Higgs production, and more

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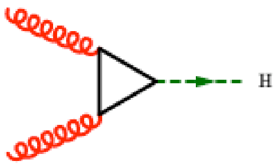
ETH & U. Zürich & INFN

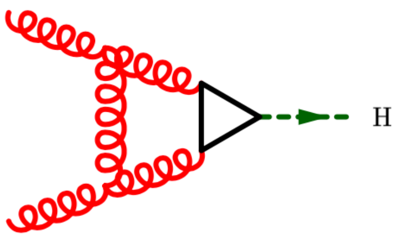
Universität Zürich 26 October 2021

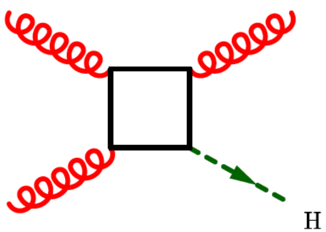
Higgs production at LHC

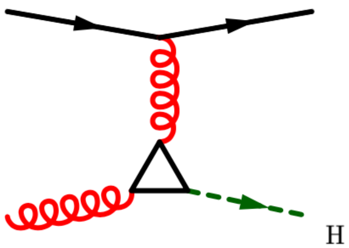
- In proton collisions, the Higgs boson is produced mostly via gluon fusion
  The gluons do not couple directly to the Higgs boson
  The coupling is mediated by a heavy quark loop
  The largest contribution comes from the top loop
  The production mode is (roughly) proportional to the top Yukawa coupling y_t



- QCD NLO corrections







Djouadi Graudenz Spira Zerwas 1992-1995

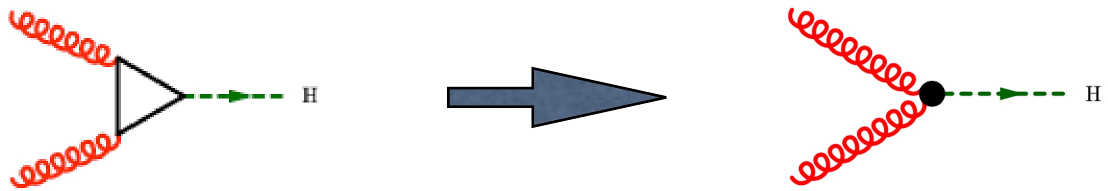
  - QCD NLO corrections are about 100% larger than leading order

- QCD NNLO corrections are known for the top-quark loop only

Czakon Harlander Klappert Niggetiedt 2021

Higgs production in HEFT

- $m_H << 2m_t$



all amplitudes are reduced by one loop

- … but, beware of quark mass effects

σ^{LO}_{EFT}	15.05 pb	σ^{NLO}_{EFT}	34.66 pb
$R_{LO}\,\sigma^{LO}_{EFT}$	16.00 pb	$R_{LO}\,\sigma^{NLO}_{EFT}$	36.84 pb
$\sigma^{LO}_{ex;t}$	16.00 pb	$\sigma^{NLO}_{ex;t}$	36.60 pb
$\sigma^{LO}_{ex;t+b}$	14.94 pb	$\sigma^{NLO}_{ex;t+b}$	34.96 pb
$\sigma^{LO}_{ex;t+b+c}$	14.83 pb	$\sigma^{NLO}_{ex;t+b+c}$	34.77 pb

Anastasiou Duhr Dulat Furlan Gehrmann Herzog Lazopoulos Mistlberger 2016

- $R_{LO} = \frac{\sigma^{LO}_{ex:t}}{\sigma^{LO}_{EFT}} = 1.063$

rescaled EFT (rEFT) does a good job (< 1%) in approximating the exact (only top) NLO σ but misses the *t*-*b* interference

Higgs production

- QCD corrections have been computed at N^3LO in HEFT

Anastasiou Duhr Dulat Herzog Mistlberger 2015
Mistlberger 2018

- including quark-mass effects and QCD-EW interference the cross section is

$$\sigma = 48.58\,\text{pb}^{+2.22\,\text{pb}\,(+4.56\%)}_{-3.27\,\text{pb}\,(-6.72\%)}\ (\text{theory}) \pm 1.56\,\text{pb}\,(3.20\%)\ (\text{PDF}+\alpha_s)$$

- The breakdown of the cross section

$$\begin{aligned}
48.58\,\text{pb} = \quad & 16.00\,\text{pb} & (+32.9\%) \quad & (\text{LO, rEFT}) \\
& +20.84\,\text{pb} & (+42.9\%) \quad & (\text{NLO, rEFT}) \\
& -\ 2.05\,\text{pb} & (-4.2\%) \quad & ((t,b,c),\ \text{exact NLO}) \\
& +\ 9.56\,\text{pb} & (+19.7\%) \quad & (\text{NNLO, rEFT}) \\
& +\ 0.34\,\text{pb} & (+0.2\%) \quad & (\text{NNLO},\ 1/m_t) \\
& +\ 2.40\,\text{pb} & (+4.9\%) \quad & (\text{EW, QCD-EW}) \\
& +\ 1.49\,\text{pb} & (+3.1\%) \quad & (\text{N}^3\text{LO, rEFT})
\end{aligned}$$

Anastasiou Duhr Dulat Furlan Gehrmann Herzog Lazopoulos Mistlberger 2016
Handbook 4 of LHC Higgs Cross Sections 2016

Higgs production

Handbook 4 of LHC Higgs Cross Sections 2016

- 6 sources of uncertainties due to:
  higher orders
  truncation of the threshold expansion
  PDFs
  NLO corrections to QCD-EW interference
  quark mass effects (2: top mass and top-b interference) at NNLO

δ(scale)	δ(trunc)	δ(PDF-TH)	δ(EW)	$\delta(t,b,c)$	$\delta(1/m_t)$
$^{+0.10}_{-1.15}$ pb	~~±0.18 pb~~	±0.56 pb	±0.49 pb	±0.40 pb	~~±0.49 pb~~
$^{+0.21\%}_{-2.37\%}$	~~±0.37%~~	±1.16%	±1%	±0.83%	~~±1%~~

δ(trunc) = 0.11 pb Mistlberger 2018

$\delta(1/m_t)$ = - 0.26% Czakon Harlander Klappert Niggetiedt 2021

QCD NNLO corrections

- Top-quark mass corrections are known at NNLO

Czakon Harlander Klappert Niggetiedt 2021

channel	$\sigma^{\rm NNLO}_{\rm HEFT}$ [pb] $\mathcal{O}(\alpha_s^2)+\mathcal{O}(\alpha_s^3)+\mathcal{O}(\alpha_s^4)$	$(\sigma^{\rm NNLO}_{\rm exact}-\sigma^{\rm NNLO}_{\rm HEFT})$ [pb] $\mathcal{O}(\alpha_s^3)$	$\mathcal{O}(\alpha_s^4)$	$(\sigma^{\rm NNLO}_{\rm exact}/\sigma^{\rm NNLO}_{\rm HEFT}-1)$ [%]
		$\sqrt{s}=8\,\mathrm{TeV}$		
gg	$7.39+8.58+3.88$	$+0.0353$	$+0.0879\pm0.0005$	$+0.62$
qg	$0.55+0.26$	-0.1397	-0.0021 ± 0.0005	-18
qq	$0.01+0.04$	$+0.0171$	-0.0191 ± 0.0002	-4
total	$7.39+9.15+4.18$	-0.0873	$+0.0667\pm0.0007$	-0.10
		$\sqrt{s}=13\,\mathrm{TeV}$		
gg	$16.30+19.64+8.76$	$+0.0345$	$+0.2431\pm0.0020$	$+0.62$
qg	$1.49+0.84$	-0.3696	-0.0115 ± 0.0010	-16
qq	$0.02+0.10$	$+0.0322$	-0.0501 ± 0.0006	-15
total	$16.30+21.15+9.79$	-0.3029	$+0.1815\pm0.0023$	-0.26

