The Higgs sector of the MSSM

Two Higgs doublets $H_u = (H_u^+, H_u^0)$, $H_d = (H_d^0, H_d^-)$

$\rightsquigarrow$ 8 scalar fields = 5 physical states + 3 Goldstones

Superpotential : $W = y_u Q H_u U + y_d Q H_d D + y_e L H_d E + \mu H_u H_d$

$\rightarrow$ F-terms : $F_{u,d} = -\dfrac{\partial W}{\partial H_{u,d}}$, $V_F = |F_{u,d}|^2 = \left|\dfrac{\partial W}{\partial H_u}\right|^2 + \left|\dfrac{\partial W}{\partial H_d}\right|^2 = |\mu|^2 \left(|H_u|^2 + |H_d|^2\right)$

$\rightarrow$ D-terms : $D^a = -\dfrac{g}{2}\left(H_u^\dagger \sigma^a H_u + H_d^\dagger \sigma^a H_d\right)$, $D_Y = -\dfrac{g'}{2}\left(|H_u|^2 - |H_d|^2\right)$

$$V_D = \frac{1}{2}\left(D^a D^a + D_Y^2\right) = \frac{g^2}{8}\sum_a \left(H_u^\dagger \sigma^a H_u + H_d^\dagger \sigma^a H_d\right)^2 + \frac{g'^2}{8}\left(|H_u|^2 - |H_d|^2\right)^2$$

$$= \frac{g^2 + g'^2}{8}\left(|H_u^+|^2 + |H_u^0|^2 - |H_d^0|^2 - |H_d^-|^2\right)^2 + \frac{g^2}{2}\left|H_u^+ H_d^{0*} + H_u^0 H_d^{-*}\right|^2$$ (check as exercise!)

Soft terms : $V_{soft} = m_u^2 |H_u|^2 + m_d^2 |H_d|^2 + b\,(H_u H_d + h.c.)$

$$\begin{aligned} V(H_u, H_d) =\ & \left(|\mu|^2 + m_u^2\right)\left(|H_u^+|^2 + |H_u^0|^2\right) + \left(|\mu|^2 + m_d^2\right)\left(|H_d^0|^2 + |H_d^-|^2\right) \\ & + b\left(H_u^+ H_d^- - H_u^0 H_d^0\right) + h.c. + \frac{g^2 + g'^2}{8}\left(|H_u^+|^2 + |H_u^0|^2 - |H_d^0|^2 - |H_d^-|^2\right)^2 \\ & + \frac{g^2}{2}\left|H_u^+ H_d^{0*} + H_u^0 H_d^{-*}\right|^2 \end{aligned}$$

Properties of the scalar potential:

•) No v.e.v. for H_u^+, H_d^- (would break electromagnetism!)

We are free to put $\langle H_u \rangle = \dfrac{1}{\sqrt{2}}\begin{pmatrix} v_u \\ 0 \end{pmatrix}$, $v_u \in \mathbb{R}$ through $SU(2)_L$ transformations (like for the SM Higgs), but can not do the same for H_d simultaneously.

$$\left.\frac{\partial V}{\partial H_u^+}\right|_{\langle H_u^+\rangle = 0} = b H_d^- + \frac{g^2}{2} H_u^{0*} H_d^{0*} H_d^- = 0$$

$$\left.\frac{\partial V}{\partial H_d^-}\right|_{\langle H_u^+\rangle = 0} = \left(|\mu|^2 + m_d^2\right) H_d^{-*} + \frac{g^2 + g'^2}{4}\left(|H_u^0|^2 - |H_d^0|^2 - |H_d^-|^2\right) H_d^{-*} = 0$$

The minimum conditions are satisfied, for generic parameters, for $H_d^- = 0$.

$\Rightarrow \langle H_d^- \rangle = \langle H_u^+ \rangle = 0$.

•) b can be taken real and ≥ 0 (again through $SU(2)_L$ rotations).

CP violation: $V(H_u, H_d) = \underbrace{-b H_u^0 H_d^0 + h.c.}$ + terms without phases

only term that depends on the phase of H_d^0, φ

$-b H_u^0 H_d^0 + h.c. \propto -b \cos\varphi \Rightarrow \langle\varphi\rangle = 0$ no spontaneous breaking of CP.

