

PHY 117 HS2026

Today:

Energy transformation / conservation

Non-conservative forces

Elastic and inelastic collisions

Momentum conservation

Relationship of momentum and force

Impulse = change in momentum

Week 3, Lecture 2
Sept. 30th, 2026
Prof. Ben Kilminster

Note: several "k"s:
I'll try to write
them differently

K : kinetic energy
units: $[N \cdot m] = [J]$

k : spring constant
units: $\frac{N}{m}$

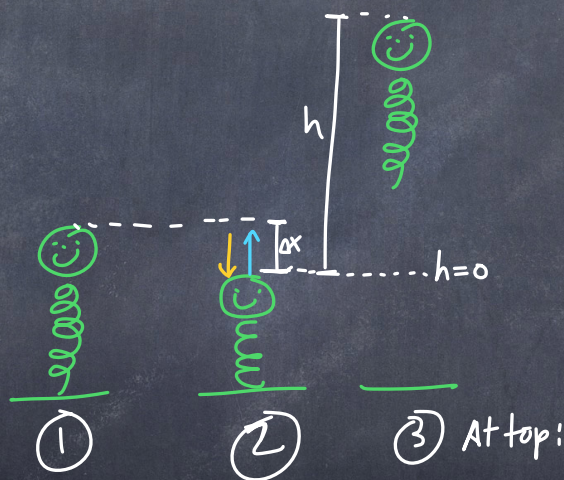
so far: work done by spring:
is $W_s = -\frac{1}{2}k(\Delta x)^2$

work done by me:
 $W_{me} = -W_s = +\frac{1}{2}k(\Delta x)^2$

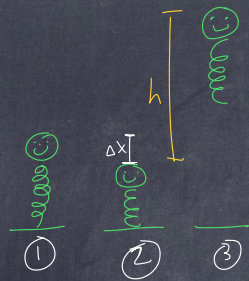
Negative work by the spring
gives it positive potential
energy

$$\Delta U_{\text{spring}} = -W_{\text{spring}} = \frac{1}{2}k(\Delta x)^2$$

Keep in mind $\begin{cases} \Delta U = U_f - U_i \\ U_f = \Delta U + U_i \end{cases}$
No initial stored
energy in spring



Energies: spring
not
compressed



Apply energy conservation:

$$\text{Total energy ②} = \text{Total energy ③}$$

Non-conservative forces: $K + U$ is no longer conserved. Work depends on path taken.

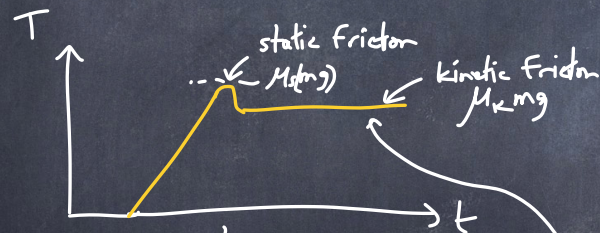
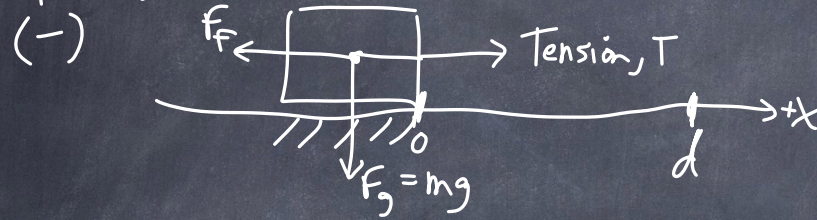
Generally, non-conservative forces convert mechanical energy into unusable forms such as heat.

Examples: friction, air resistance, drag, deformation, breaking

Consider friction:

What is work done by force of friction?