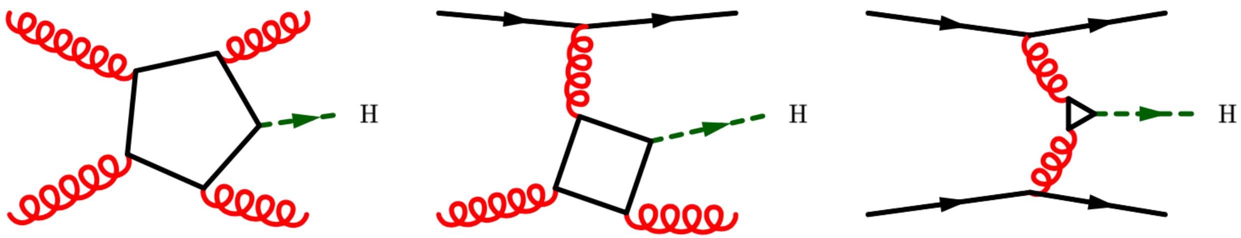
- HEFT not so good for *qg* and *qq* channels

- even accounting for the low luminosity, *qg* and *qq* channels would make the error to be > 1% if absolute values were taken

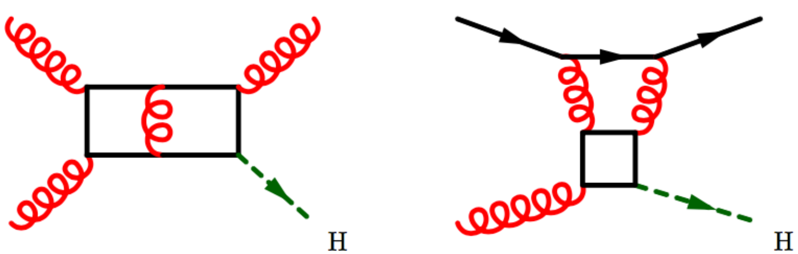
QCD NNLO corrections

- Higgs + 4-parton amplitudes at one loop

VDD Kilgore Oleari Schmidt Zeppenfeld 2001
Budge Campbell De Laurentis K. Ellis Seth 2020



- Higgs + 3-parton amplitudes at two loops



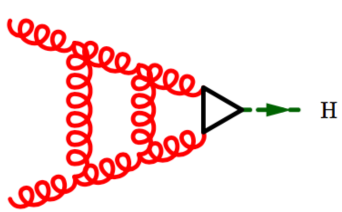
top loop: Jones Kerner Luisoni 2018
Czakon Harlander Klappert Niggetiedt 2021(?)

arbitrary heavy quark masses (only Master Int):

Bonciani VDD Frellesvig Henn Moriello V. Smirnov 2016
all above + Hidding Maestri Salvatori 2019

multi-scale problem with complicated analytic structure
elliptic iterated integrals appear

- gg→Higgs amplitudes at three loops



one scale: one & two top loops
one top loop + light-quark loop

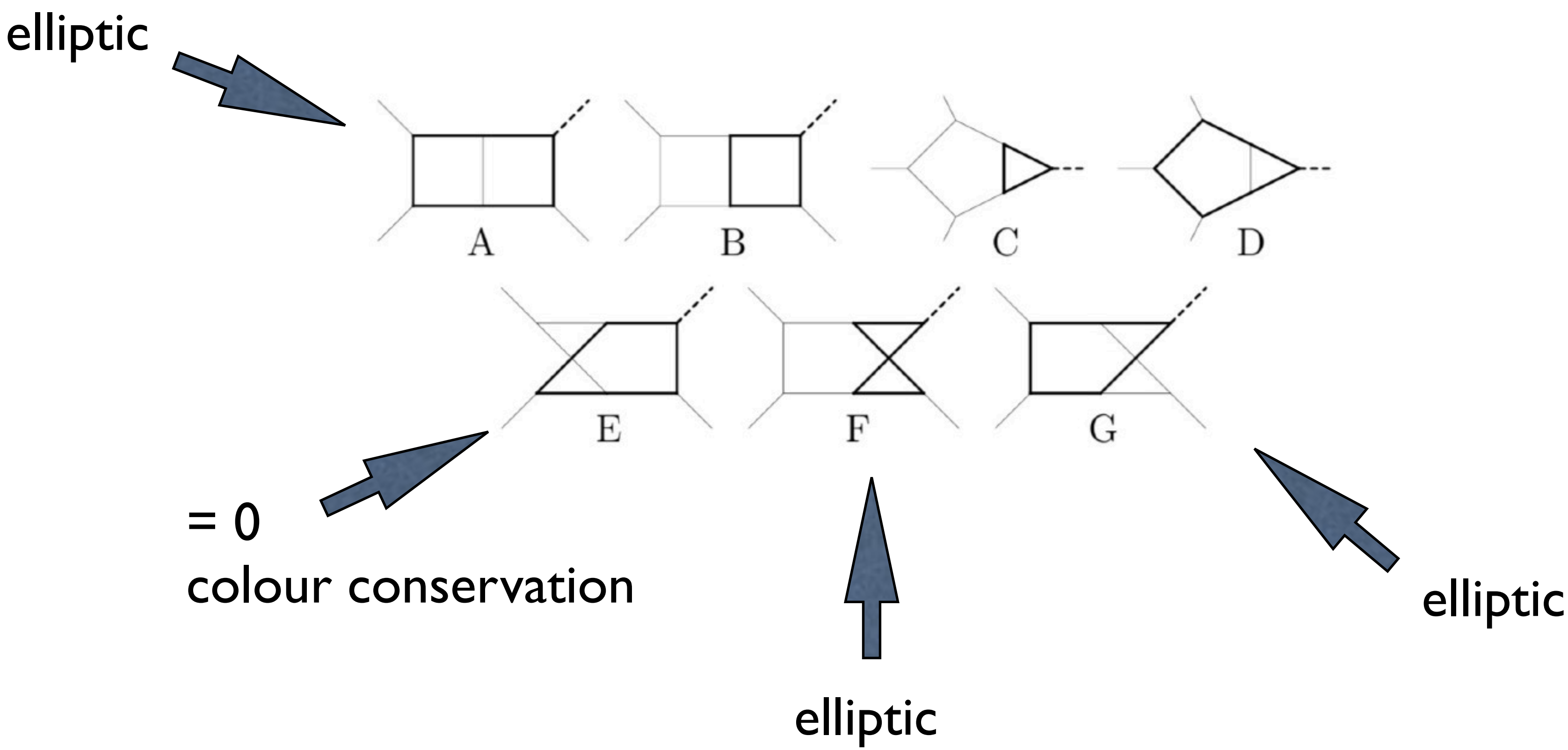
Czakon Niggetiedt 2020
Harlander Prausa Usovitsch 2019

two scales: one top loop + *b*-quark loop

Higgs+3-parton Master Integrals at two loops

4 scales, s, t, m_H, m_t → 3 external parameters
6 seven-propagator integral families

Bonciani VDD Frellesvig Henn Moriello V. Smirnov 2016 (A, B, C, D)
Bonciani VDD Frellesvig Henn Hidding Maestri Moriello Salvatori V. Smirnov 2019 (F)
Frellesvig Hidding Maestri Moriello Salvatori 2019 (G)



Family F: 73 MIs (65 in the polylogarithmic sector, 8 in the elliptic sector)
alphabet: 69 independent letters, with 12 independent square roots

solved through generalised power series expansion Moriello 2019
of the differential equations, defining the parameter n-ples along a contour