•) Must have a negative mass term to have Electroweak Symmetry Breaking

$$\det\left[\frac{\partial^2 V}{\partial H_i \partial H_j}\right] = (|\mu|^2 + m_u^2)(|\mu|^2 + m_d^2) - b^2 < 0$$

•) In the direction $H_u^0 = H_d^{0*}$ the quartic interaction vanishes $\left(\frac{g^2+g'^2}{8}(|H_u^0|^2 - |H_d^0|^2)^2 \equiv 0\right)$

Stability of the potential $\Rightarrow$ positive mass term along this flat direction

$$\Rightarrow 2|\mu|^2 + m_u^2 + m_d^2 - 2b = \left.\frac{\partial^2 V}{\partial H^* \partial H}\right|_{H \equiv H_u^0 \equiv H_d^{0*}} > 0 .$$

(notice that the last two conditions can not be satisfied if $m_u = m_d$)

$$(|\mu|^2 + m_u^2)(|\mu|^2 + m_d^2) < b^2 < \left(|\mu|^2 + \frac{m_u^2 + m_d^2}{2}\right)^2$$

μ-problem: why is the supersymmetric parameter μ related to the soft terms $b, m_{u,d}$?

$$\langle H_u^0\rangle = \frac{v_u}{\sqrt{2}}, \qquad \langle H_d^0\rangle = \frac{v_d}{\sqrt{2}}$$

$|D_\mu H_u|^2 + |D_\mu H_d|^2 \rightsquigarrow v^2 = v_u^2 + v_d^2$ (fixed by the gauge boson masses)

$\tan\beta \equiv \frac{v_u}{v_d}$, $\qquad v_u = v \sin\beta$, $v_d = v\cos\beta$

$$V(H_d^0, H_u^0) = (|\mu|^2 + m_u^2)|H_u^0|^2 + (|\mu|^2 + m_d^2)|H_d^0|^2 - b H_u^0 H_d^0 + h.c. + \frac{g^2+g'^2}{8}(|H_u^0|^2 - |H_d^0|^2)^2$$

Minimisation conditions:

$$\begin{cases} \sqrt{2}\left.\frac{\partial V}{\partial H_u^0}\right|_{\langle H_{u,d}\rangle} = (|\mu|^2 + m_u^2) v_u - b v_d + \frac{g^2+g'^2}{8}\cancel{2} v_u \times \frac{1}{\cancel{2}}(v_u^2 - v_d^2) = 0 \\ \sqrt{2}\left.\frac{\partial V}{\partial H_d^0}\right|_{\langle H_{u,d}\rangle} = (|\mu|^2 + m_d^2) v_d - b v_u - \frac{g^2+g'^2}{8}\cancel{2} v_d \times \frac{1}{\cancel{2}}(v_u^2 - v_d^2) = 0 \end{cases}$$

$$\rightsquigarrow 2|\mu|^2 + m_u^2 + m_d^2 = b(\tan\beta + \cot\beta) = \frac{2b}{\sin 2\beta}$$ μ-problem

$$\boxed{\begin{aligned} m_Z^2 \cos 2\beta &= m_u^2 - m_d^2 + b\left(\tan\beta - \cot\beta\right) \\ \sin 2\beta &= \frac{2b}{2|\mu|^2 + m_u^2 + m_d^2} \end{aligned}}$$

Pseudo-scalar (CP-odd) states

$$H_u^0 = \frac{1}{\sqrt{2}}\left(v_u + h_u + i\phi_u\right) \quad , \quad H_d^0 = \frac{1}{\sqrt{2}}\left(v_d + h_d + i\phi_d\right)$$

$$\begin{aligned} V = {} & \left(|\mu|^2 + m_u^2\right)\frac{h_u^2}{2} + \left(|\mu|^2 + m_d^2\right)\frac{h_d^2}{2} - 2b\,\frac{h_u h_d}{2} - \frac{m_Z^2}{2}\cos 2\beta\,\frac{h_u^2 - h_d^2}{2} + \frac{m_Z^2}{2}\left(h_u s_\beta - h_d c_\beta\right)^2 \\ & + \left(|\mu|^2 + m_u^2\right)\frac{\phi_u^2}{2} + \left(|\mu|^2 + m_d^2\right)\frac{\phi_d^2}{2} + 2b\,\frac{\phi_u \phi_d}{2} - \frac{m_Z^2}{2}\cos 2\beta\,\frac{\phi_u^2 - \phi_d^2}{2} + \text{interactions} \ldots \end{aligned}$$

