Sheet 7 solution session: June 19th morning ?

PHY127 FS2023

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Lecture 13
June 2nd, 2023

Last time (Lecture 12):

$$\text{Nuclear binding energy} = Z(m_p c^2) + N(m_n c^2) - \underset{\uparrow}{mc^2}$$

m_p : mass of proton

m_n : mass of neutron

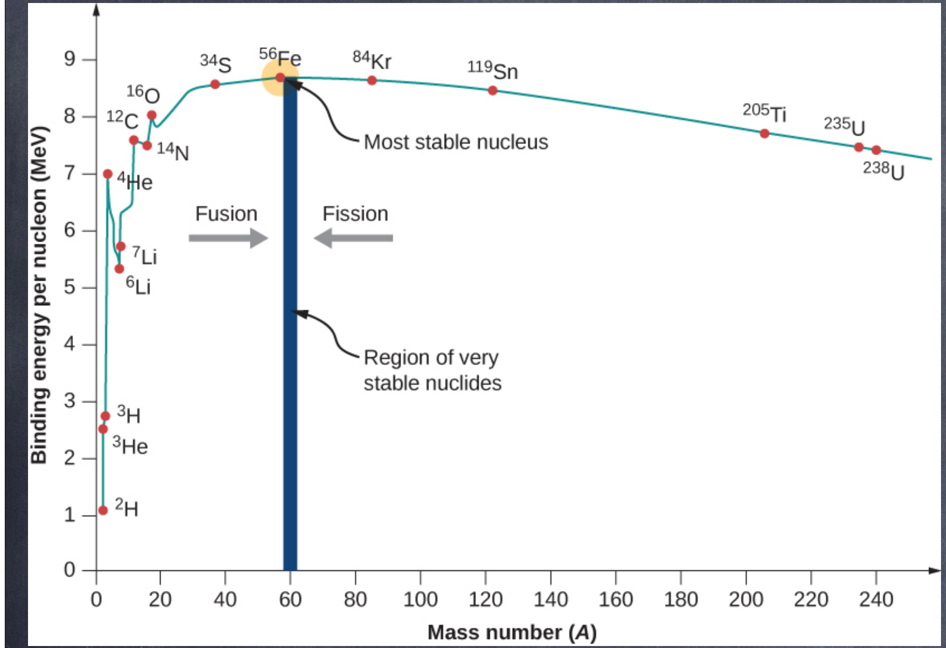
mass of the nucleus.

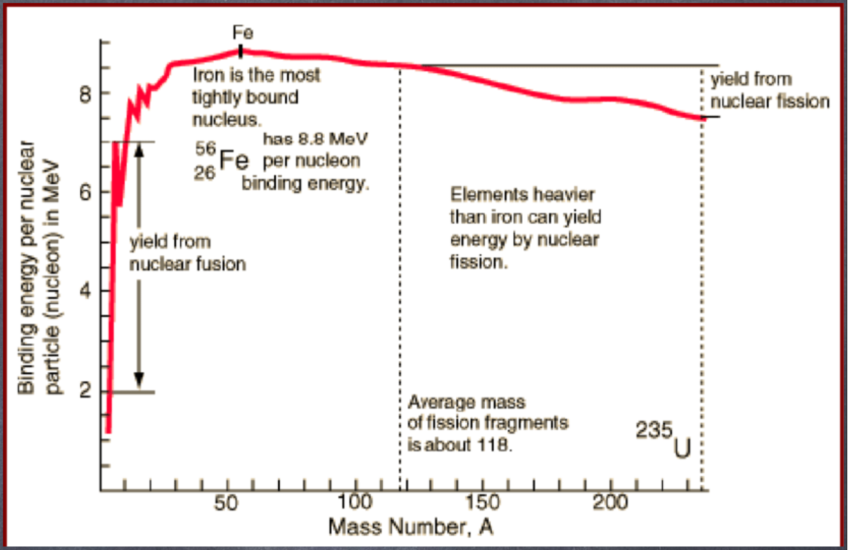
This binding energy is about 1% of the mass

$$\frac{8 \text{ MeV}}{1 \text{ u}} = \frac{8 \text{ MeV}}{930 \text{ MeV}} \sim 1\%$$

