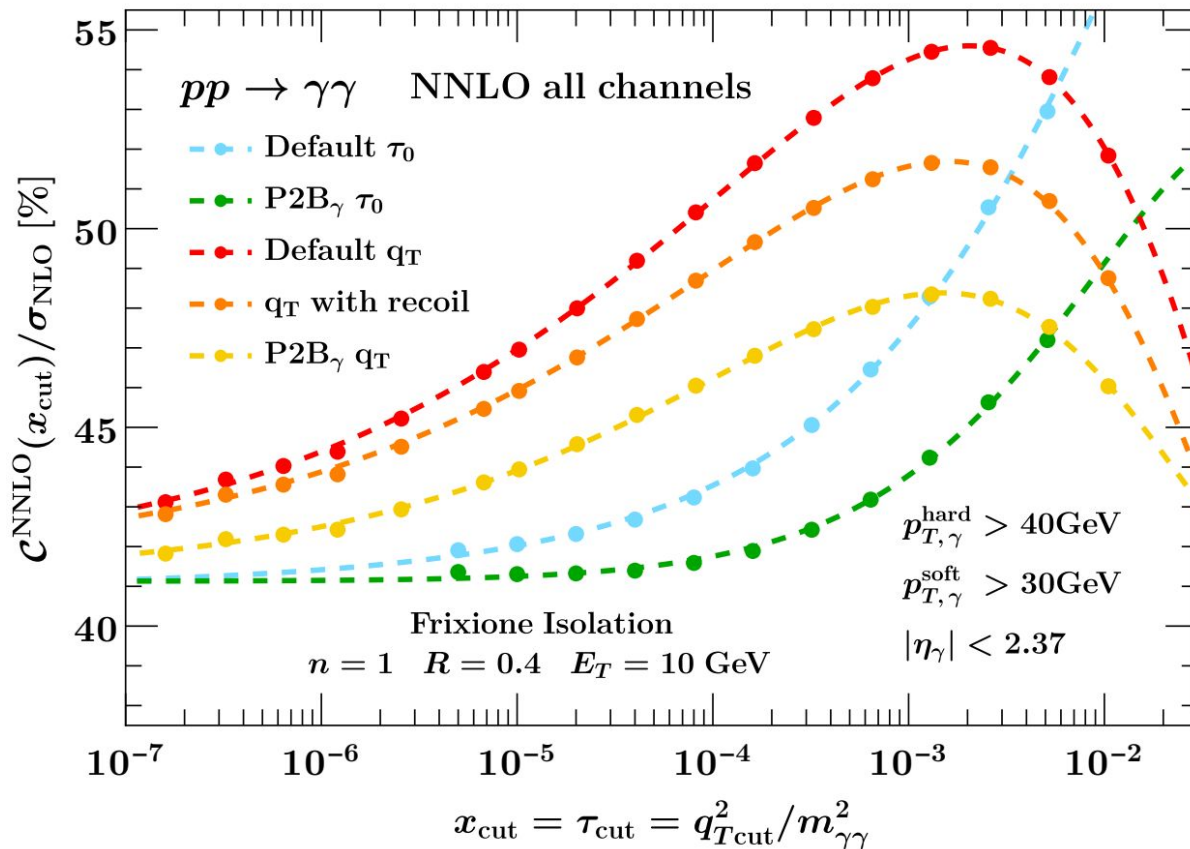
- The point is that for realistic values of the isolation parameters the quark inside the cone of radius  $R$  must be soft so one can directly calculate this contribution numerically without a counterterm.