Mass matrix in the (ϕ_u, ϕ_d) basis: (using the minimum conditions)

$$M_A^2 = \begin{pmatrix} b\cot\beta & b \\ b & b\tan\beta \end{pmatrix}$$

$$\det M_A^2 = b^2\left(\cot\beta\tan\beta - 1\right) = 0$$

$\Rightarrow$ massless eigenstate, Goldstone boson π^0

$$\begin{pmatrix} \pi^0 \\ A^0 \end{pmatrix} = \begin{pmatrix} s_\beta & -c_\beta \\ c_\beta & s_\beta \end{pmatrix}\begin{pmatrix} \phi_u \\ \phi_d \end{pmatrix} \quad \text{mass eigenstates}$$

$$\boxed{m_A^2 = b\left(\cot\beta + \tan\beta\right) = \frac{b}{\sin\beta\cos\beta}}$$

Charged Higgses

Additional interaction term $\frac{g^2}{2}\left|H_u^+ H_d^{0*} + H_u^0 H_d^{-*}\right|^2$ in the potential

$$\Rightarrow \quad M_\pm^2 = M_A^2 + m_W^2 \begin{pmatrix} c_\beta^2 & s_\beta c_\beta \\ s_\beta c_\beta & s_\beta^2 \end{pmatrix}$$

is diagonalised by the same rotation $R\left(\frac{\pi}{2} - \beta\right)$ as M_A^2

$\Rightarrow \quad m_\pm^2 = m_A^2 + m_W^2$, the massless eigenstate $\pi^\pm$ is the charged Goldstone.

Scalar (CP-even) states

Rewrite the potential in terms of m_A, m_Z:

$$V(H_u^0, H_d^0) = \left(|\mu|^2 + \frac{m_u^2 + m_d^2}{2}\right)\left(|H_u^0|^2 + |H_d^0|^2\right) + \frac{m_u^2 - m_d^2}{2}\left(|H_u^0|^2 - |H_d^0|^2\right) - b H_u^0 H_d^0 + \text{h.c.} + \frac{g^2 + g'^2}{8}\left(|H_u^0|^2 - |H_d^0|^2\right)^2$$

$$= \frac{m_A^2}{2}\left(|H_u^0|^2 + |H_d^0|^2\right) + \frac{m_A^2 + m_Z^2}{2}\cos 2\beta\left(|H_u^0|^2 - |H_d^0|^2\right) - \frac{m_A^2}{2}\sin 2\beta\, H_u^0 H_d^0 + \text{h.c.} + \frac{g^2 + g'^2}{8}\left(|H_u^0|^2 - |H_d^0|^2\right)^2$$

expanding around $H^c_{u,d} = \frac{v_{u,d} + h_{u,d}}{\sqrt{2}}$ $\rightsquigarrow$ CP-even mass matrix in the (h_u, h_d) basis

$$M_H^2 = \begin{pmatrix} b\cot\beta + m_Z^2 s_\beta^2 & -b - m_Z^2 s_\beta c_\beta \\ -b - m_Z^2 s_\beta c_\beta & b\tan\beta + m_Z^2 c_\beta^2 \end{pmatrix} = \begin{pmatrix} m_A^2 c_\beta^2 + m_Z^2 s_\beta^2 & -(m_A^2 + m_Z^2) s_\beta c_\beta \\ -(m_A^2 + m_Z^2) s_\beta c_\beta & m_A^2 s_\beta^2 + m_Z^2 c_\beta^2 \end{pmatrix}$$