f_f is opposite movement



spring stretching
 $F = k \Delta x \rightarrow W = \frac{1}{2} k (\Delta x)^2$

At constant velocity,

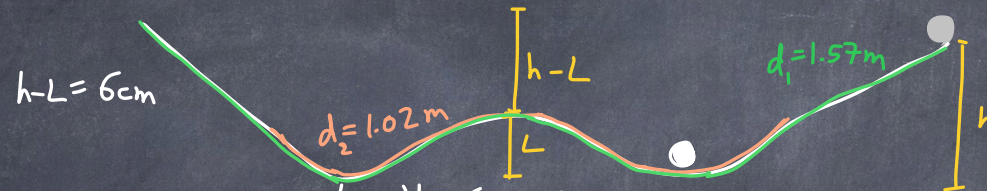
$$\sum F = 0$$

$$T - F_f = 0$$

$$T = F_f \text{ is constant}$$

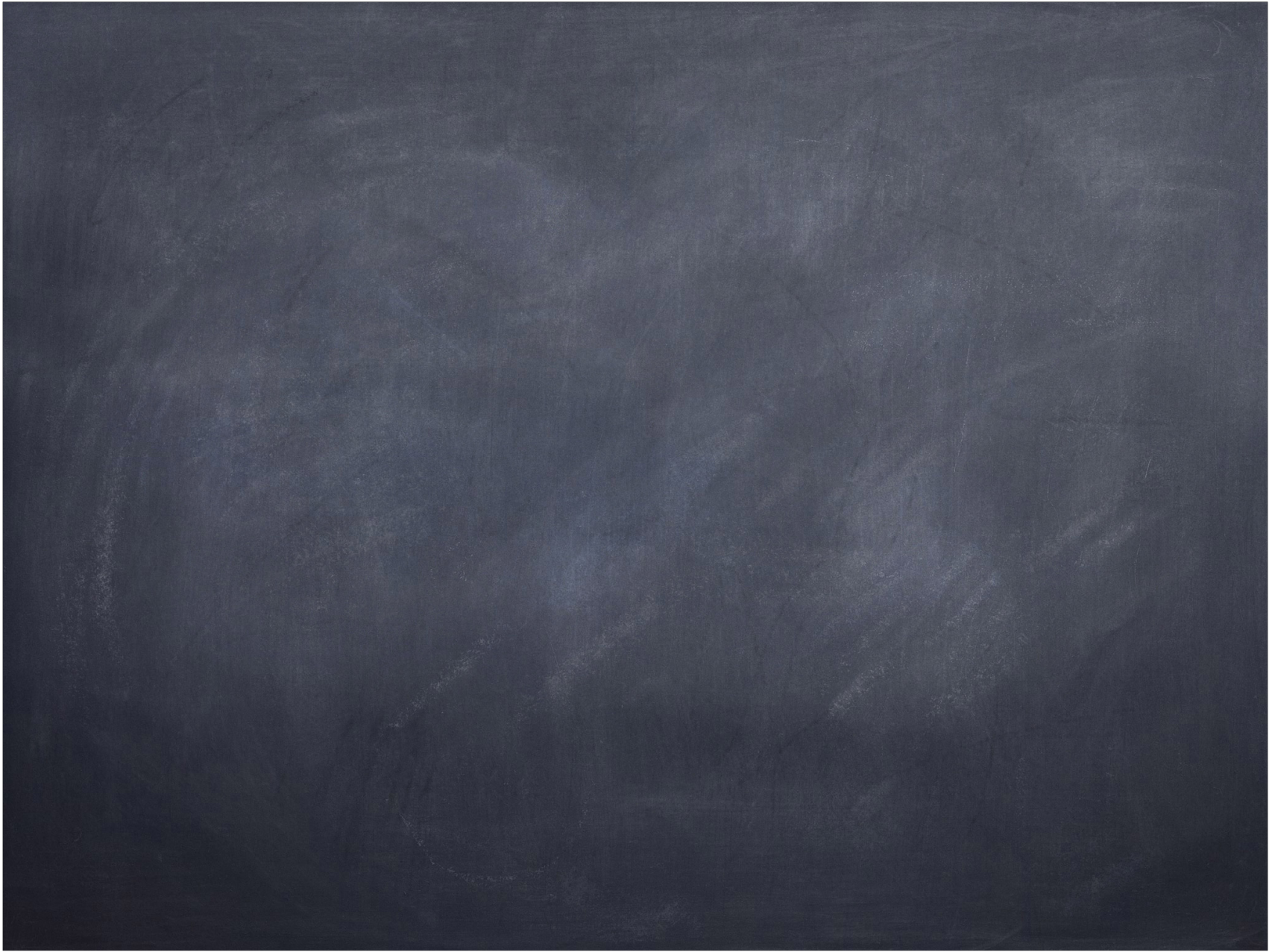


How many times will a steel ball roll across this track before getting stuck?