# Di-photon at NNLO: fragmentation channel

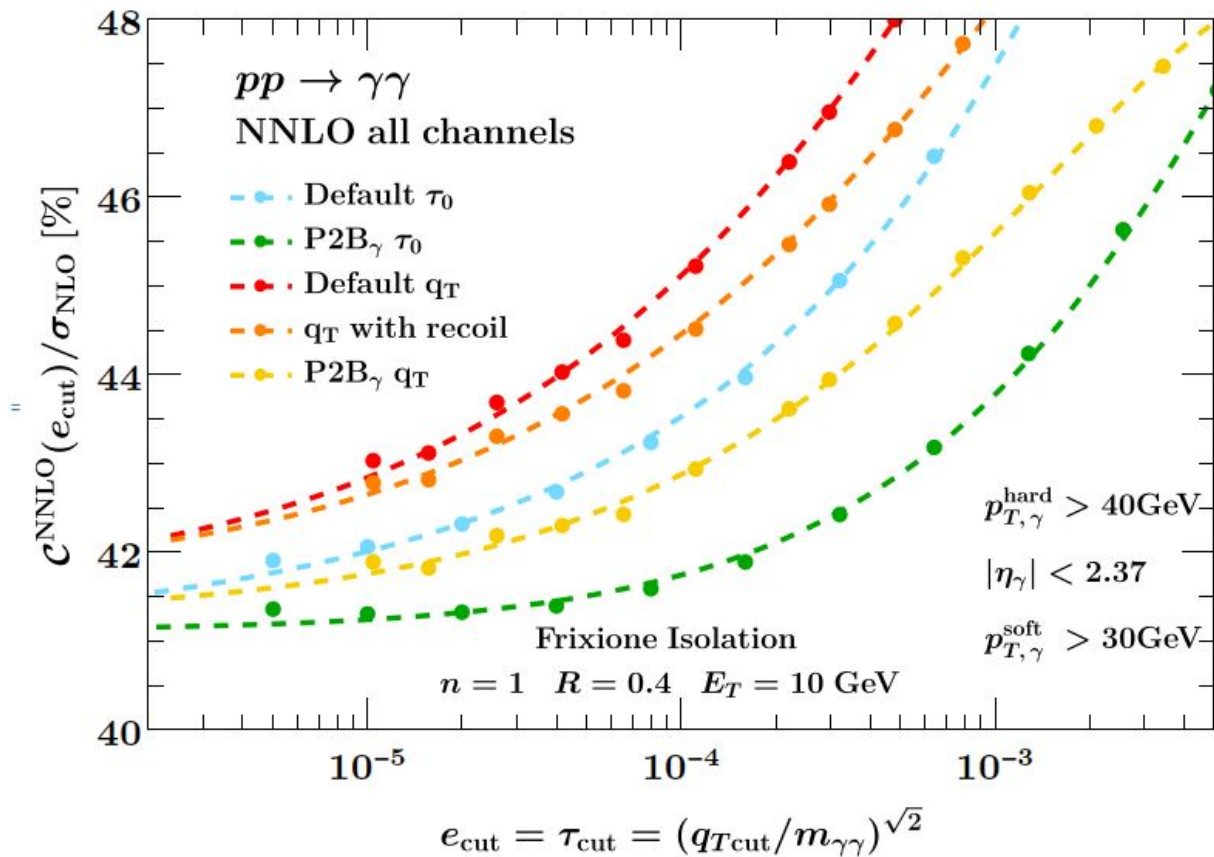
- For this channel, the P2B and recoil corrections are small
- Numerically it seems that  $\mathbf{P2B}_\gamma$  is capturing an important amount of the effect both for 0-jettiness as well as for  $q_T$  subtraction.



# Di-photon at NNLO: all channels



# Di-photon at NNLO: all channels



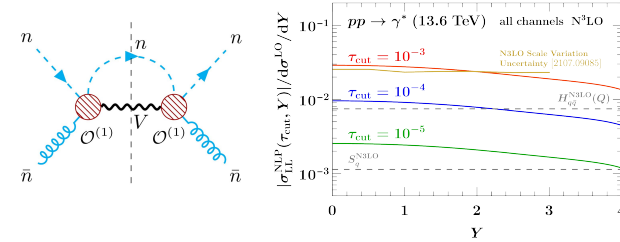
# Conclusion

➤ Discussed challenges of N3LO calculations and slicing methods

$$\sigma(X) = \int_0^{\tau_{\text{cut}}} dq_T \frac{d\sigma^{\text{sing}}(X)}{dq_T} + \int_{\tau_{\text{cut}}} dq_T \frac{d\sigma(X)}{dq_T} + \Delta\sigma(X, \tau_{\text{cut}})$$

Below the cut region      Above the cut region      Residual

➤ Illustrated impact of the LL power corrections at N3LO for 0-jettiness for Drell-Yan and Higgs production



