Lecture 1: Probability

Frequentism

Bayesian

Motivation

Probability is a fundamental building block for data analysis

Experimental uncertainties are statements of probability

What is probability? Two common definitions:

Frequentism & Bayesian

Frequentist probability:

Probability of outcome A is defined as fraction of the outcomes in N identical measurements.

$$P(A) = \lim_{N \to \infty} \frac{N_A}{N}$$

E.g: coin flipping

Key advantage of frequentism: Objective. No need for prior beliefs

Frequentist interpretation

Probability is a property of the ensemble, not of the coin itself.

e.g. probability to have long hair.

Count 15 people with a class from a class of 30 with long hair

$$P(\text{long hair}) = 50\%$$

I cannot say that the probability for a particular person to have long hair is 50%.

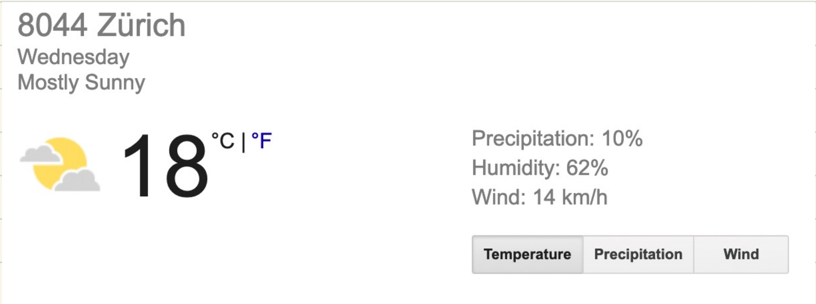
I can say that a sample representative of the class, 50% of them will have long hair.

Frequentist limitations

Frequentist probability only works when one can perform “identical trials”.

Example: will it rain tomorrow?

No way to repeat “tomorrow” many times - frequentist interpretation impossible?



Frequentist definition: under the current conditions of my weather equipment, 10% of days afterwards had rain.

What about truly unique events: Is this physics theory correct?

Bayesian probability

Bayesian probability represents degree of belief that outcome A is true.

It is subjective by nature.

Unlike frequentism, it can be used for any outcome.

Central equation in Bayesian probability is Bayes' theorem:

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

Relating conditional probabilities

We use this equation instinctively all the time (eg pack of cards).

Bayesian probability in science

In science we use the following form of Bayes' theorem:

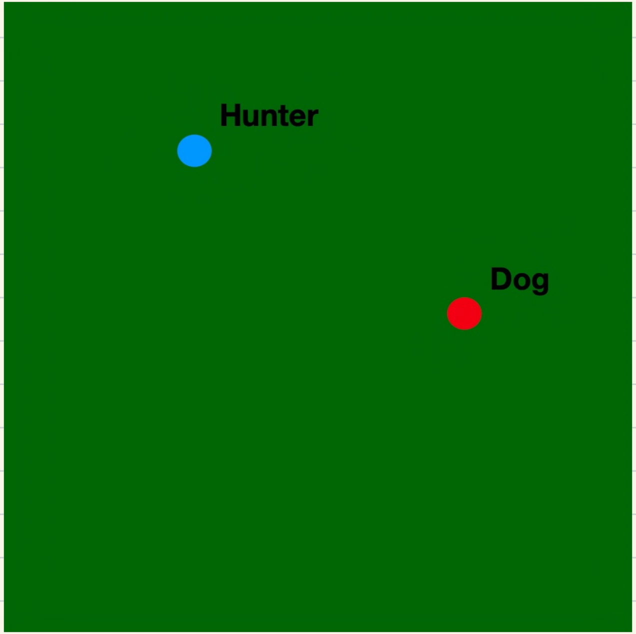
$$P(theory|result) = \frac{P(result|theory)P(theory)}{P(result)}$$

Our prior belief is updated by the result to become the posterior probability.

Normally this is automatic, but sometimes intuitions fail (Monty Hall).

Boundary conditions

Imagine a hunter and their dog, the hunter is uniformly distributed in a field.



Given the hunters location, probability for the dog to be closer than 100m is 90%.

Given the dogs location, probability for the hunter to be closer than 100m =??

What if there is a river that only the dog can cross?

Summary

Two mutually incompatible ways of interpreting probability.

Frequentism: Objective

Bayesian: Subjectibe

How to interpret measurement under these approaches?

$$\text{Measurement} = \text{central value} \pm \text{uncertainty}\,[\text{unit}]$$

Not important to know which one is better.

Important to know they both exist and lead to completely different branches of statistics.

Also important: in most situations you get the same statistical answer for both frequentist and Baysesian approaches.