HS 21
Prof. Ph. Jetzer

MMP I
Exercise Sheet 4

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https://www.physik.uzh.ch/en/teaching/PHY312

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Exercise 1 [Vector space (4 points)]

Consider a $\mathbb{C}$-vector space V in which a scalar product $\langle\cdot|\cdot\rangle : V \times V \to \mathbb{C}$ and a norm $||x|| = \sqrt{\langle x|x\rangle}$ are defined.
Show that for any orthonormal system $\{y_k \in V | \langle y_k|y_l\rangle = \delta_{kl}\}$ the following relations are valid:

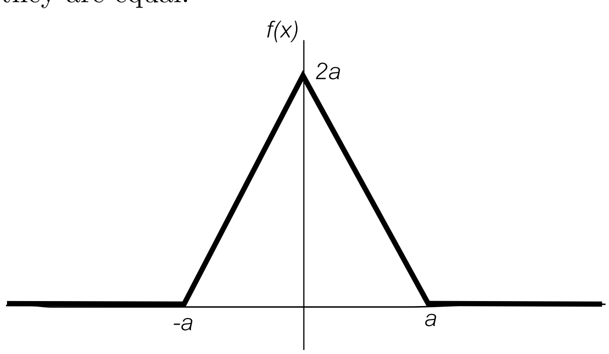
a) Bessel identity:
$$||x||^2 = \sum_{k=1}^{n} |\langle x|y_k\rangle|^2 + ||x - \sum_{k=1}^{n}\langle x|y_k\rangle y_k||^2.$$

b) Bessel inequality:
$$||x||^2 \geq \sum_{k=1}^{n} |\langle x|y_k\rangle|^2.$$

c) Schwarz inequality:
$$||x|| \cdot ||y|| \geq |\langle x|y\rangle| \quad \forall x, y \in V.$$

Exercise 2 [Fourier Transform and Fourier Integral (2 points)]

Find both cosine and exponential Fourier transforms of the function $f(x)$ and explain why they are equal.



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Exercise 3 [Finite Wave (4 points)]

a) Find the Fourier transform $g(\omega)$ of a finite wave $f(t)$ with angular frequency ω_0, given by

$$f(t) = \begin{cases} \sin\omega_0 t, & \text{for} \quad -t_0 < t < t_0 \\ 0, & \text{otherwise} \end{cases} .$$

$t_0 = N\pi/\omega_0$ is chosen so that the wave contains precisely N full oscillations.

b) Why is $g(\omega)$ not just a peak at ω_0?

c) Estimate $g(\omega)$ for the following special cases:

i) $|\omega| \ll |\omega_0|$

ii) $|\omega - \omega_0| \ll |\omega_0|$

iii) $|\omega| \gg |\omega_0|$,

and draw a sketch of the function. Consider also the zeroes of $g(\omega)$.

d) What happens when $N \to \infty$?