is <u>not</u> diagonalised by $R\left(\frac{\pi}{2} - \beta\right)$

Mass eigenstates : $\begin{pmatrix} h_1 \\ h_2 \end{pmatrix} \equiv R(\alpha) \begin{pmatrix} h_u \\ h_d \end{pmatrix}$

$$m_{1,2}^2 = \frac{1}{2}\left\{ m_A^2 + m_Z^2 \pm \sqrt{(m_A^2 + m_Z^2)^2 - 4 m_A^2 m_Z^2 \cos^2 2\beta} \right\}$$

$$\tan 2\alpha = \frac{2(m_A^2 + m_Z^2) s_\beta c_\beta}{m_A^2(c_\beta^2 - s_\beta^2) - m_Z^2(c_\beta^2 - s_\beta^2)} = \frac{m_A^2 + m_Z^2}{m_A^2 - m_Z^2} \tan 2\beta$$

Define the new doublets $\begin{pmatrix} h \\ H^* \end{pmatrix} \equiv R\left(\beta - \frac{\pi}{2}\right) \begin{pmatrix} H_u \\ H_d^* \end{pmatrix} = \begin{pmatrix} s_\beta & c_\beta \\ -c_\beta & s_\beta \end{pmatrix} \begin{pmatrix} H_u \\ H_d^* \end{pmatrix}$

(notice that this is consistent with the definitions of $A^0, H^\pm$!)

CP-even components : $\begin{cases} h \equiv h_u \sin\beta + h_d \cos\beta \\ H \equiv -h_u \cos\beta + h_d \sin\beta \end{cases}$

$\langle h \rangle = v_u \sin\beta + v_d \cos\beta = v$

$\langle H \rangle = -v_u \cos\beta + v_d \sin\beta = v(-s_\beta c_\beta + s_\beta c_\beta) = 0$

h is the doublet that takes all the v.e.v., and contains the Goldstone bosons $\pi^0, \pi^\pm$ eaten by Z, W ; H contains the physical CP-odd and charged Higgses.

CP-even mass matrix in this basis :

$$\begin{pmatrix} m_Z^2 \cos^2 2\beta & \frac{m_Z^2}{2} \sin 4\beta \\ \frac{m_Z^2}{2} \sin 4\beta & m_A^2 + m_Z^2 \sin^2 2\beta \end{pmatrix} \xrightarrow[m_A \to \infty]{} \begin{pmatrix} 0 & 0 \\ 0 & m_A^2 \end{pmatrix} + \mathcal{O}(m_Z^2/m_A^2)$$

<u>Decoupling limit</u> $(m_A \gg m_Z)$

$$\begin{cases} m_1^2 = m_Z^2 \cos^2 2\beta \left[1 + \mathcal{O}(m_Z^2/m_A^2)\right] \\ m_2^2 = m_A^2 \left[1 + \mathcal{O}(m_Z^2/m_A^2)\right] \end{cases}$$

$\tan 2\alpha \longrightarrow \tan 2\beta + \mathcal{O}(m_Z^2/m_A^2)$

Higgs potential in the (h,H) basis:

$$V(h,H) = \frac{m_A^2}{2}\left(|h|^2+|H|^2\right) + \frac{m_A^2+m_Z^2}{2}\cos 2\beta\left[(c_\beta^2-s_\beta^2)(|H|^2-|h|^2) - 2s_\beta c_\beta(Hh^*+H^*h)\right]$$

$$+ b\left[(c_\beta^2-s_\beta^2)(Hh^*+H^*h) + 2s_\beta c_\beta(|H|^2-|h|^2)\right] + \frac{g^2+g'^2}{8}\left[(c_\beta^2-s_\beta^2)(|H|^2-|h|^2) - 2s_\beta c_\beta(Hh^*+H^*h)\right]^2$$

$$= -\frac{m_Z^2}{2}\cos^2 2\beta\,|h|^2 + \left(m_A^2+\frac{m_Z^2}{2}\cos^2 2\beta\right)|H|^2 - \frac{m_Z^2}{4}\sin 4\beta\,(Hh^*+H^*h) + \text{interactions}$$

In the decoupling limit $m_A \to \infty$:

SM Higgs field h, with $\lambda_h = \frac{g^2+g'^2}{8}\cos^2 2\beta$ + heavy scalar $m_A^2|H|^2$

$$(h_1,h_2) = R(\alpha)\cdot(h_u,h_d) = R(\alpha)\cdot R\left(\beta-\tfrac{\pi}{2}\right)^{-1}\cdot(h,H) \equiv R_\delta\cdot(h,H)\,, \qquad \delta \equiv \alpha-\beta+\frac{\pi}{2}$$

h couples to vectors like the SM Higgs (it takes the v.e.v. v)

couplings of the mass eigenstates h_1, h_2:

$$g_{h_1VV} = \cos\delta\cdot g_{hVV}^{SM}\,, \qquad g_{h_2VV} = \sin\delta\cdot g_{hVV}^{SM}$$

exercise: compute the couplings of h_1, h_2 to fermions

Bound on the Higgs boson mass: $\boxed{m_{h_1} \le m_Z\cos 2\beta \le m_Z}$

→ this is in conflict with the observation $m_h \simeq 125$ GeV $(m_Z \simeq 91.2$ GeV$)$

Loop corrections

$$m_h^2 \le 2\lambda_h v^2 = m_Z^2\cos^2 2\beta$$

corrections to m_h with v fixed (i.e. m_Z fixed) ⇝ corrections to quartic coupling λ

The largest coupling of the Higgs bosons is to top / stop quarks (unless $\tan\beta$ very large)

$\Rightarrow$ H_u^4 quartic coupling.

$$W \supset y_t H_u^0 \bar{t}_L t_R - \mu H_u^0 H_d^0$$

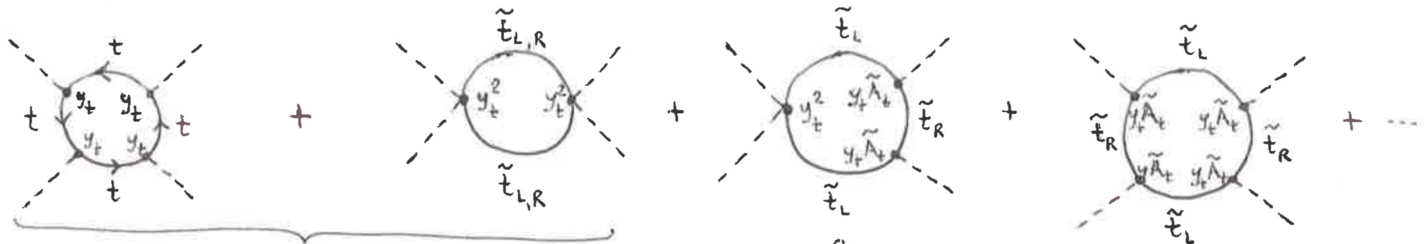
$$V_F(H_u^0,H_d^0,\tilde t_L,\tilde t_R) = \left|\frac{\partial W}{\partial H_u^0}\right|^2 + \left|\frac{\partial W}{\partial \tilde t_L}\right|^2 + \left|\frac{\partial W}{\partial \tilde t_R}\right|^2 = |y_t\tilde t_L\tilde t_R - \mu H_d^0|^2 + y_t^2|H_u^0\tilde t_L| + y_t^2|H_u^0\tilde t_R|^2$$

$$V_{soft}(H_u^0,\tilde t_L,\tilde t_R) = \tilde m_Q^2\tilde t_L^2 + \tilde m_U^2\tilde t_R^2 + y_t A_t\tilde t_L\tilde t_R H_u^0$$

$$\Rightarrow V_{stop} = (\tilde m_Q^2+m_t^2)|\tilde t_L|^2 + (\tilde m_U^2+m_t^2)|\tilde t_R|^2 + m_t(A_t-\mu\cot\beta)\tilde t_L\tilde t_R + \text{h.c.}$$

$$+ y_t^2|h_u\tilde t_L|^2 + y_t^2|h_u\tilde t_R|^2 + \dots$$

1-loop corrections to H_u^4 vertex



$\propto \; y_t^4 \log\left(\frac{\tilde{m}_t^2}{m_t^2}\right)$ $\qquad$ $\simeq \; y_t^4 \frac{\tilde{A}_t^2}{\tilde{m}_t^2}$ $\qquad$ $\simeq \; y_t^4 \frac{\tilde{A}_t^4}{\tilde{m}_t^4}$

