The Minimal Supersymmetric Standard Model

We now have all the ingredients to write $\mathcal{L}_{SUSY} + \mathcal{L}_{soft}$ describing the minimal SUSY extension of the SM

$\uparrow$

particle content

$\Downarrow$

SM "matter" content

$\cap$

SUSY chiral superfield

Spin ϕ	Spin $\frac{1}{2}$	$SU(3)\times SU(2)\times U(1)_Y$
	$Q_L = (u_L, d_L)^T$	$(3, 2, \frac{1}{2})$
	u_R	$(3, 1, +\frac{2}{3})$
	d_R	$(3, 1, -\frac{1}{3})$
	$L_L = (\nu, e_L)^T$	$(1, 2, -\frac{1}{2})$
	e_R	$(1, 1, -1)$
H		$(1, 2, -\frac{1}{2})$

$\Rightarrow$ One could try to put in the same chiral multiplet L & H (same gauge quantum numbers) but this does not work for various phenomenological reasons ($\rightarrow$ violation of lept. number)

$\Rightarrow$ $\mathcal{L}^{SM}_{Yukawa}$ contains several conjugate fields, while we need to construct it via the holomorphic W (superpotential) $\rightarrow$ not a problem for the fermion fields (identify u_R^+, d_R^+, e_R^+ with the basic chiral fields), but problematic for the Higgs $\rightarrow$ need a 2nd Higgs field

$$\left(\bar{u}_R Q_L H^+ + \bar{d}_R Q_L H \rightarrow u^c Q H_u + d^c Q H_d \right)$$

A second Higgs is also needed for completely independent reason (cancellation of gauge anomalies due to the fermion component of the superfield)

MSSM matter content

	spin 0	spin 1/2	$SU(3)\times SU(2)\times U(1)_Y$
Q	$\tilde{Q}_L=(\tilde{u}_L,\tilde{d}_L)^T$	$Q_L=(u_L\ d_L)^T$	$(3,2,\frac{1}{2})$
u^c	$\tilde{u}_R^{+}$	u_R^{+}	$(\bar{3},1,-\frac{2}{3})$
d^c	$\tilde{d}_R^{+}$ ("squarks")	d_R^{+}	$(\bar{3},1,+\frac{1}{3})$
L	$\tilde{L}_L=(\tilde{\nu}_L,\tilde{e}_L)^T$	$L_L=(\nu_L,e_L)^T$	$(1,2,-\frac{1}{2})$
e^c	$\tilde{e}_R^{+}$	e_R^{+}	$(1,1,1)$
H_u	$(H_u^+,H_u^0)^T$	$(\tilde{H}_u^+,\tilde{H}^0)^T$	$(1,2,\frac{1}{2})$
H_d	$(H_d^0,H_d^-)^T$	$(\tilde{H}_d^0,\tilde{H}^-)^T$ "higgsinos"	$(1,2,-\frac{1}{2})$

MSSM gauge sector

Spin 1/2	Spin 1	
$\tilde{g}^a$	g^a	$(8,1,0)$
$\tilde{W}^\pm$ $\tilde{W}^3$	$W^\pm, W^3$	$(1,3,0)$
$\tilde{B}^0$	B^0	$(1,1,0)$

↑ "gauginos"

The MSSM Superpotential:

Given the MSSM field content, the superpotential that allow us to reproduce the Yukawa sector of the SM and gives a mass term to the Higgs is

$$W_{MSSM} = u^c \, Y_u \, Q \, H_u + d^c \, Y_d \, Q \, H_d + e^c \, Y_e \, L \, H_d + \mu \, H_u H_d$$