Differential Equations

- Differential Equation method to solve the MIs

$$\partial_i f(x_n;\varepsilon) = A_i(x_n;\varepsilon) f(x_n;\varepsilon)$$

f: N-vector of MIs, A_i: NxN matrix, i=1,…,n external parameters

but in some cases ε-independent form

$$\partial_i f(x_n;\varepsilon) = \varepsilon A_i(x_n) f(x_n;\varepsilon)$$

Henn 2013

solution in terms of iterated integrals

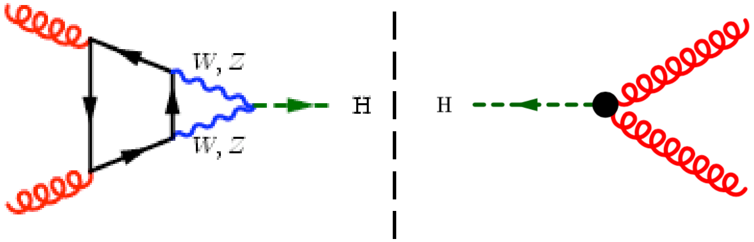
- Take two points (a_1, …, a_n) and (b_1,…, b_n) in the *n*-dim parameter space, and parametrise the contour $\gamma(t)$ that connects the two points

$$\gamma(t) : t \to \{x_1(t), \ldots, x_n(t)\} \qquad \vec{x}(0) = \vec{a}\,, \qquad \vec{x}(1) = \vec{b}$$

and write the differential equation with respect to *t*.
Then find a solution about a point τ by series expanding the coefficient matrix *A* and then iteratively integrating it.
The procedure works in general, for canonical or elliptic sectors

Moriello 2019

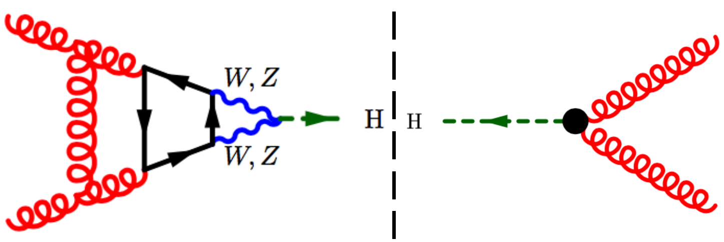
QCD-EW interference



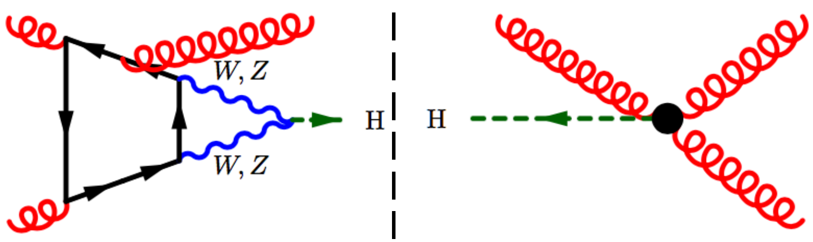
Aglietti Bonciani Degrassi Vicini 2004; Degrassi Maltoni 2004
(light fermion loop)
Actis Passarino Sturm Uccirati 2008
(heavy fermion loop)

- gg-initiated QCD NLO corrections (light fermion loop)

Becchetti Bonciani VDD Hirschi Moriello Schweitzer 2020



Bonetti Melnikov Tancredi 2016



Becchetti Bonciani Casconi VDD Moriello 2018 (only planar MIs)
Bonetti Panzer V. Smirnov Tancredi 2020
Becchetti Moriello Schweitzer 2021(?)

LO $\sigma^{(\alpha_s^2\alpha^2)}_{gg\to H+X} = 0.68739^{+23.4\%+2.0\%}_{-17.3\%-2.0\%}$ pb

NLO $\sigma^{(\alpha_s^2\alpha^2+\alpha_s^3\alpha^2)}_{gg\to H+X} = 1.467(2)^{+18.7\%+2.0\%}_{-14.6\%-2.0\%}$ pb i.e. NLO 110% wrt LO

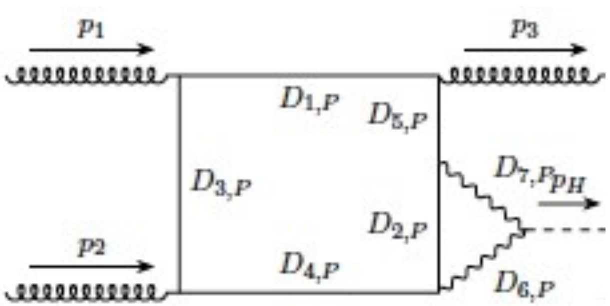
gg-initiated NLO corrections in HEFT $\sigma^{(\mathrm{HEFT},\alpha_s^2\alpha+\alpha_s^3\alpha)}_{gg\to H+X} = 30.484^{+19.8\%+1.9\%}_{-15.3\%-1.9\%}$ pb

thus our NLO result 4.8% wrt gg-initiated NLO HEFT

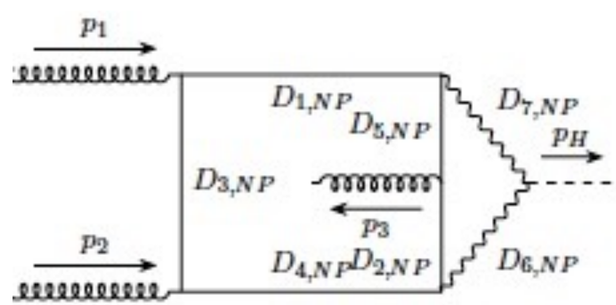
QCD-EW Higgs+3-parton master integrals at two loops

4 scales, s, t, m_H, m_V → 3 external parameters
7 seven-propagator integral families
48 MIs (planar), 61 MIs (non-planar)
alphabet: square roots are present, but an MPL representation is possible

Becchetti Bonciani Casconi VDD Moriello 2018 (planar MIs)
Becchetti Moriello Schweitzer 2021(?) (non-planar MIs)