➤ Used P2B improved slicing to account for fiducial power corrections

$$\sigma_{h, \text{N}^3\text{LO}}(\mathcal{O}) = \sigma_{h, \text{N}^3\text{LO}}(\tilde{\mathcal{O}}) + \sigma_{h+j, \text{NNLO}}(\mathcal{O} - \tilde{\mathcal{O}}) \quad \text{P2B correction factor}$$

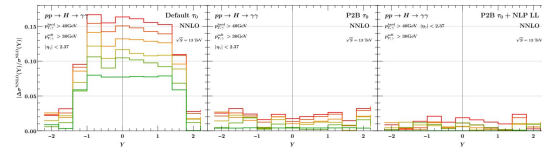
$$= \int_0^{\tau_{\text{cut}}} d\tau \frac{d\sigma_{h, \text{N}^3\text{LO}}^{\text{sub}}}{d\tau}(\tilde{\mathcal{O}}) + \int_{\tau > \tau_{\text{cut}}} d\sigma_{h+j, \text{NNLO}}^{\text{full}}(\tilde{\mathcal{O}})$$

Below the cut term      Above the cut term

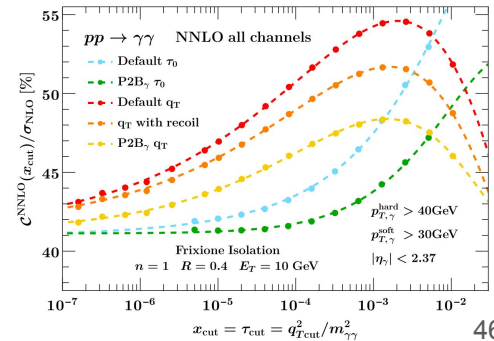
$$+ \int_0^{\tau_{\text{cut}}} d\tau \left[ \frac{d\sigma_{h, \text{N}^3\text{LO}}^{\text{full}}}{d\tau} - \frac{d\sigma_{h, \text{N}^3\text{LO}}^{\text{sub}}}{d\tau} \right](\tilde{\mathcal{O}}) \quad \text{Residual Error}$$

$$+ \int d\sigma_{h+j, \text{NNLO}}^{\text{full}}(\mathcal{O} - \tilde{\mathcal{O}}) \quad \text{P2B correction factor} \quad \int (\Upsilon - \tilde{\Upsilon})$$

Slicing calculation for Born projected observable



➤ Presented prescription for isolation corrections in Diphoton production due to soft quark emissions



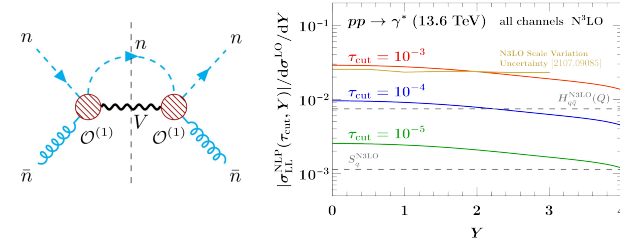
# Conclusion

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$$\sigma(X) = \int_0^{\tau_{\text{cut}}} dq_T \frac{d\sigma^{\text{sing}}(X)}{dq_T} + \int_{\tau_{\text{cut}}} dq_T \frac{d\sigma(X)}{dq_T} + \Delta\sigma(X, \tau_{\text{cut}})$$

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➤ Illustrated impact of the LL power corrections at N3LO for 0-jettiness for Drell-Yan and Higgs production



➤ Used P2B improved slicing to account for fiducial power corrections

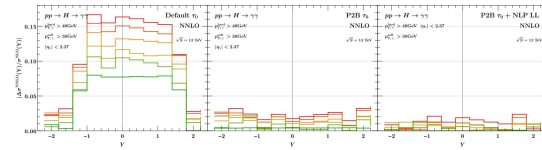
$$\sigma_{h, \text{N}^3\text{LO}}(\mathcal{O}) = \underbrace{\sigma_{h, \text{N}^3\text{LO}}(\tilde{\mathcal{O}})}_{\text{Slicing calculation for Born projected observable}} + \underbrace{\sigma_{h+j, \text{NNLO}}(\mathcal{O} - \tilde{\mathcal{O}})}_{\text{P2B correction factor}}$$

$$= \int_0^{\tau_{\text{cut}}} d\tau \frac{d\sigma_{h, \text{N}^3\text{LO}}^{\text{sub}}}{d\tau}(\tilde{\mathcal{O}}) + \int_{\tau > \tau_{\text{cut}}} d\sigma_{h+j, \text{NNLO}}^{\text{full}}(\tilde{\mathcal{O}})$$