Binding energy per nucleon as a function of A

Release of energy occurs when binding energy per nucleon moves to higher energ

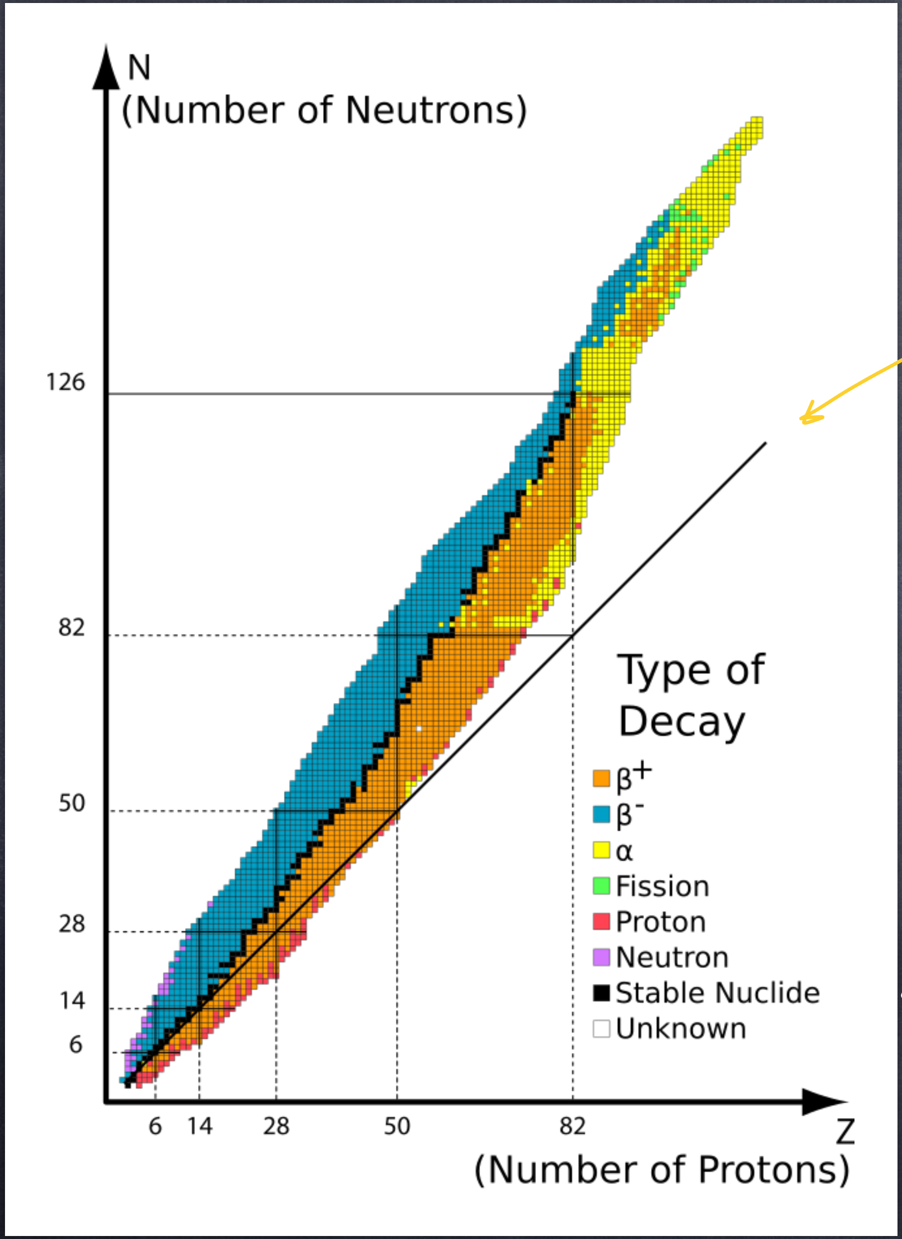




fusion from lighter elements to heavier elements produces energy below Fe

Fission from heavier elements to lighter elements produces energy above Fe.

Note: ^{4}He has a very high (relative) binding energy *



Black curve are stable nuclides.

Z = N

← stable : slightly more neutrons than protons

Pauli exclusion principle: the nucleus has protons + neutrons } Fermions with spin $= \frac{1}{2}$

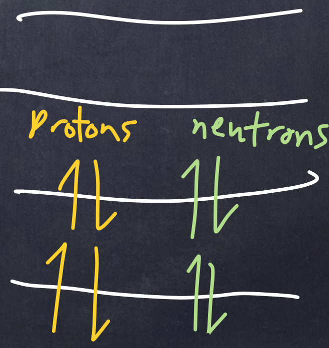
As we add protons + neutrons to a nucleus, they occupy higher energy levels, But protons + neutrons do this separately.

Compare
a) nucleus with Z protons, 0 neutrons
b) nucleus with $\frac{Z}{2}$ protons, $\frac{Z}{2}$ neutrons.

Both have the same $A = Z + N$.
But the energy of (a) is larger than (b)

Example:
a) $Z = 8, N = 0$
b) $Z = 4, N = 4$

a)


b)


Nuclides with $Z \cong N$ tend to have lower energies + be more stable.

