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Issued: 22.05.2020

Website: <http://www.physik.uzh.ch/en/teaching/PHY519/>

Discussion: 29.05.2020

**Exercise 1** [Post-Newtonian Lagrangian for a binary system]

The gravitational action for a particle  $a$  propagating in the metric  $g_{\mu\nu}$  is given by

$$S_a = -m_a c \int dt \left( -g_{\mu\nu} \frac{dx_a^\mu}{dt} \frac{dx_a^\nu}{dt} \right)^{1/2}. \quad (1)$$

In this problem we are considering two masses  $m_1$  and  $m_2$  with separation  $r$ . Note that the total action is given by  $S = S_1 + S_2$  and that particle 1 propagates in the metric generated by particle 2 and vice versa.

(i) Expand the action to order 1PN.

(ii) Expand the energy momentum tensor

$$T_a^{\mu\nu} = \frac{1}{\sqrt{-g}} \frac{dt}{d\tau_a} m_a \frac{dx_a^\mu}{dt} \frac{dx_a^\nu}{dt} \delta^{(3)}(\mathbf{x} - \mathbf{x}_a(t)) \quad (2)$$

to order 1PN. Here  $\tau_a$  is the proper time of particle  $a$ .

(iii) Calculate the required metric corrections from the above energy momentum tensor. Recall that the metric is given order by order as

$${}^{(2)}g_{00} = -2\phi \quad {}^{(2)}g_{ij} = -2\phi\delta_{ij} \quad (3)$$

$${}^{(3)}g_{0i} = \zeta_i + \partial_i \partial_0 \chi \quad {}^{(4)}g_{00} = -2(\phi^2 + \psi) \quad (4)$$

where  $\phi$ ,  $\zeta_i$  and  $\psi$  are defined as

$$\phi(t, \mathbf{x}) = -\frac{G}{c^4} \int \frac{d^3x'}{|\mathbf{x} - \mathbf{x}'|} {}^{(0)}T^{00}(t, \mathbf{x}') \quad (5)$$

$$\zeta_i(t, \mathbf{x}) = -\frac{4G}{c^4} \int \frac{d^3x'}{|\mathbf{x} - \mathbf{x}'|} {}^{(1)}T^{0i}(t, \mathbf{x}') \quad (6)$$

$$\psi(t, \mathbf{x}) = -\frac{G}{c^4} \int \frac{d^3x'}{|\mathbf{x} - \mathbf{x}'|} \left[ {}^{(2)}T^{00}(t, \mathbf{x}') + {}^{(2)}T^{ii}(t, \mathbf{x}') \right] \quad (7)$$

$$\chi(t, \mathbf{x}) = -\frac{G}{2c^4} \int d^3x' |\mathbf{x} - \mathbf{x}'| {}^{(0)}T^{00}(t, \mathbf{x}') \quad (8)$$

Note that these definitions follow the book of Straumann whereas the script follows Weinberg/Maggiore.

(iv) Plug the metric perturbations into the action and identify the Newtonian and leading order post-Newtonian Lagrangian.