Y_u: 3×3 matrices in flavor space

μ: so-called μ-term

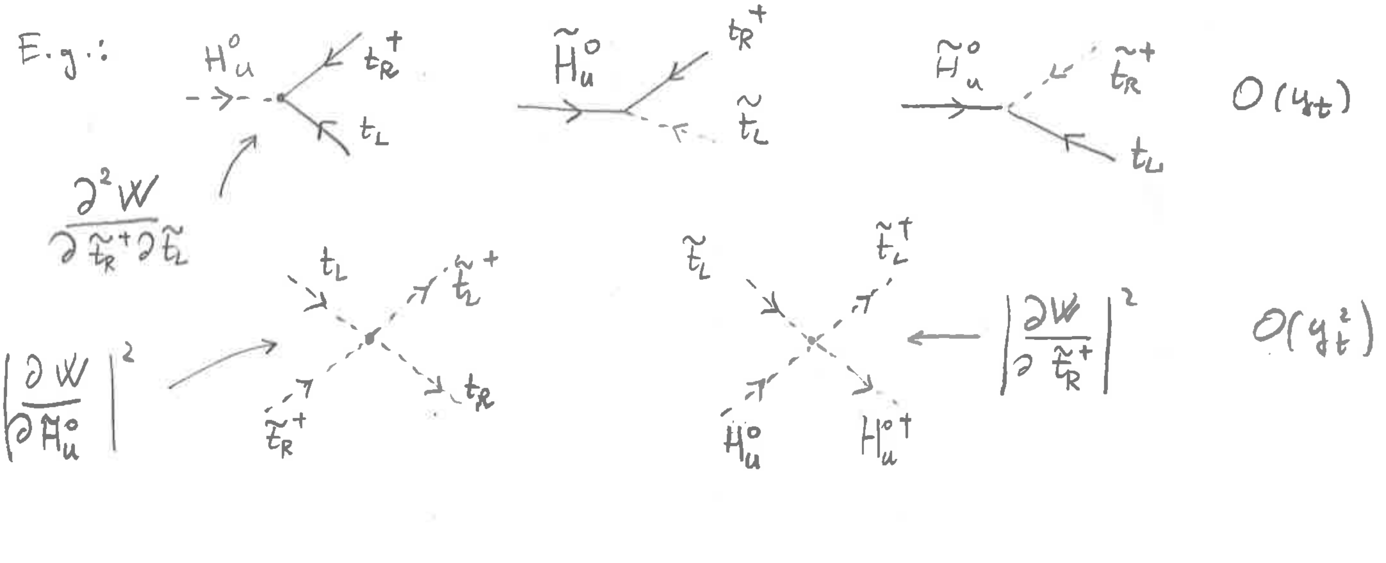
(flavor, SU(3) & SU(2) indices omitted)

Some observations:

1) Less parameters than in the SM, at this stage ($\rightarrow$ no analog of $\lambda (H^\dagger H)^2$) despite many more fields ($\rightarrow$ re-call no non-holomorphic terms)

2) Given the general rules of $\mathcal{L}_{SUSY}$, the Yukawa couplings generates both "true-Yukawa" interaction but also quartic interaction $\rightarrow$ but no quartic Higgs appears

E.g.:



3) The only dimensional parameter in W_{MSSM} is μ

↳ Higgsino mass term $\qquad \delta\mathcal{L} = -\mu(\tilde{H}_u^+ \tilde{H}_d^- - \tilde{H}_u^0 \tilde{H}_d^0)$

↳ Higgs mass term $\qquad \delta\mathcal{L} = -|\mu|^2 \left(|\tilde{H}_u^0|^2 + |\tilde{H}_d^0|^2 + |\tilde{H}_u^+|^2 + |\tilde{H}_d^-|^2\right)$
(always positive)

↳ need soft-breaking terms
to generate a negative Higgs mass terms

$\Updownarrow$

μ must be $O(M_{soft})$

R-parity

Can we add other terms to W_{MSSM} compatible with the SM gauge symmetry?

In principle yes:

$$W_{\Delta L=1} = \frac{1}{2}\lambda_1^{ijk} L_i L_j e_k^c + \lambda_2^{ijk} L_i Q_j d_k^c + (\mu')^i L_i H_u$$

(i, j, k = flavor indices)

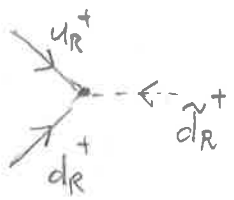
(N.B.: This is because L has the same gauge quantum numbers as H_d)

$$W_{\Delta B=1} = \frac{1}{2}\lambda_3^{ijk} u_i^c d_j^c d_k^c$$

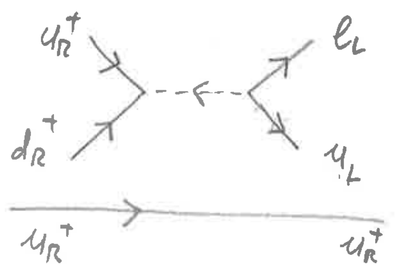
These extra terms are very dangerous since they violate Lepton and Baryon numbers, that are naturally (accidentally) conserved in the SM.