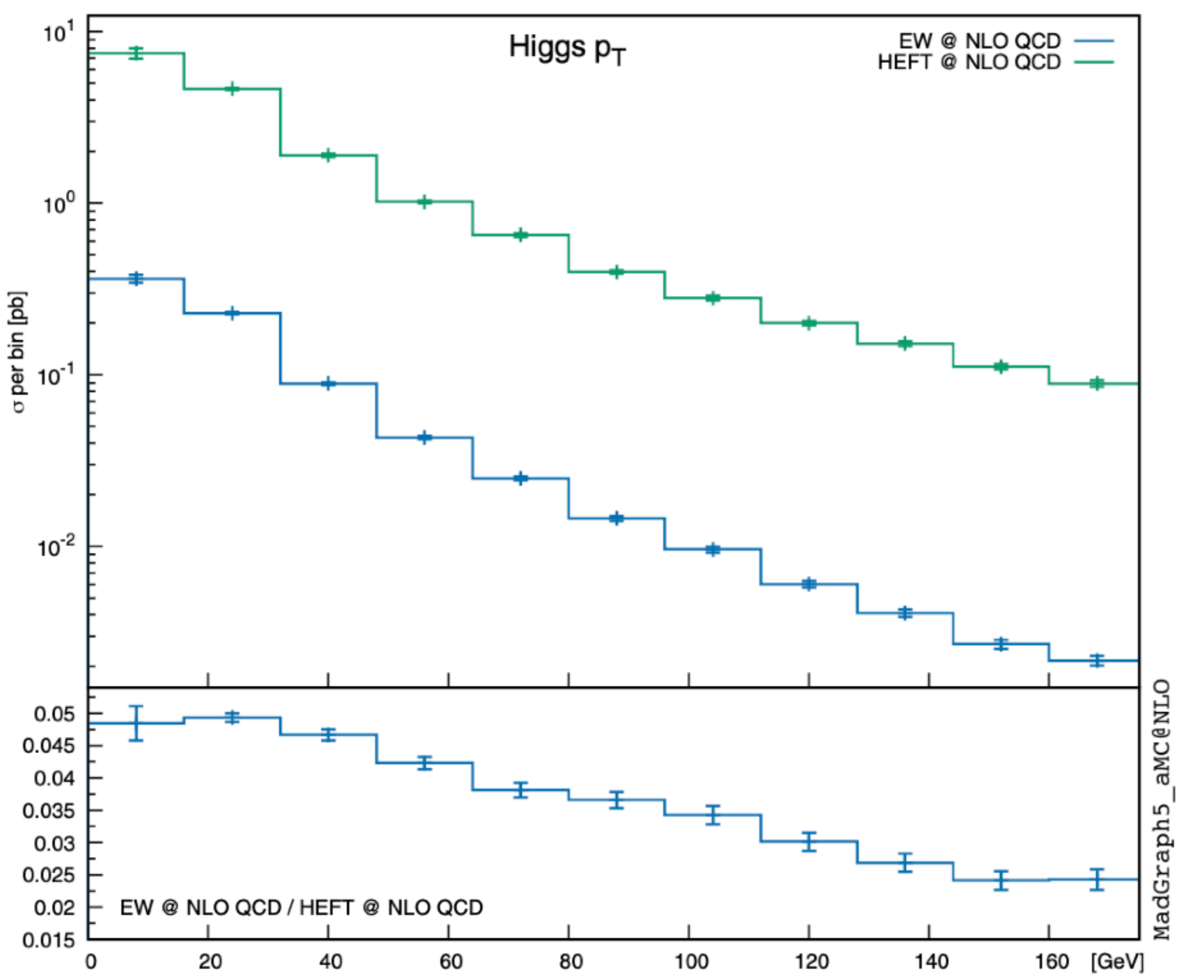
planar



non-planar

solved through generalised power series expansion Moriello 2019

Higgs p_T distribution due to QCD-EW interference



QCD-EW p_T spectrum harder than HEFT

- gg-initiated QCD NLO corrections (light fermion loop) computed in various approximations:
  — $m_{w,z} \to \infty$ limit — Anastasiou Boughezal Petriello 2009
  — soft approximation — Bonetti Melnikov Tancredi 2018
  — $m_{w,z} \to 0$ limit — Anastasiou VDD Furlan Mistlberger Moriello Schweitzer Specchia 2018

  and found to be about 5% wrt NLO (HEFT) cross section

- *qg*-initiated QCD-EW interference ?

NLO production rates

Process-independent procedure devised in the 90's

- slicing — Giele Glover Kosower 1992-93
- subtraction — Frixione Kunszt Signer 1995
  - dipole — Catani Seymour 1996
  - antenna — Kosower 1997; Campbell Cullen Glover 1998

based on universal collinear and soft currents

$$\sigma^{\rm NLO} = \int_{m+1} d\sigma^{\rm R}_{m+1} J_{m+1} + \int_m d\sigma^{\rm V}_m J_m$$

the 2 terms on the rhs are divergent in d=4

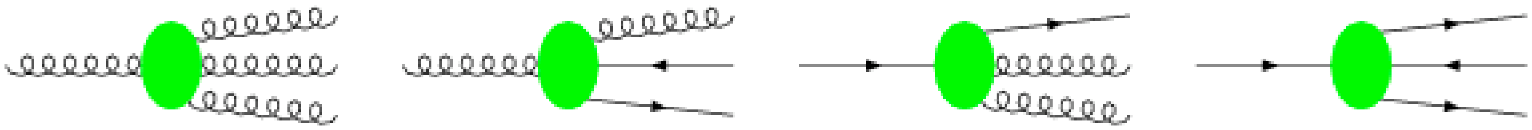
use universal IR structure to subtract divergences

$$\sigma^{\rm NLO} = \int_{m+1} \left[d\sigma^{\rm R}_{m+1} J_{m+1} - d\sigma^{\rm R,A}_{m+1} J_m \right] + \int_m \left[d\sigma^{\rm V}_m + \int_1 d\sigma^{\rm R,A}_{m+1} \right] J_m$$

the 2 terms on the rhs are finite in d=4

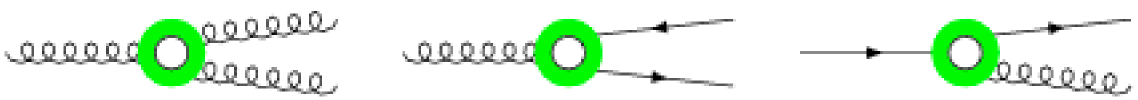
Collinear and soft currents at NNLO

- universal collinear and soft currents
  - tree 3-parton splitting functions and 2-soft-parton eikonal factors



J. Campbell N. Glover 1997; S. Catani M. Grazzini 1999; A. Frizzo F. Maltoni VDD 1999; D. Kosower 2002

  - one-loop 2-parton splitting functions and soft-gluon eikonal factor



Z. Bern L. Dixon D. Dunbar D. Kosower 1994; Z. Bern W. Kilgore C. Schmidt VDD 1998-99; D. Kosower P. Uwer 1999; S. Catani M. Grazzini 2000

NNLO cross section methods

- use universal IR structure to subtract double-real and real-virtual divergences

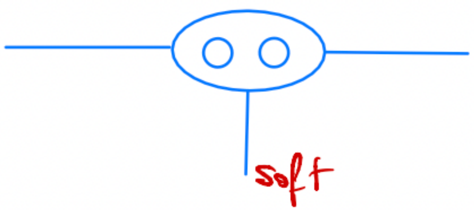
Sector decomposition	Denner Roth 1996; Binoth Heinrich 2000 Anastasiou, Melnikov, Petriello 2004
Antenna	Gehrmann-De Ridder, Gehrmann, Glover 2005
Colorful	Somogyi, Trocsanyi, VDD 2005; 2016
q_T	Catani, Grazzini 2007
Residue improved	Czakon 2010
N-jettiness	Boughezal, Focke, Liu, Petriello 2015 Gaunt Stahlhofen Tackmann Walsh 2015
Projection to Born	Cacciari, Dreyer, Karlberg, Salam, Zanderighi 2015
Nested soft-collinear	Caola Melnikov Röntsch 2017

Collinear and soft currents at N^3LO

- two-loop 2-parton splitting functions

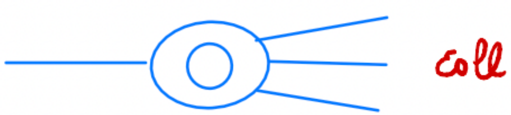
Z. Bern L. Dixon D. Kosower 2004; S. Badger N. Glover 2004;
C. Duhr T. Gehrmann M. Jacquier 2014

- two-loop soft-gluon eikonal factor



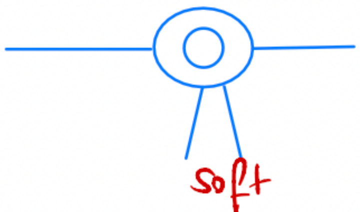
C. Duhr T. Gehrmann 2013; Y. Li H.X. Zhu 2013;
L. Dixon E. Herrmann K. Yan H.X. Zhu 2019

- one-loop 3-parton splitting functions



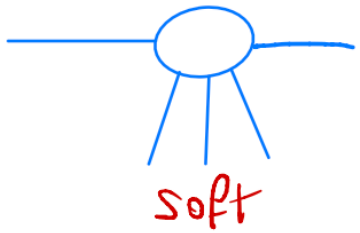
S. Catani D. de Florian G. Rodrigo 2003
S. Badger F. Buciuni T. Peraro 2015