d_1 : Full length of track

d_2 : length of track when ball gets stuck

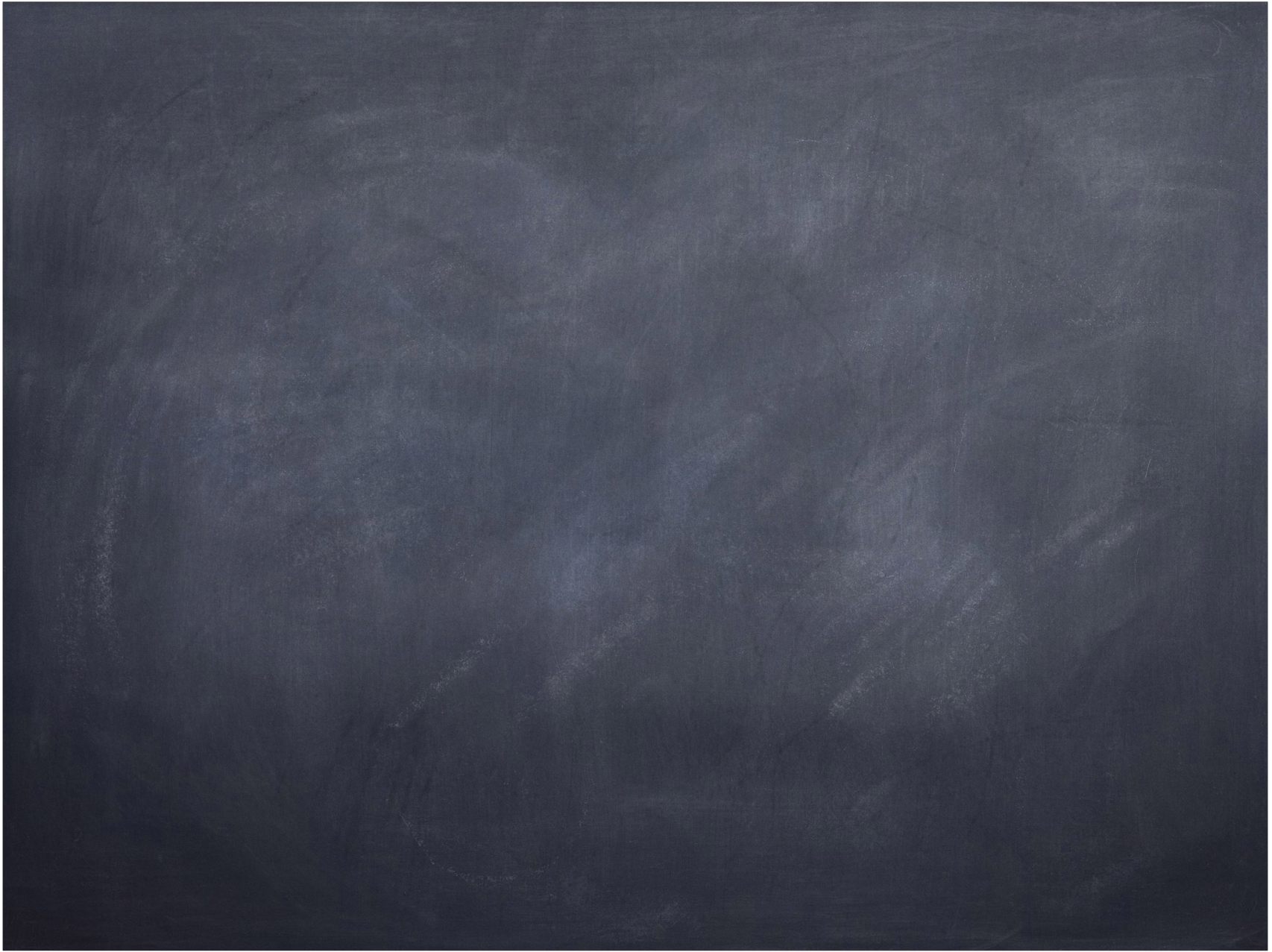


collisions : elastic + inelastic collisions

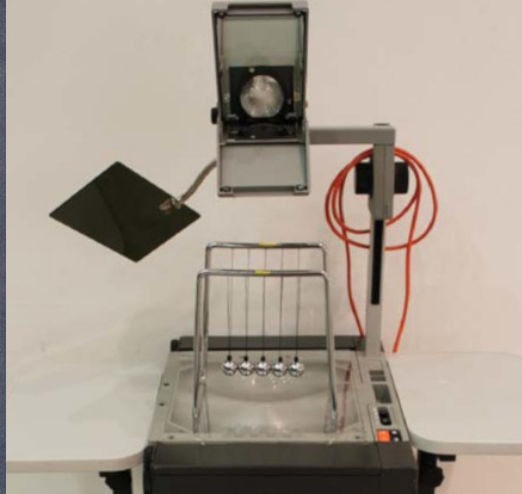
A collision is elastic if there is no work done by the forces of friction, ^{permanent} deformation, or sticking or breaking. Otherwise, it is inelastic.