N.B: All vertices generated by $W_{\Delta L=1}$ & $W_{\Delta B=1}$ involve at least 1 "super-partner"

e.g.



$\Rightarrow$



$\bar{p} \to \ell^- \pi^0$

A way out is to postulate a new symmetry

$$\boxed{P_M = (-1)^{3(B-L)}}$$

$P_M \{Q, L, u^c, d^c, e^c\} = -1 * \{\ \}$ on superfields

$P_M \{H_u, H_d, \text{gauge}\} = +1 * \{\ \}$

And to impose it as an exact symmetry on $\mathcal{L}_{SUSY}$. It commutes with SUSY (imposed on superfields) and is not broken by chiral anomalies (contrary to B and L) $\Rightarrow$ good candidate for exact symm. of Nature

$P_M \; W_{MSSM} = + W_{MSSM}$

$\left.\begin{array}{l} P_M \; W_{\Delta L=1} = - W_{\Delta L=1} \\ P_M \; W_{\Delta B=1} = - W_{\Delta B=1} \end{array}\right\}$ forbidden

It is often convenient to "trade" P_M (matter parity) with R-parity, defined as

$$\boxed{P_R = (-1)^{3(B-L)+2S}}$$ on each comp. of the superfield

Because of Lorentz invariance, $(-1)^{2S}$ is conserved in any vertex of the theory $\Rightarrow$ P_R is equivalent to P_M

$\Downarrow$

Easy to check that P_R (SM particles) $= +1$, P_R (gauginos, higgsinos, squarks, sleptons) $= -1$

$\Downarrow$

If we impose R-parity (or equivalently P_M discrete symmetry) the lightest SUSY particle is stable $\rightarrow$ Dark matter candidate

Soft-breaking terms in the MSSM:

Following the general rules for soft susy breaking and retaining only the soft-breaking terms compatible with the SM gauge symmetry we have

$$\mathcal{L}^{MSSM}_{soft} = -\frac{1}{2}\left(M_3 \tilde{g}\tilde{g} + M_2 \tilde{W}\tilde{W} + M_1 \tilde{B}\tilde{B}\right) + h.c.$$

$$- \left(\tilde{Q}_L^{+} m_Q^2 \tilde{Q}_L + \tilde{L}_L^{+} m_L^2 \tilde{L}_L + \tilde{u}_R^{+} m_u^2 \tilde{u}_R + \tilde{d}_R^{+} m_d^2 \tilde{d}_R + \tilde{e}_R^{+} m_e^2 \tilde{e}_R \right)$$

$$- \left(\tilde{u}_R^{+} a_u \tilde{Q}_L H_u + \tilde{d}_R^{+} a_d \tilde{Q}_L H_d + \tilde{e}_R^{+} a_e \tilde{L}_L H_d + h.c.\right)$$

$$- \left(m^2_{H_u} H_u^{+} H_u + m^2_{H_d} H_d^{+} H_d \right) - \left(b\, H_u H_d + h.c.\right)$$

gaugino mass terms

Squark & slepton mass terms

Higgs-sfermion trilinear couplings

Higgs soft mass terms

N.B.: $\{ m_Q^2, m_d^2, m_u^2, m_e^2, m_L^2 \}$ are 3×3 hermitian matr. in flavor space

$\{ a_u, a_d, a_e \}$ are 3×3 non-hermitian matr. in flavor space

↳ Several free parameters

⇓

$$\boxed{105 \text{ new free parameters}}$$

$m_Q^2, m_d^2, m_u^2, m_e^2, m_L^2 \oplus m^2_{H_u}, m^2_{H_d}, b \sim m^2_{soft}$

$M_1, M_2, M_3, a_u, a_d, a_e \sim m_{soft}$

(if common origin)