- one-loop 2-soft-parton eikonal factor



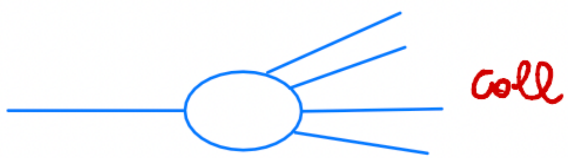
S. Catani L. Cieri 2021 (qqbar)

- tree 3-soft-parton eikonal factor



S. Catani D. Colferai A. Torrini 2019 (ggg)

- tree 4-parton splitting functions

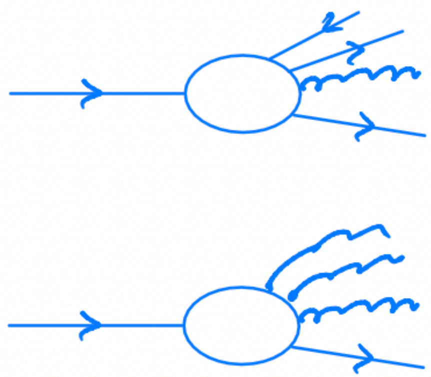


A. Frizzo F. Maltoni VDD 1999
T. Birthwright N. Glover V. Khoze P. Marquard 2005
C. Duhr 2006
C. Duhr R. Haindl A. Lazopoulos M. Michel VDD 2019-20

Tree 4-parton splitting functions

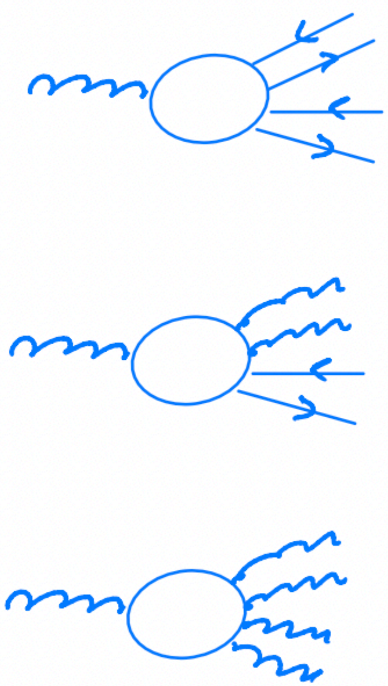
quark-parent splitting functions

C. Duhr R. Haindl A. Lazopoulos M. Michel VDD 2019

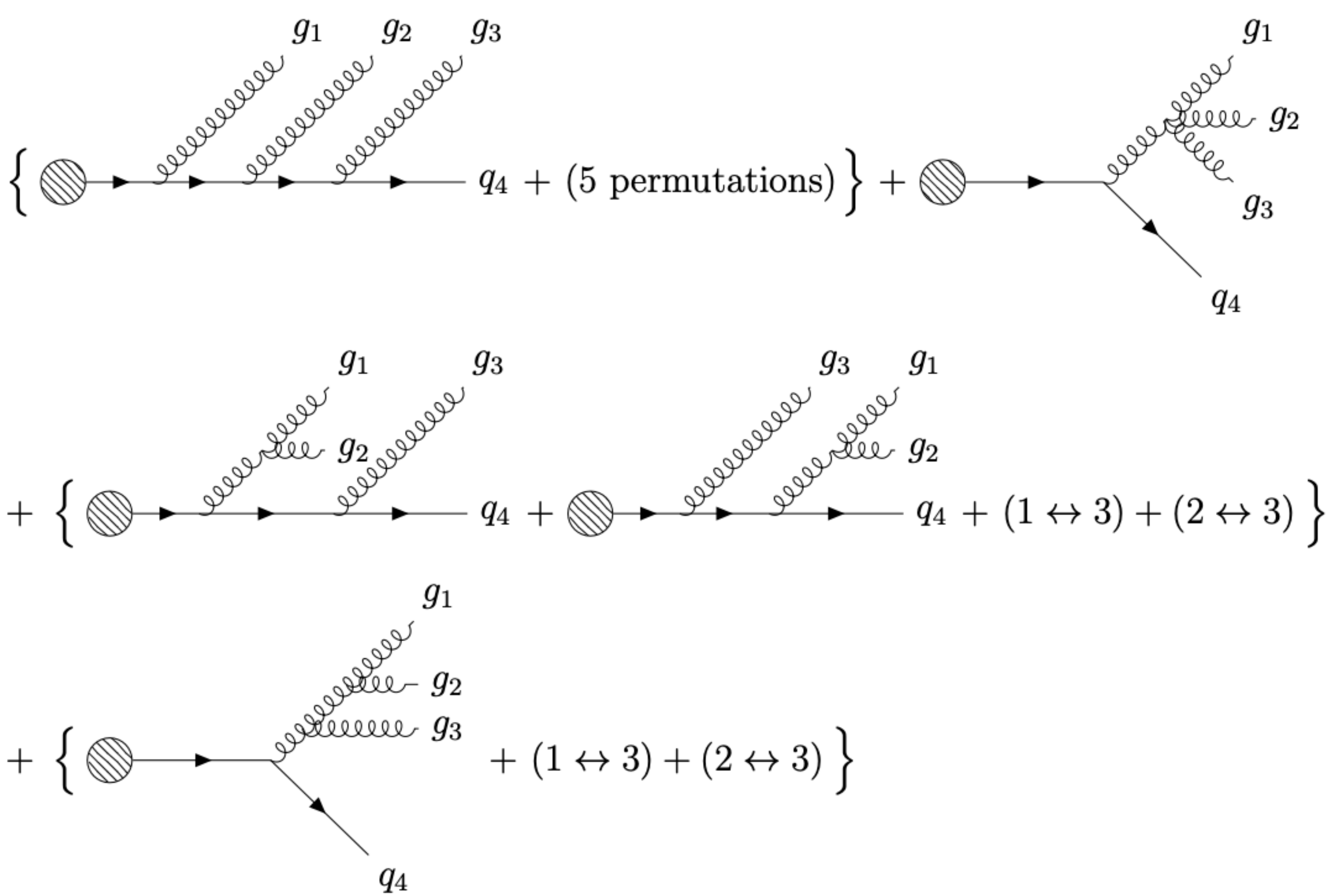


gluon-parent splitting functions

C. Duhr R. Haindl A. Lazopoulos M. Michel VDD 2020



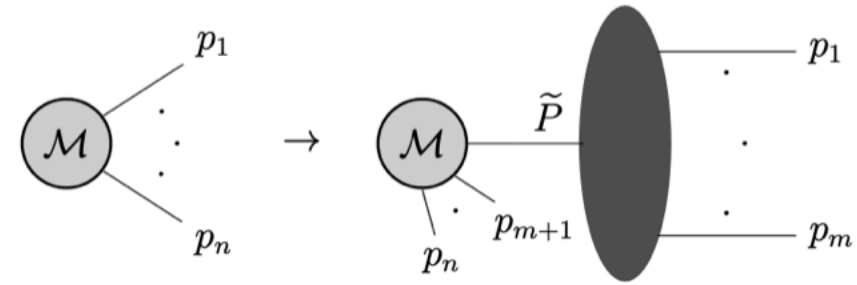
A sample: $q \to qggg$ splitting function



$$\langle \hat{P}_{g_1 g_2 g_3 q_4} \rangle = C_F^3 \langle \hat{P}^{(\mathrm{ab})}_{g_1 g_2 g_3 q_4} \rangle + C_F^2 C_A \langle \hat{P}^{(\mathrm{nab})_1}_{g_1 g_2 g_3 q_4} \rangle + \frac{3}{2} C_A^2 C_F \langle \hat{P}^{(\mathrm{nab})_2}_{g_1 g_2 g_3 q_4} \rangle$$

