MMP I
Tutorial 4

HS 2019
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Exercise 1: Fourier Transform (3 Pts.)

Compute the Fourier transform of the following functions:

a) $f(x) = \begin{cases} 1 - \dfrac{|x|}{2} & \text{if} \quad |x| \leq 2 \\ 0 & \text{if} \quad |x| > 2 \, ; \end{cases}$

b) $g(x) = \begin{cases} 1 - \dfrac{x}{2} & \text{if} \quad |x| \leq 2 \\ 0 & \text{if} \quad |x| > 2 \, ; \end{cases}$

c) $h(x) = f(x) * f(x)$, where $*$ indicates the convolution.

Exercise 2: Differential equations (5 Pts.)

Find the general solution (in the form $y = y(x)$ or $f(x, y) = 0$) for the following differential equations

a) $y' = \frac{x}{y} + \frac{y}{x}$ (1 Pt.);

b) $y' = xy^2 e^x$ (1 Pt);

c) $y' = \frac{y}{2x+y}$ (1 Pt.);

d) $y' = -\frac{x+y\sin(xy)}{1+x\sin(xy)}$ (2 Pts.).

– please turn over –

Exercise 3: Parachutists (7 Pts.)

Once a parachutist jumps from an airplane, there are two forces that determine his motion: the pull of the earth's gravity and the opposing force of air resistance. At high speeds (in the regime of turbulent flow, which corresponds to a high Reynold's number), the strength of the air resistance force can be expressed as $K_1 v^2$, where v is the speed with which the jumper descends and K_1 is a proportionality constant. Once the parachute opens, the descent speed decreases greatly. In this case, the air flow is called "laminar", and the Reynold's number is smaller than 1; the strength of the air resistance force is then given by $K_2 v$.

a) Write down and solve the differential equation for $v(t)$ for freefalling without a parachute. Use the initial condition $v(t = 0) = 0$. After a certain amount of time, the parachutist reaches a stationary velocity v_s. Find v_s and express your solution in terms of it.

b) What is the stationary velocity $v_{s,1}$ for a freefalling parachutist of total mass 70 kg if $K_1 = 1/4$ kg/m? How long does it take to reach this velocity? How long does it take to reach 99% of the stationary velocity?

c) Write down and solve the differential equation for $v(t)$ once the parachute opens, if $v(0) = v_{s,1}$. What is the parachutist's stationary velocity $v_{s,2}$ under the parachute if $K_2 = 110$ kg/s? Check whether your result for $v(t)$ satisfies your expectations for the limits $t \to 0$ and $t \to \infty$. Express the solution in terms of $v_{s,1}$ and $v_{s,2}$.

d) Write down and solve the differential equation for the parachutist's altitude $y(t)$ under the parachute (once it has opened), if $y(0) = 0$.