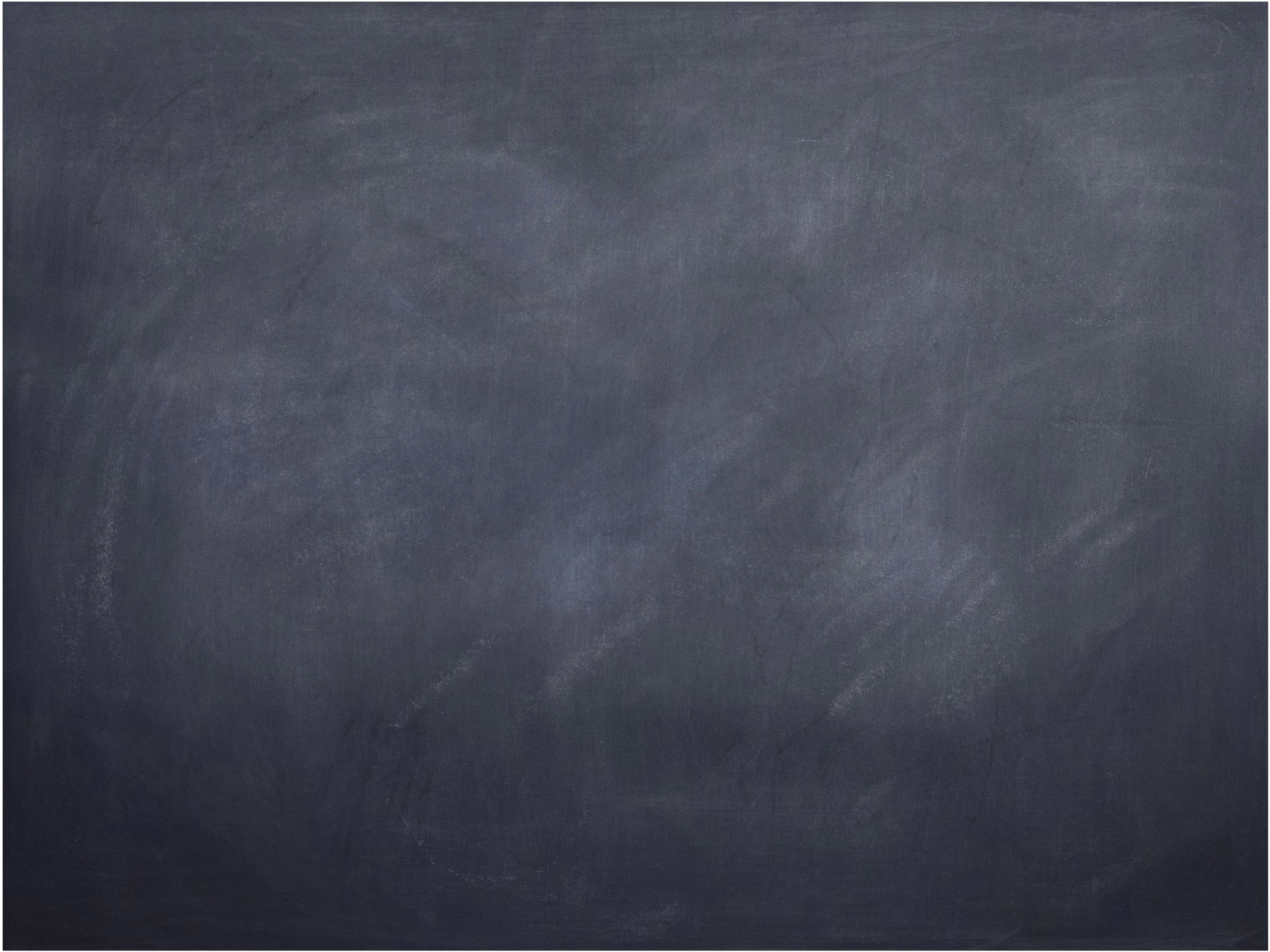






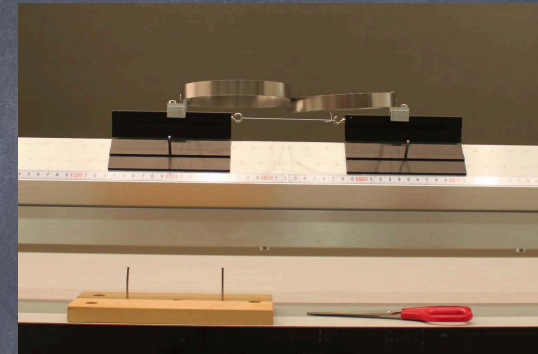
Elastic collisions!





momentum conservation examples:

①

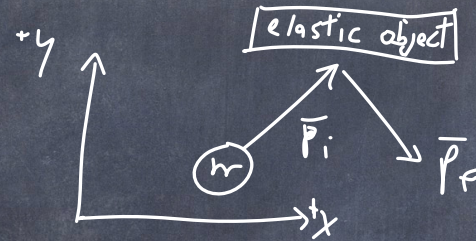


②

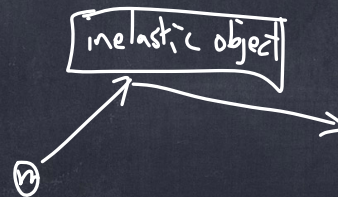


$$m_2 > m_1$$

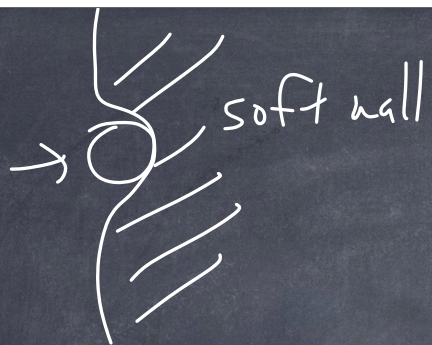
All real objects deform during a collision.
The question is whether the energy stored during that deformation comes back as kinetic energy.