Collinear limit

- perform uniform rescaling $k_{\perp i} \to \lambda k_{\perp i}$
  keep leading term in the 1/λ expansion



$$\mathscr{C}_{1\ldots m}\mathcal{M}^{c_1\ldots c_n;s_1\ldots s_n}_{f_1\ldots f_n}(p_1,\ldots,p_n) = \mathbf{Sp}^{c,c_1\ldots c_m;s,s_1\ldots s_m}_{ff_1\ldots f_m}\,\mathcal{M}^{c,c_{m+1}\ldots c_n;s,s_{m+1}\ldots s_n}_{ff_{m+1}\ldots f_n}(\widetilde{P},p_{m+1},\ldots,p_n)$$

splitting amplitude

$$\mathscr{C}_{1\ldots m}\left|\mathcal{M}_{f_1\ldots f_n}(p_1,\ldots,p_n)\right|^2 = \left(\frac{2\mu^{2\epsilon}g_s^2}{s_{1\ldots m}}\right)^{m-1}\hat{P}^{ss'}_{f_1\ldots f_m}\,\mathcal{T}^{ss'}_{ff_{m+1}\ldots f_n}(\widetilde{P},p_{m+1},\ldots,p_n)$$

$$s_{1\ldots m} \equiv (p_1+\ldots+p_m)^2$$

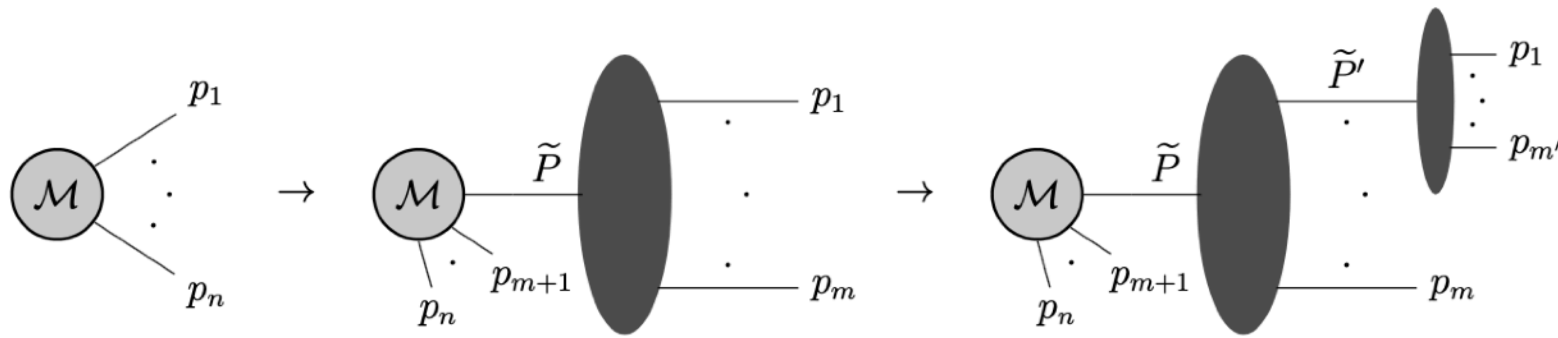
splitting function

helicity tensor

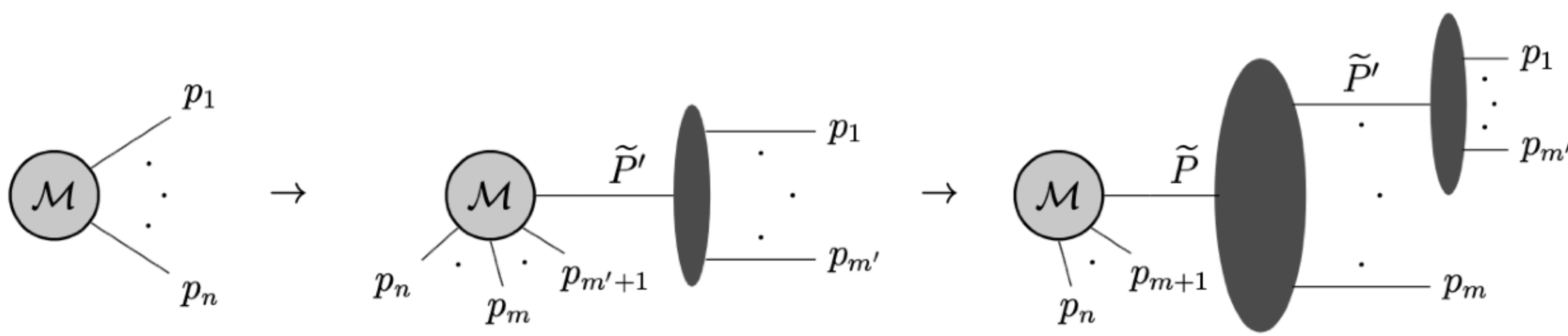
$$\left(\frac{2\mu^{2\epsilon}g_s^2}{s_{1\ldots m}}\right)^{m-1}\hat{P}^{ss'}_{f_1\ldots f_m} = \frac{1}{\mathcal{C}_f}\sum_{\substack{(s_1,\ldots,s_m)\\(c,c_1,\ldots,c_m)}}\mathbf{Sp}^{c,c_1\ldots c_m;s,s_1\ldots s_m}_{ff_1\ldots f_m}\left[\mathbf{Sp}^{c,c_1\ldots c_m;s',s_1\ldots s_m}_{ff_1\ldots f_m}\right]^*$$

Nested collinear limits

- $k_{\perp i} \to \lambda k_{\perp i}, \quad \kappa_{\perp i} \to \lambda' \kappa_{\perp i}$ **λ ≫ λ'**



iterated $\quad f \to f_1 + \ldots + f_{m'} + \ldots + f_m \to (f_1 + \ldots + f_{m'}) + \ldots + f_m$



strongly ordered $\quad f_{(1\ldots m')} \to f_1 + \ldots + f_{m'}$ and $f \to (f_1 + \ldots + f_{m'}) + \ldots + f_m$

different kinematic approaches, but same phase space region

$$\mathscr{C}_{1\ldots m'} \mathscr{C}_{1\ldots m} |\mathcal{M}_{f_1\ldots f_n}(p_1,\ldots,p_n)|^2 = \mathscr{C}_{(1\ldots m')\ldots m} \mathscr{C}_{1\ldots m'} |\mathcal{M}_{f_1\ldots f_n}(p_1,\ldots,p_n)|^2$$

iterated $\qquad\qquad$ strongly ordered

Strongly ordered collinear limit

C. Duhr R. Haindl A. Lazopoulos M. Michel VDD 2019

- $$\mathcal{C}_{(1\dots m')\dots m}\mathcal{C}_{1\dots m'}|\mathcal{M}_{f_1\dots f_n}(p_1,\dots,p_n)|^2$$
$$= \left(\frac{2g_s^2\mu^{2\epsilon}}{s_{1\dots m'}}\right)^{m'-1}\left(\frac{2g_s^2\mu^{2\epsilon}}{s_{[1\dots m']\dots m}}\right)^{m-m'} \hat{P}^{hh'}_{f_1\dots f_{m'}}\,\hat{H}^{hh';ss'}_{f_{(1\dots m')}f_{m'+1}\dots f_m}\,\mathcal{T}^{ss'}_{ff_{m+1}\dots f_n}(\tilde{P},p_{m'+1},\dots,p_n)$$

splitting function — splitting tensor — helicity tensor

if compare to the collinear limit,
this can be cast as the collinear factorisation of the splitting function

$$\mathcal{C}_{1\dots m'}\hat{P}^{ss'}_{f_1\dots f_m} = \left(\frac{s_{[1\dots m']\dots m}}{s_{1\dots m'}}\right)^{m'-1} \hat{P}^{hh'}_{f_1\dots f_{m'}}\,\hat{H}^{hh';ss'}_{f_{(1\dots m')}f_{m'+1}\dots f_m}$$

splitting tensor has *m-m'+1* flavour indices and plays the same role as the helicity tensor in the collinear limit, so summed over helicities yields the splitting function

$$\delta^{hh'}\,\hat{H}^{hh';ss'}_{f_{(1\dots m')}f_{m'+1}\dots f_m} = \hat{P}^{ss'}_{f_{(1\dots m')}f_{m'+1}\dots f_m}$$

m=3, *m'*=2, *m-m'*+1=2 — G. Somogyi Z. Trocsanyi VDD 2005

m=4, *m'*=3, *m-m'*+1=2
m'=2, *m-m'*+1=3 — C. Duhr R. Haindl A. Lazopoulos M. Michel VDD 2019-2020