$$\delta\lambda_u \simeq \frac{y_t^4 N_c}{16\pi^2}\left\{\log\frac{\tilde{m}_t^2}{m_t^2} + a\,\frac{\tilde{A}_t^2}{\tilde{m}_t^2} + b\,\frac{\tilde{A}_t^4}{\tilde{m}_t^4}\right\}, \qquad \tilde{A}_t \equiv A_t - \mu\cot\beta$$

due to $\tilde{t}_L$-$\tilde{t}_R$ mixing

$$\delta M^2_{u,u} = 2v^2\delta\lambda^2 \simeq \frac{3v^2 y_t^4}{16\pi^2}\{\cdots\} = \frac{3m_t^4}{4\pi^2 v^2\sin^2\beta}\left\{\log\frac{\tilde{m}_t^2}{m_t^2} + 2\,\frac{\tilde{A}_t^2}{\tilde{m}_t^2} - \frac{1}{6}\,\frac{\tilde{A}_t^4}{\tilde{m}_t^4}\right\} \equiv \frac{\Delta_t^2}{\sin^2\beta}$$

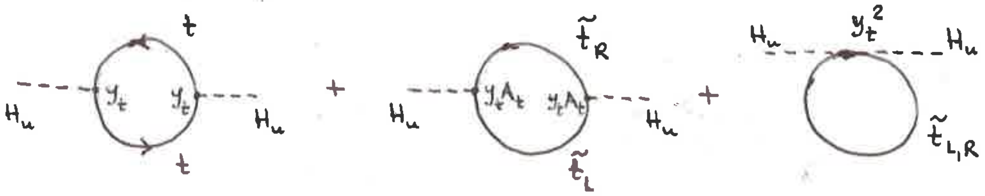
$m_h^2 \leq m_Z^2\cos^2 2\beta + \Delta_t^2$ $\qquad$ large $\tilde{m}_t \Rightarrow$ large correction to m_h

Corrections to v

$$m_Z^2\cos 2\beta = m_u^2 - m_d^2 + b(\tan\beta - \cot\beta)$$

$\hookrightarrow$ corrections from top / stop loops

mass parameter $\rightsquigarrow$ quadratic dependence on $\tilde{m}$ (unlike λ_u) $\longrightarrow$ fine-tuning



$$\delta m_u^2 = -\frac{3y_t^2}{16\pi^2}\left(\tilde{m}_Q^2 + \tilde{m}_U^2 + A_t^2\right)\log\frac{\Lambda^2}{\tilde{m}_t^2} \longleftarrow$$

logarithmic dependence on cut-off Λ (soft SUSY breaking)

$\uparrow$ quadratic dependence on soft masses

(m_t^2 term cancels between top and stop contribution: no correction in the exact SUSY limit!)

$$\delta v^2 = \frac{\partial v^2}{\partial m_u^2}\delta m_u^2 + \cdots \qquad \frac{\partial v^2}{\partial m_u^2} = \frac{1}{\cos 2\beta}\frac{4}{g^2+g'^2} \overset{t_\beta\to\infty}{\approx} -\frac{4}{g^2+g'^2}$$

(bounded by gauge couplings)

$$\delta v^2 \geq \frac{3y_t^2}{16\pi^2}\frac{v^2}{m_Z^2}\left(\tilde{m}_Q^2 + \tilde{m}_U^2 + A_t^2\right)\log\frac{\Lambda^2}{\tilde{m}_t^2}$$

Fine-tuning: $\quad \Delta_{F.T.} \equiv \sum_i\left|\frac{\partial\log v^2}{\partial\log\tilde{m}_i^2}\right| = \frac{3y_t^2}{16\pi^2}\,\frac{\tilde{m}_Q^2 + \tilde{m}_U^2 + A_t^2}{m_Z^2}\,\log\frac{\Lambda^2}{\tilde{m}_t^2}$

large $\tilde{m}_t \Rightarrow$ large fine-tuning