Here, the object deforms permanently:



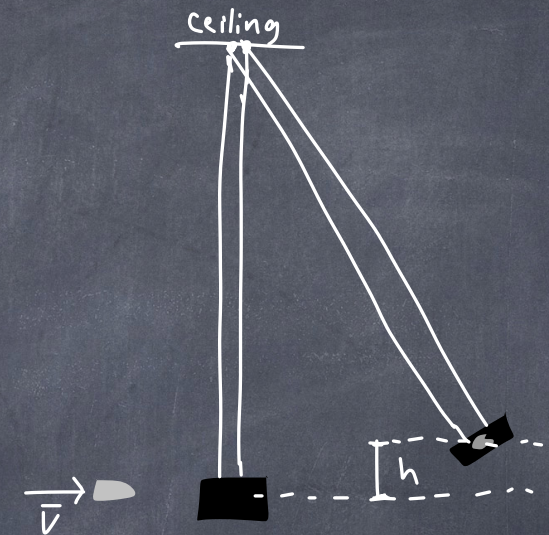
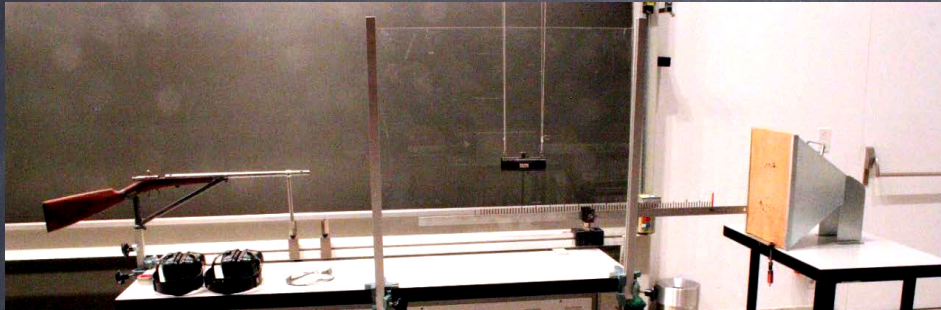
Let's look at forces changing with time.



Note: $\bar{p}$ is momentum

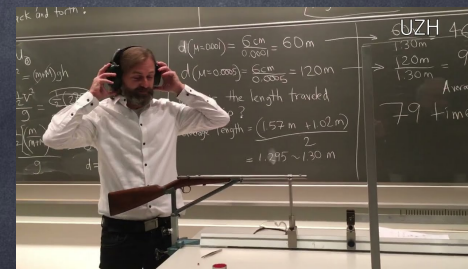


Ballistic pendulum experiment



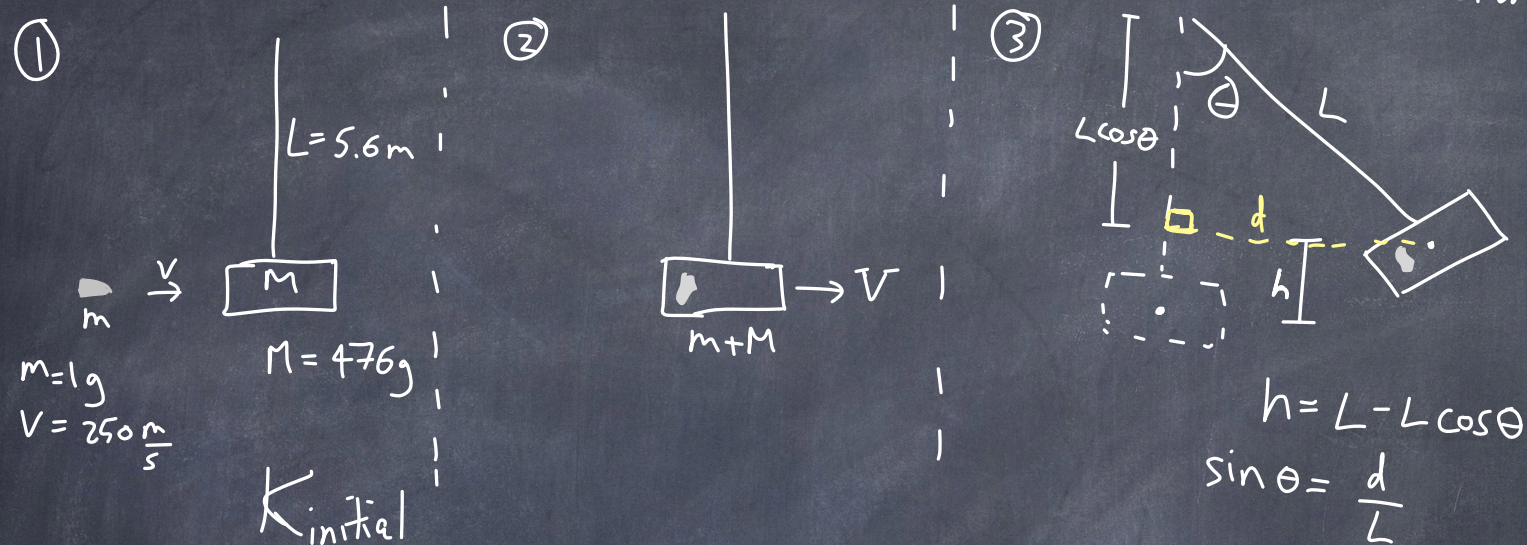
How high will the block go?