Soft limit

- soft gluon of momentum p_1

$$\mathscr{S}_1|\mathcal{M}_{g_1 f_2 \ldots f_n}|^2$$

$$= \mu^{2\epsilon} g_s^2 \sum_{j,k=2}^{n} \mathcal{S}_{jk}(p_1) \left[\mathcal{M}_{f_2 \ldots f_n}^{c_2 \ldots c'_j \ldots c_k \ldots c_n; s_2 \ldots s_n}\right]^* \mathbf{T}_{c'_j c_j}^{c_1} \mathbf{T}_{c_k c'_k}^{c_1} \mathcal{M}_{f_2 \ldots f_n}^{c_2 \ldots c_j \ldots c'_k \ldots c_n; s_2 \ldots s_n}$$

colour-correlated amplitudes

with eikonal function

$$\mathcal{S}_{jk}(p_1) = -\frac{2\, s_{jk}}{s_{1j} s_{1k}}$$

the short-hand is

$$\mathbf{T}_j \cdot \mathbf{T}_k \, |\mathcal{M}_{(j,k)}|^2 \equiv \left[\mathcal{M}_{f_2 \ldots f_n}^{c_2 \ldots c'_j \ldots c_k \ldots c_n; s_2 \ldots s_n}\right]^* \mathbf{T}_{c'_j c_j}^{c_1} \mathbf{T}_{c_k c'_k}^{c_1} \mathcal{M}_{f_2 \ldots f_n}^{c_2 \ldots c_j \ldots c'_k \ldots c_n; s_2 \ldots s_n}$$

Soft limit of splitting functions

- soft gluon of momentum p_1 in a set of m collinear partons

$$\mathcal{C}_{1\dots m}\mathcal{S}_1|\mathcal{M}_{g_1 f_2\dots f_n}|^2$$

$$= \mu^{2\epsilon} g_s^2\, \mathcal{T}^{ss'} \frac{1}{\mathcal{C}_f} \sum_{j,k=2}^{m} U_{jk;1} \left[\mathbf{Sp}_{f f_2\dots f_m}^{c,c_2\dots c_j'\dots c_k\dots c_m;s'}\right]^* \mathbf{T}^{c_1}_{c_j' c_j} \mathbf{T}^{c_1}_{c_k c_k'} \mathbf{Sp}_{f f_2\dots f_m}^{c,c_2\dots c_j\dots c_k'\dots c_m;s}$$

obtained using colour coherence: hard amplitude factorises

with $$U_{jk;l} \equiv 2\left(-\frac{s_{jk}}{s_{jl}s_{kl}} + \frac{z_k}{z_l s_{kl}} + \frac{z_j}{z_l s_{jl}}\right)$$

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- $$\mathcal{S}_1\left[\left(\frac{2\mu^{2\epsilon}g_s^2}{s_{1\dots m}}\right)^{m-1} \hat{P}^{ss'}_{g_1 f_2\dots f_m}\right]$$

$$= \mu^{2\epsilon} g_s^2 \frac{1}{\mathcal{C}_f} \sum_{j,k=2}^{m} U_{jk;1} \left[\mathbf{Sp}_{f f_2\dots f_m}^{c,c_2\dots c_j'\dots c_k\dots c_m;s'}\right]^* \mathbf{T}^{c_1}_{c_j' c_j} \mathbf{T}^{c_1}_{c_k c_k'} \mathbf{Sp}_{f f_2\dots f_m}^{c,c_2\dots c_j\dots c_k'\dots c_m;s}$$

$m=2$ $$\mathcal{S}_1\left[\left(\frac{2\mu^{2\epsilon}g_s^2}{s_{12}}\right) \hat{P}^{ss'}_{g_1 f_2}\right] = \mu^{2\epsilon} g_s^2 \frac{4(1-z_1)}{z_1 s_{12}} C_2\, \delta^{ss'}$$ DGLAP

$$C_2 = C_A, C_F$$

m=3

$$\mathcal{S}_1\left[\left(\frac{2\mu^{2\epsilon}g_s^2}{s_{123}}\right)^2\hat{P}^{ss'}_{g_1f_2f_3}\right]$$
$$=\mu^{2\epsilon}g_s^2\left[U_{22;1}\mathbf{T}_2^2+U_{23;1}(\mathbf{T}_P^2-\mathbf{T}_2^2-\mathbf{T}_3^2)+U_{33;1}\mathbf{T}_3^2\right]\left(\frac{2\mu^{2\epsilon}g_s^2}{s_{23}}\right)\hat{P}^{ss'}_{f_2f_3}$$

colour coefficient of parent parton $\mathbf{T}_P=\mathbf{T}_2+\mathbf{T}_3$

$q\to g_1g_2q_3$

$$\mathcal{S}_1\left[\left(\frac{2\mu^{2\epsilon}g_s^2}{s_{123}}\right)^2\hat{P}^{ss'}_{g_1g_2q_3}\right]=2\mu^{2\epsilon}g_s^2\left[\frac{2z_3}{z_1s_{13}}C_F+\left(\frac{s_{23}}{s_{12}s_{13}}+\frac{z_2}{z_1s_{12}}-\frac{z_3}{z_1s_{13}}\right)C_A\right]\left(\frac{2\mu^{2\epsilon}g_s^2}{s_{23}}\right)\hat{P}^{ss'}_{g_2q_3}$$

$g\to g_1\bar{q}_2q_3$

$$\mathcal{S}_1\left[\left(\frac{2\mu^{2\epsilon}g_s^2}{s_{123}}\right)^2\hat{P}^{ss'}_{g_1\bar{q}_2q_3}\right]=2\mu^{2\epsilon}g_s^2\left[2\frac{s_{23}}{s_{12}s_{13}}C_F+\left(-\frac{s_{23}}{s_{12}s_{13}}+\frac{z_2}{z_1s_{12}}+\frac{z_3}{z_1s_{13}}\right)C_A\right]\left(\frac{2\mu^{2\epsilon}g_s^2}{s_{23}}\right)\hat{P}^{ss'}_{\bar{q}_2q_3}$$

$g\to g_1g_2g_3$

$$\mathcal{S}_1\left[\left(\frac{2\mu^{2\epsilon}g_s^2}{s_{123}}\right)^2\hat{P}^{ss'}_{g_1g_2g_3}\right]=2\mu^{2\epsilon}g_s^2\left(\frac{s_{23}}{s_{12}s_{13}}+\frac{z_2}{z_1s_{12}}+\frac{z_3}{z_1s_{13}}\right)C_A\left(\frac{2\mu^{2\epsilon}g_s^2}{s_{23}}\right)\hat{P}^{ss'}_{g_2g_3}$$

G. Somogyi Z. Trocsanyi VDD 2005

m=4

simplest case is $q \to g_1 \bar{q}'_2 q'_3 q_4$

$$\mathcal{S}_1 \left[\left(\frac{2\mu^{2\epsilon} g_s}{s_{1234}} \right)^3 \hat{P}^{ss'}_{g_1 \bar{q}'_2 q'_3 q_4} \right] = \mu^{2\epsilon} g_s^2 \left[C_F B^{(q)}_{23,4} + C_A A^{(q)} \right] \left(\frac{2\mu^{2\epsilon} g_s^2}{s_{234}} \right)^2 \hat{P}^{ss'}_{\bar{q}'_2 q'_3 q_4}$$