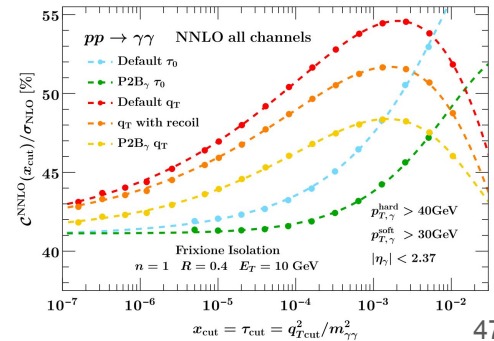
Below the cut term      Above the cut term

$$+ \int_0^{\tau_{\text{cut}}} d\tau \left[ \frac{d\sigma_{h, \text{N}^3\text{LO}}^{\text{full}}}{d\tau} - \frac{d\sigma_{h, \text{N}^3\text{LO}}^{\text{sub}}}{d\tau} \right](\tilde{\mathcal{O}}) \quad \text{Residual Error}$$

$$+ \int d\sigma_{h+j, \text{NNLO}}^{\text{full}}(\mathcal{O} - \tilde{\mathcal{O}}) \quad \text{P2B correction factor} \quad \int (\Upsilon - \tilde{\Upsilon})$$



➤ Presented prescription for isolation corrections in Diphoton production due to soft quark emissions

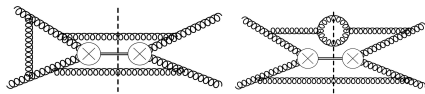


Thank you

**Backup**

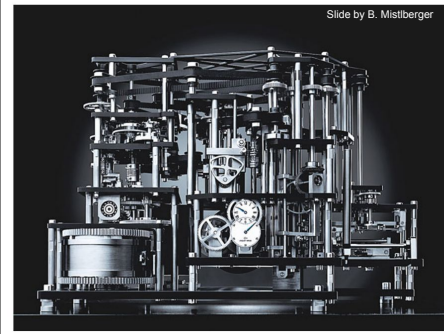
# Beam Functions at N3LO

1 million 3-loop Feynman Diagrams



THE CROSS SECTION CALCULATION MACHINE

Side by B. Mistlberger



Collinear expansion of the partonic cross section for  
Drell Yan and Higgs at N3LO differential in  $(Q_T, \tau, z)$

$$B_a(t_a, x_1^B, \mu)$$

project to  $\tau$

project to  $q_T$

$$\tilde{B}_i\left(x_1^B, b_T, \mu, \frac{\nu}{\omega_a}\right)$$

“N-Jettiness Beam Functions at N3LO”

M.Ebert, B.Mistlberger, *GV* [2006.03056]

“Transverse Momentum Dependent PDFs at N3LO”

M.Ebert, B.Mistlberger, *GV* [2006.05329]

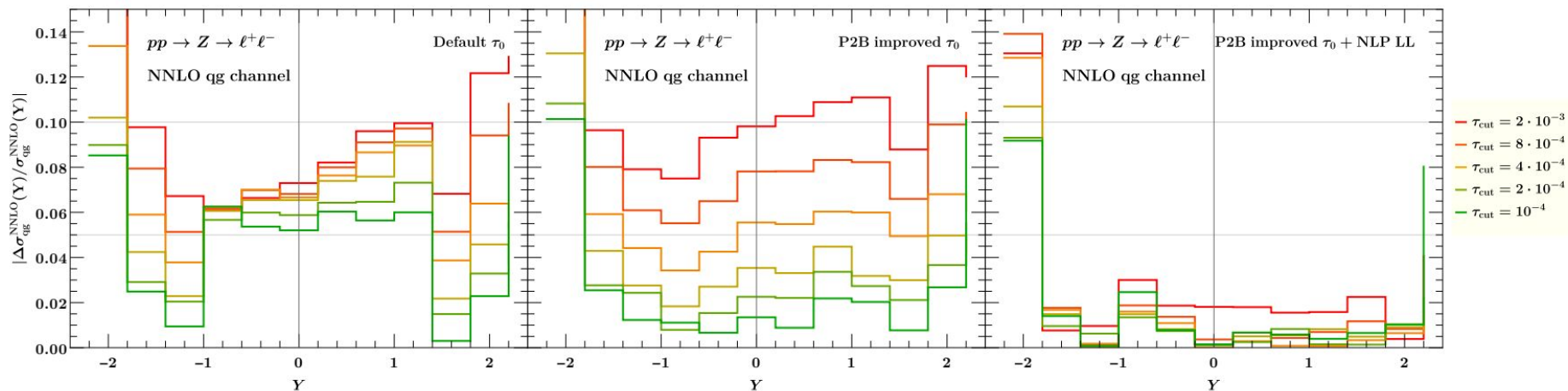
○ Quark  $\tau$  Beam Functions (i.e. Quark N-Jettiness BF)