video posted



<https://www.youtube.com/watch?v=v2s6Nxc6Dr8>

Ballistic Pendulum - Momentum conservation in inelastic collision.



Between ① + ②, momentum is conserved. Inelastic collision
 Kinetic energy is not conserved.
 $mv = (m+M)V$ V : velocity of block + bullet.

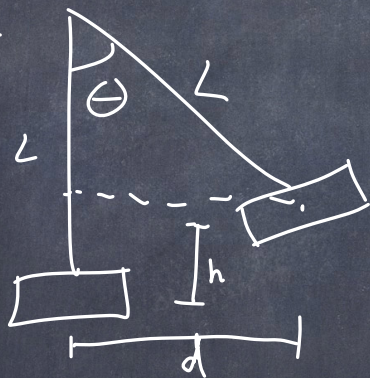
Between ② + ③, this is elastic.
 $K_{\text{②}} = U_{\text{③}}$

$$\underbrace{\frac{1}{2}(\cancel{m+M})V^2}_{K_2} = \underbrace{(\cancel{m+M})gh}_{U_3}$$

solve for $h = \frac{\frac{1}{2}V^2}{g} = \frac{\frac{1}{2}\left[\frac{m}{m+M}V\right]^2}{g} =$

predicted h
0.014 m

$L = 5.6 \text{ m}$



we measure d .

$$h = L - L \cos \theta$$

$$\sin \theta = \frac{d}{L} \Rightarrow \theta = \sin^{-1}\left(\frac{d}{L}\right)$$

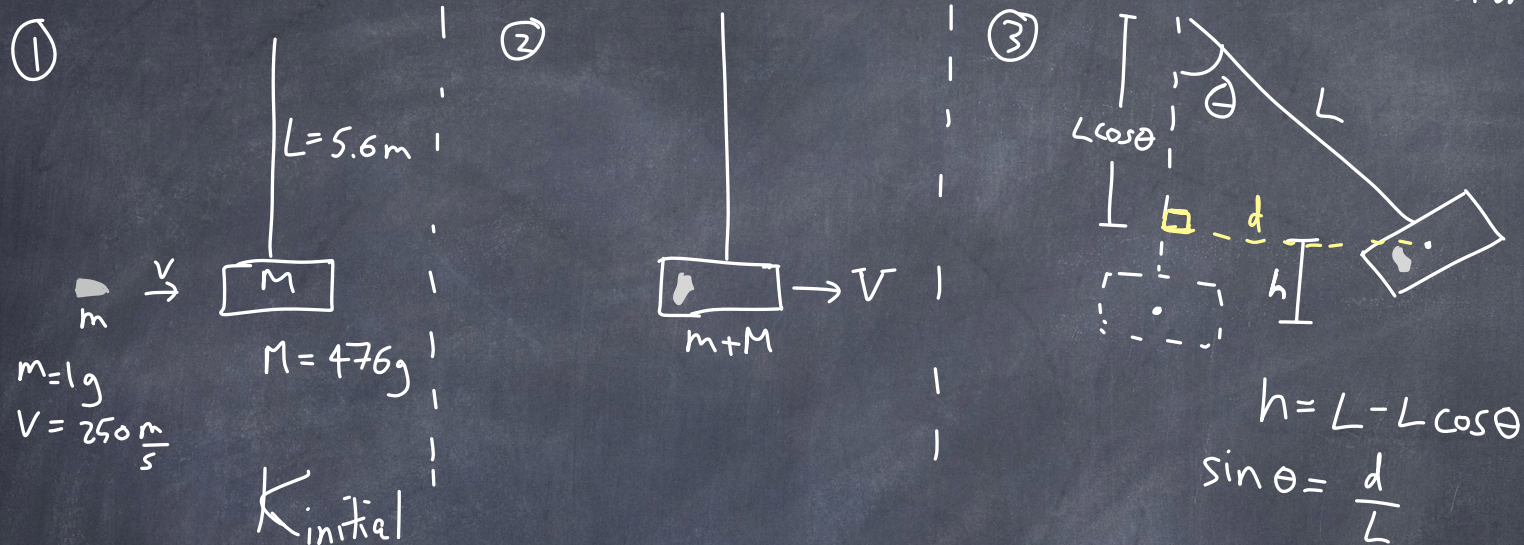
we can measure d , figure out θ , then figure out h .

