Applications of General Relativity
Sheet 8

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Exercise 1 [Hamiltonian geodesic formulation in PN framework]

Geodesics are commonly expressed in terms of the Lagrangian,

$$L = \frac{1}{2} g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu \, , \tag{1}$$

where the dots are derivatives with respect to the affine parameter. An equivalent, yet generally more numerically stable formulation, exists in terms of the Hamiltonian

$$H = \frac{1}{2} g^{\mu\nu} p_\mu p_\nu \, , \tag{2}$$

with the conjugate momenta

$$p_\mu = \frac{\partial L}{\partial \dot{x}^\mu} \, . \tag{3}$$

The solutions to Hamilton's equations

$$\dot{x}^\mu = \frac{\partial H}{\partial p_\mu} \, , \quad \dot{p}_\mu = -\frac{\partial H}{\partial x^\mu} \, , \tag{4}$$

are geodesics.

a) Write down the Lagrangian and Hamiltonian of the Schwarzschild metric.

b) Write down Hamilton's equations for a test particle trajectory in the Schwarzschild spacetime.

c) Expand the equations of motion for both a massive and massless particle to first post-Newtonian order.

d) Integrate these equations numerically. Produce a plot of a few trajectories. Compare the full solution to the expanded one.