○ Quark **TMDPDF** (Quark  $q_T$  Beam Function)

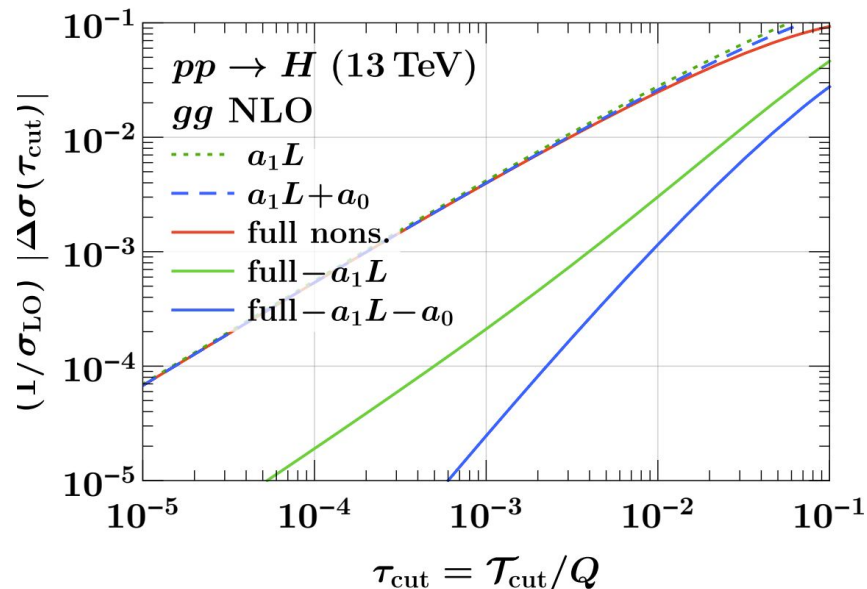
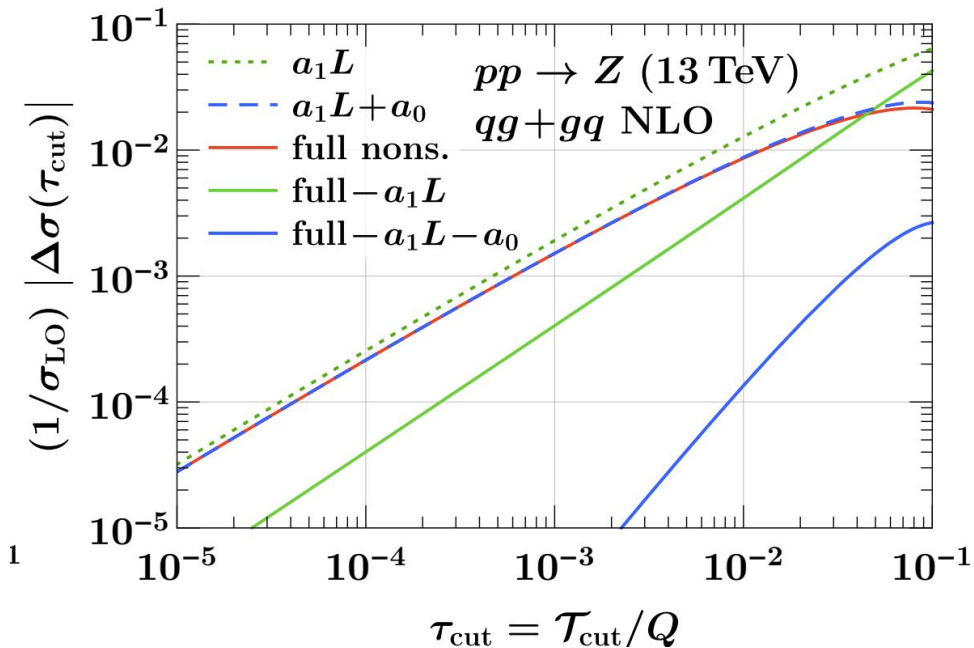
○ Gluon  $\tau$  Beam Functions (i.e. Gluon N-Jettiness BF)