measure $d = 0.4 \text{ m}$, $\theta = \sin^{-1}\left(\frac{0.4 \text{ m}}{5.6 \text{ m}}\right) = 0.07 \text{ radians}$

and calculate $h = L - L \cos(0.07 \text{ rad}) = 5.6 \text{ m} - 5.585$

measured $h = 0.014 \text{ m}$

Ballistic Pendulum - Momentum conservation in inelastic collision.



Note:

If kinetic energy was conserved, then

$$K_1 = U_3$$

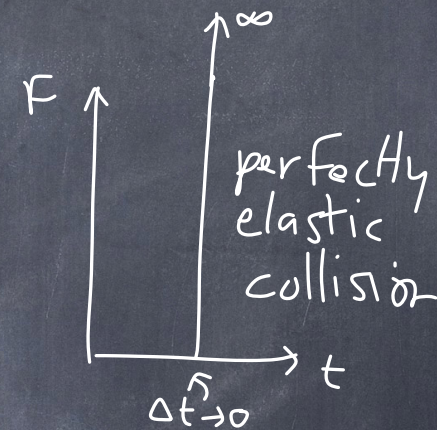
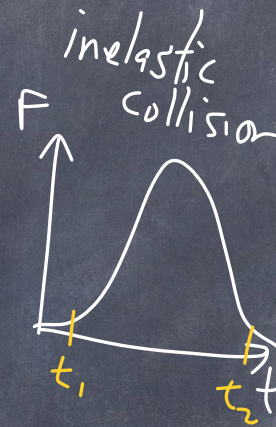
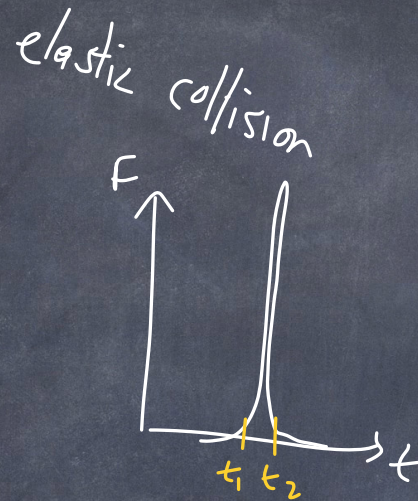
$$\frac{1}{2}mv^2 = mgh \rightarrow h = \frac{v^2}{2g}$$

$h = \underline{\underline{6.7m}}$ ← wrong!
(collision is inelastic)

Questions from students:

- 1) what is the difference between elastic & inelastic collisions in terms of f vs. t ?
- 2) If momentum changes directions, wouldn't the average force always be the same if the momentum change was the same?
- 3) How can something change directions? Isn't momentum not conserved if this happens?
- 4) What if the problem is 2-dimensional?
How is momentum conserved?

1) what is the difference between elastic & inelastic collisions in terms of F vs. t ?

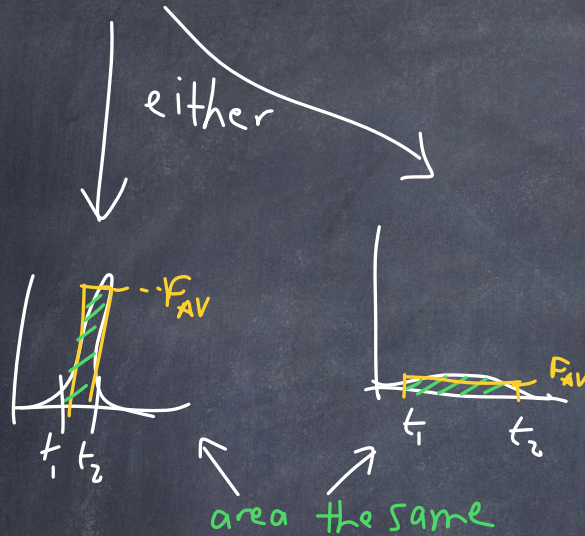


for perfectly elastic collisions,
 $\Delta t \rightarrow 0$, $F \rightarrow \infty$
(these don't happen in the real world.)
All interactions have some Δt

$$\Delta p = I = F_{AV} \Delta t$$

2) If momentum changes directions, isn't F_{AV} always the same?

The average force depends on Δt .