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with

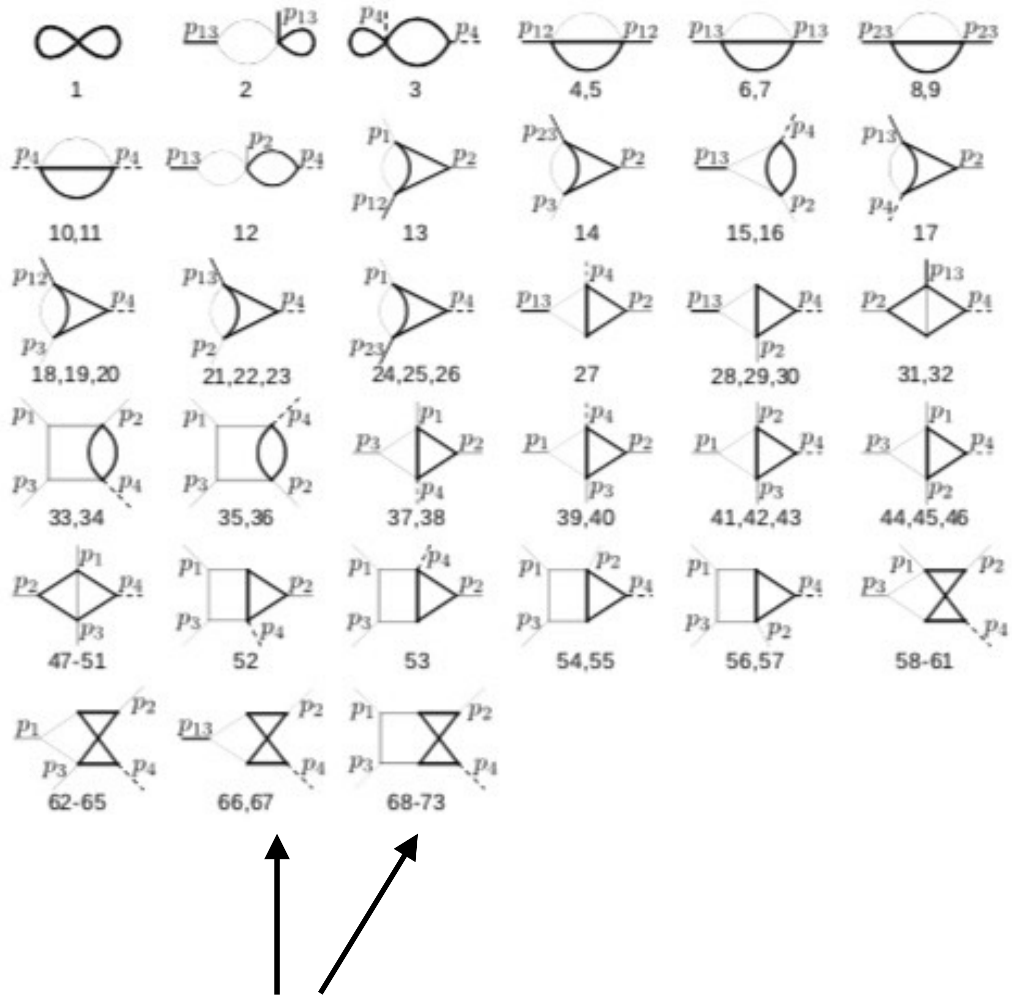
$$A^{(q)} = \frac{4z_2}{s_{12} z_1} - \frac{2z_3}{s_{13} z_1} - \frac{2z_4}{s_{14} z_1} - \frac{2s_{23}}{s_{12} s_{13}} - \frac{2s_{24}}{s_{12} s_{14}} + \frac{4s_{34}}{s_{13} s_{14}}$$

$$B^{(q)}_{ij,k} = \frac{4s_{ij}}{s_{1i} s_{1j}} + \frac{8s_{ik}}{s_{1i} s_{1k}} - \frac{8s_{jk}}{s_{1j} s_{1k}} - \frac{8z_i}{s_{1i} z_1} + \frac{8z_j}{s_{1j} z_1} + \frac{4z_k}{s_{1k} z_1}$$

up to replacing $U_{jk;l}$ with the suitable factor,
the soft-gluon limit of splitting functions is valid for any soft emission characterised by two-parton colour correlations
(soft *qqbar* pair, non-abelian part of two soft gluons, etc.)

Back-up slides

73 Master Integrals



elliptic

gg-initiated NLO corrections in HEFT $\sigma_{gg}^{NLO,QCD} = 33.24 \text{ pb}$

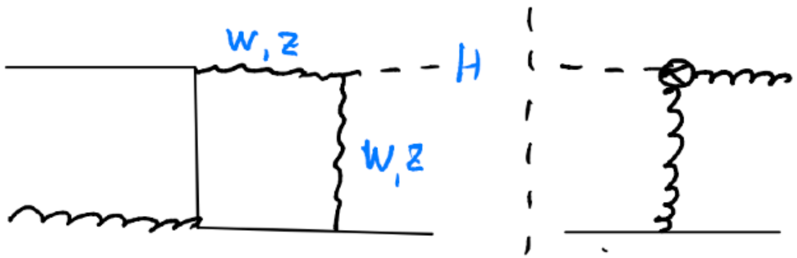
gg-initiated QCD NLO corrections (light fermion loop)

$m_{w,z} \to \infty$ limit $\sigma_{gg,R(hb)}^{NLO,QCD-EW} = 34.98 \text{ pb}, (= \sigma_{gg}^{NLO,QCD} + 5.23\%)$

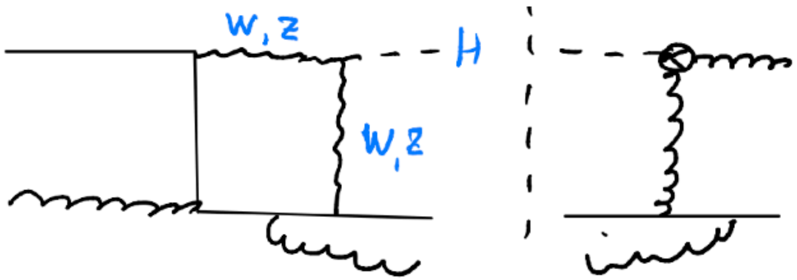
$m_{w,z} \to 0$ limit $\sigma_{gg,R(lb)}^{NLO,QCD-EW} = 35.03 \text{ pb}, (= \sigma_{gg}^{NLO,QCD} + 5.39\%)$

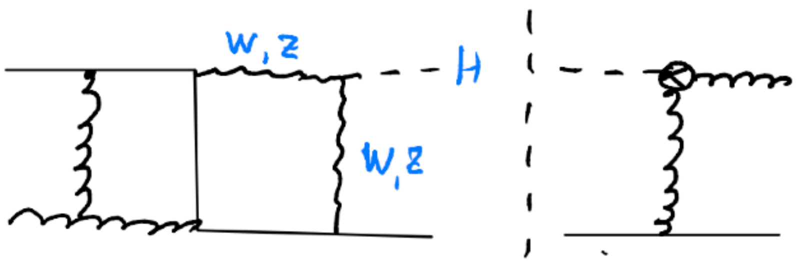
Anastasiou VDD Furlan Mistlberger Moriello Schweitzer Specchia 2018

qg-initiated QCD-EW interference



$\mathcal{O}(\alpha_s^2\alpha^2)$





$\mathcal{O}(\alpha_s^3\alpha^2)$

Becchetti Bonciani Casconi VDD Moriello 2018 (planar MIs)
Becchetti Moriello Schweitzer 2021(?) (non-planar MIs)