○ Unpolarized Gluon **TMDPDF** (Gluon  $q_T$  Beam Function)

# Projection-to-Born-improved Subtractions at NNLO



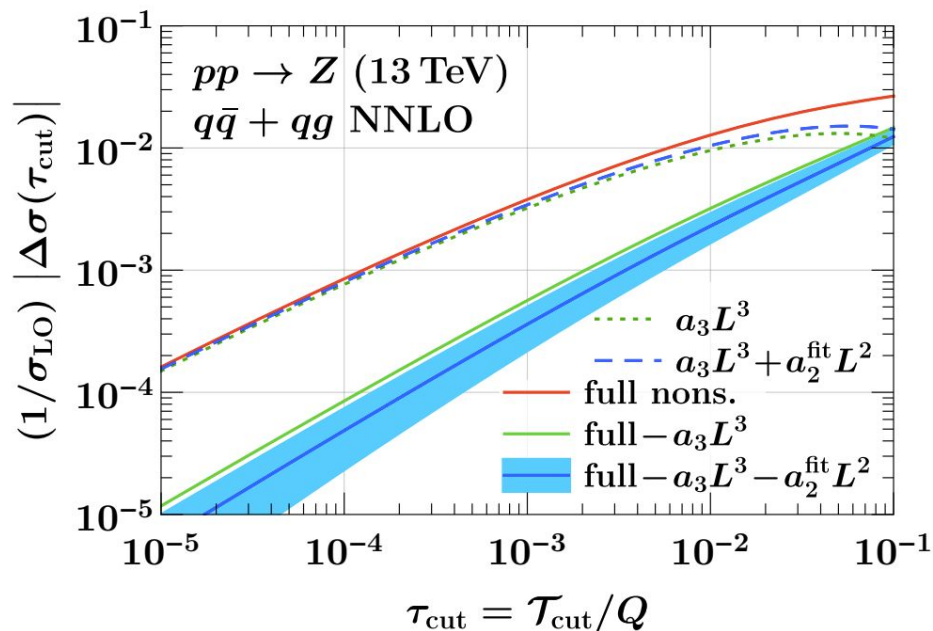
# Log behaviour at NLP NLO



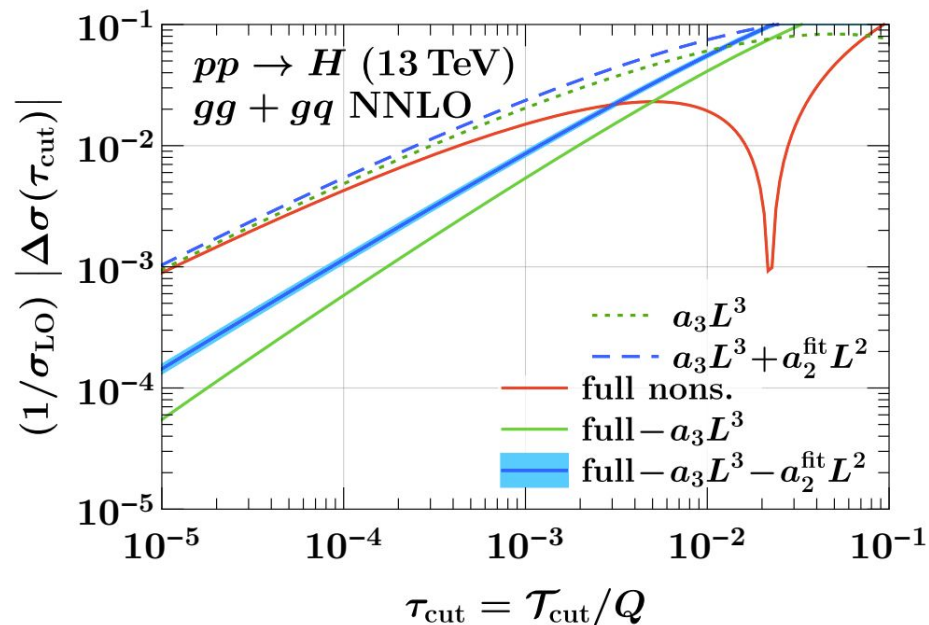
[1807.10764]