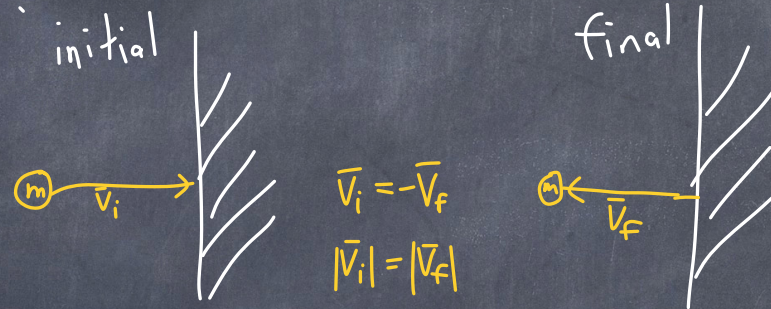


Notice, we can have the same $\Delta p = p_2 - p_1$, in cases where Δt is small and F_{AV} is large, or when Δt is large and F_{AV} is small.

In both cases Δp is the same
(area is the same)

How can something change directions?
Isn't momentum not conserved if this happens?

Consider this case:



The momentum changes from mv_i to mv_f

$$\bar{p}_f = -\bar{p}_i \quad \text{①}$$

What happened?

$$\Delta \bar{p} = \bar{p}_2 - \bar{p}_1 = p_f - p_i = -\bar{p}_i - \bar{p}_i = -2\bar{p}_i$$

The momentum changed.

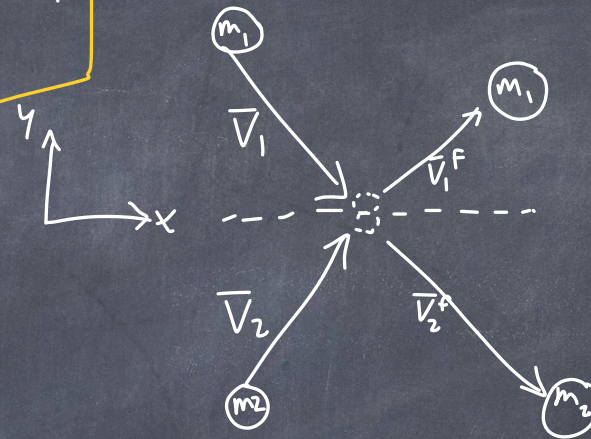
How? The wall provides a force F for a time Δt

$$\Delta p = F_{AV} \Delta t \Rightarrow \bar{F}_{AV} = \frac{-2\bar{p}_i}{\Delta t}$$

If we know Δt , we can calculate F_{AV}

④ What if the problem is 2-dimensional?
How is momentum conserved?

consider this inelastic collision



Momentum will be conserved in all directions:

Therefore, $\sum \vec{p}_i = \sum \vec{p}_f$

$$\sum p_{xi} = \sum p_{xf}$$

$$\sum p_{yi} = \sum p_{yf}$$

we have these equations.

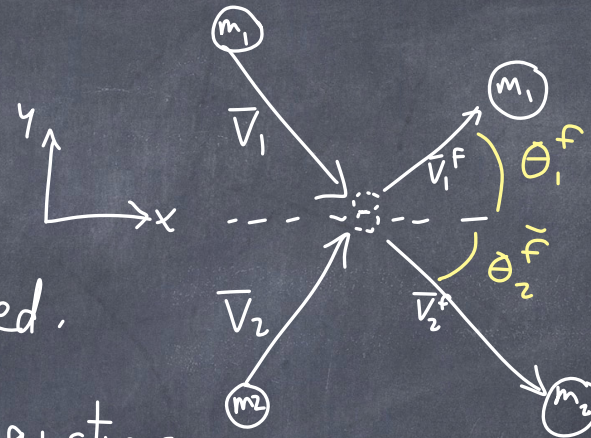
$$\begin{cases} m_1 v_{1x} + m_2 v_{2x} = m_1 v_{1x}^f + m_2 v_{2x}^f \\ m_1 v_{1y} + m_2 v_{2y} = m_1 v_{1y}^f + m_2 v_{2y}^f \end{cases}$$

If we know the masses and initial velocities,
we would have 4 unknowns and 2 equations.
So we need more information to solve this.

④ Continued...

If the collision in 2D were also elastic, then K would be conserved.

Then we would have 3 equations and 4 unknowns:



Kinetic energy conservation: $\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_1v_1^f{}^2 + \frac{1}{2}m_2v_2^f{}^2$

Momentum conservation in x : $m_1v_{1x} + m_2v_{2x} = m_1v_{1x}^f + m_2v_{2x}^f$

Momentum conservation in y : $m_1v_{1y} + m_2v_{2y} = m_1v_{1y}^f + m_2v_{2y}^f$

Knowing one of the angles, θ_1^f or θ_2^f , or one final velocity component ($v_{1x}^f, v_{2x}^f, v_{1y}^f, v_{2y}^f$) would be enough information to solve the problem.