# Log behaviour at NLP NNLO

[1612.00450]



[1710.03227]



# A word on linear vs quadratic power corrections

$$0\text{-jettiness: } \Delta\sigma^{N3LO}(\tau_{\text{cut}}) \sim \left(\frac{\alpha_s}{4\pi}\right)^3 \int_0^{\tau_{\text{cut}}} d\tau (c_{3,5}^{\text{NLP}} \ln^5 \tau + c_{3,4}^{\text{NLP}} \ln^4 \tau + c_{3,3}^{\text{NLP}} \ln^3 \tau + \dots)$$

$$q_T: \Delta\sigma^{N3LO}(q_{T\text{cut}}) \sim \left(\frac{\alpha_s}{4\pi}\right)^3 \int_0^{q_{T\text{cut}}^2/Q^2} dr (d_{3,5}^{\text{NLP}} \ln^5 r + d_{3,4}^{\text{NLP}} \ln^4 r + d_{3,3}^{\text{NLP}} \ln^3 r + \dots)$$

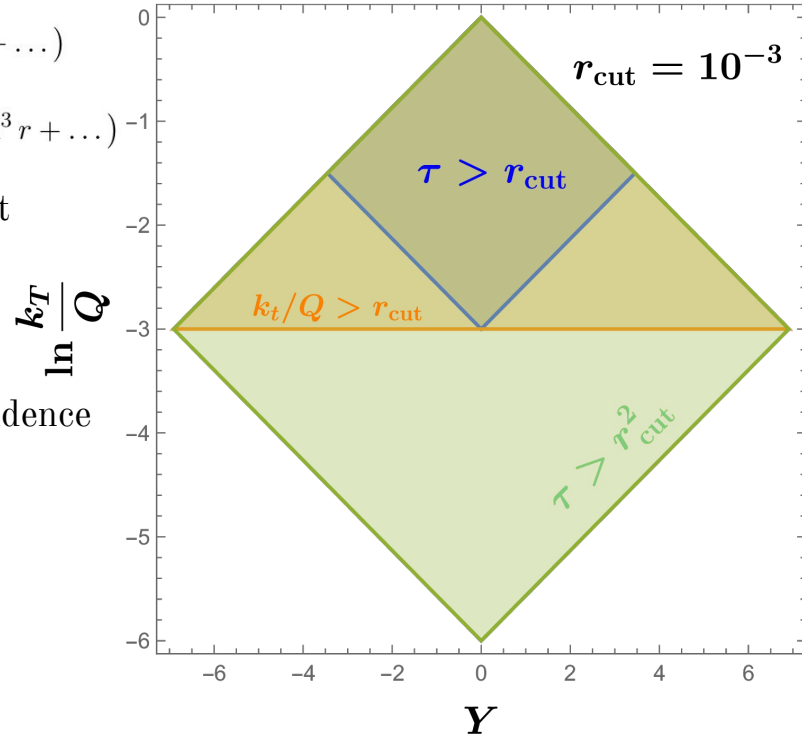
- Scaling in  $q_T$  of the slicing param. may lead to the impression that  $q_T$  subtraction has *quadratic* power corrections, while jettiness has *linear* power corrections.

- But it all comes down to how one decides to treat the angle dependence

$$\tau = \frac{q_T}{Q} e^{-|Y|} \sim \begin{cases} \frac{q_T}{Q} & \text{soft emissions} \\ \frac{q_T^2}{Q^2} & \text{collinear emissions} \end{cases}$$

- In practice, key point is what is more challenging numerically for the above the cut code:

- 0-jettiness: better suppression of collinear emissions
- $q_T$ : better suppression of wide angle soft emissions



Note: fiducial p.c. generating *linear* terms in  $q_T$ , go as  $\sqrt{\tau_{\text{cut}}}$  in the case of 0-jettiness

# Differential color singlet production at N3LO

- Two methods for differential N3LO predictions for color singlet:

## Projection to Born

[Cacciari, Dreyer, Karlberg, Salam, Zanderighi '15]

$$\frac{d\sigma_F^{N^k\text{LO}}}{d\mathcal{O}} = \left( \frac{d\sigma_{F+\text{jet}}^{N^{(k-1)}\text{LO}}}{d\mathcal{O}} - \frac{d\sigma_{F+\text{jet}}^{N^{(k-1)}\text{LO}}}{d\tilde{\mathcal{O}}} \right) + \frac{d\sigma_F^{N^k\text{LO}}}{d\tilde{\mathcal{O}}}$$

Locally subtracted real emissions
Integrated counterterm

- PRO:** Local counterterm is the full Matrix Element => Great numerical efficiency
- Cons:** Integrated counterterm is very hard to obtain (analytic differential distribution at N3LO in full kinematics)

## $q_T$ or 0-jettiness subtraction

$q_T$  Subtraction: [Catani, Grazzini '07]      N-Jettiness: [Boughezal, Focke, Liu, Petriello '15]  
 [Gaunt, Stahlhofen, Tackmann, Walsh '15]

$$\sigma(X) = \int_0^{q_{T\text{cut}}} dq_T \frac{d\sigma^{\text{sing}}(X)}{dq_T} + \int_{q_{T\text{cut}}} dq_T \frac{d\sigma(X)}{dq_T} + \Delta\sigma(X, q_{T\text{cut}})$$

Below the cut region
Above the cut region
Residual

- PRO:** Analytic control of IR divergences from EFT factorization thm. at Leading Power
- Cons:** numerically challenging

# Beam Functions calculation at N<sup>3</sup>LO

(Ebert, Mistlberger, GV)

[2006.05329], [2006.03056]

- Calculation of the **collinear expansion of the partonic cross section** for DY and Higgs @N<sup>3</sup>LO **differential** in  $(Q_T, \tau, z)$

- $\sim 100k$  Feynman diagrams
- Reverse unitarity for phase space integrals
- Collinear Expansion at the XS level

“Collinear expansion for color singlet cross sections” [Ebert, Mistlberger, GV]

$$\begin{array}{c} p_2 \\ \diagdown \\ p_3 \\ \diagup \\ p_1 \end{array} \rightarrow \lambda^{2-4\epsilon} \left[ \begin{array}{c} p_2 \\ \diagdown \\ p_4 \\ \diagup \\ p_1 \end{array} - \lambda^2 \begin{array}{c} p_2 \\ \diagdown \\ p_3 \\ \diagup \\ p_1 \end{array} + \mathcal{O}(\lambda^3) \right]$$

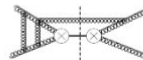
- Reduction to basis of **Master Integrals** via Integration By Parts (IBPs) using Water

Expanded diagrams admit (simplified) IBPs identities

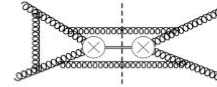
$$\begin{array}{c} p_2 \\ \diagdown \\ p_3 \\ \diagup \\ p_1 \end{array} = -\frac{1-2\epsilon}{\epsilon(p_2^+ k^-)^2} \times \begin{array}{c} p_1 \\ \diagdown \\ p_4 \\ \diagup \\ p_1 \end{array}$$

$$\begin{array}{c} p_2 \\ \diagdown \\ p_3 \\ \diagup \\ p_1 \end{array} = -\frac{k^+ x}{p_2^+} \frac{1-2\epsilon}{\epsilon(p_2^+ k^-)^2} \times \begin{array}{c} p_1 \\ \diagdown \\ p_4 \\ \diagup \\ p_1 \end{array}$$

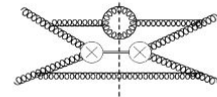
- **RVV**: known in full kinematics [Duhr, Gehrmann] [Duhr, Gehrmann, Jaquier] [Dulat, Mistlberger]



- **RRV**: 170 Collinear Master Integrals



- **RRR**: 320 Collinear Master Integrals



- Derived system of Differential Equations for the Master Integrals

- System has 2 non trivial scales with algebraic dependence on the variables (not something solvable algorithmically)

- Algebraic sectors: constructed dlog integrand basis via calculation of **leading singularities** of candidate integrals on maximal cut surface

- Boundaries from soft integrals [Anastasiou, Duhr, Dulat, Mistlberger] and constraints on singular behavior