Atoms are more stable when electron shells (s, p, d, ..) are filled → "closed shells"

Also true for nuclei with "magic" numbers of nucleons: 2, 8, 20, 28, ...
(Applies to both protons + neutrons separately)

For instance, ^4_2He $(Z=2, N=2)$ is "doubly magic" → extremely stable

Helium

(see binding energy on p.3)

Types of radiation (from unstable nuclei decaying) are classified by their penetration power

α radiation : stopped by a few pieces of paper

β radiation : stopped by a few pieces of aluminum foil

γ radiation: can penetrate several cm of lead or a concrete wall.

Later, it was found that

α : ${}^{4}_{2}He$ nucleus

β : high-energy electron

γ : high-energy photon.

Energy released in a decay is called Q

$$Q = \Big(\underset{\text{mass of nucleus}}{M} - \underset{\cancel{\text{mass}}\ \text{sum of } Z + N}{\sum m_i}\Big)c^2$$

For a radioactive decay to happen, Q must be > 0.
(natural radioactivity)

An unstable nuclei will decay by α, β, or γ radiation

E: parent nucleus

E': daughter nucleus

α decay: $${}^{A}_{Z}E \rightarrow {}^{A-4}_{Z-2}E' + \underbrace{{}^{4}_{2}He}_{\text{has kinetic energy}}$$

$$Q = (M_E - M_{E'} - M_{He})c^2$$

$Q > 0$, energy becomes the energy of ${}^{4}_{2}He$

α particle: interacts very strongly with matter because $Z = 2$ $q = +2e$
(feels stronger Coulomb force)

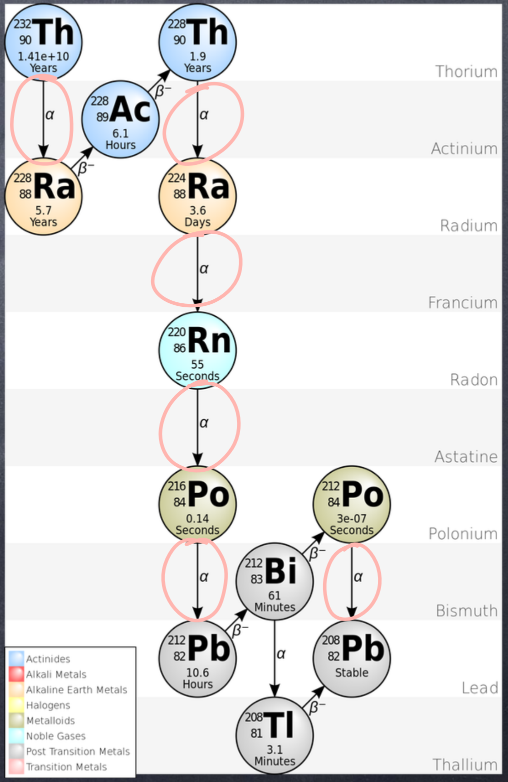
Often, ϵ' also decays by α decay, causing a succession of α decays.

unstable Nuclides with $A=4n, 4n+1, 4n+2, \ldots$ (where n is an integer) typically exhibit multiple α decays.

For instance, $^{232}_{90}\mathrm{Th}$.

$(4 \cdot 58 = 232)$

We see $^{232}_{90}\mathrm{Th}$ decays by α emission to $^{228}_{88}\mathrm{Ra}$. Later $^{228}_{90}\mathrm{Th}$ decays to $^{224}_{88}\mathrm{Ra}$, $\cdots$ many α decays happen.



Beta decay: β

"—" means anti-neutrino

$$^A_Z E \rightarrow {}^{A}_{Z+1}E' + e^- + \bar{\nu}_e$$

anti-neutrino

Neutrinos interact extremely weakly so they are typically ignored.

A neutrino can travel through a light-year of lead and not interact

Note: A neutron is being converted into a proton.

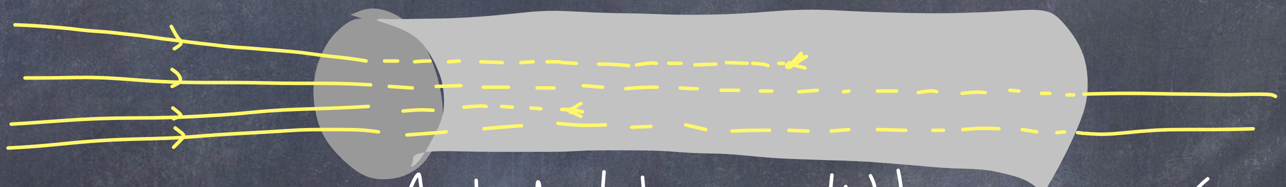
$$n \rightarrow p^+ + e^- + \bar{\nu}_e$$

This is happening because of the weak nuclear force.

what is a neutrino?

Symbol : ν (nu)

It is a neutral lepton, like the electron but with no electric charge. It only interacts through the weak nuclear interaction.



A lead tube one lightyear long would only stop ½ of neutrinos.

(9 trillion km
9E12 km)

wow!

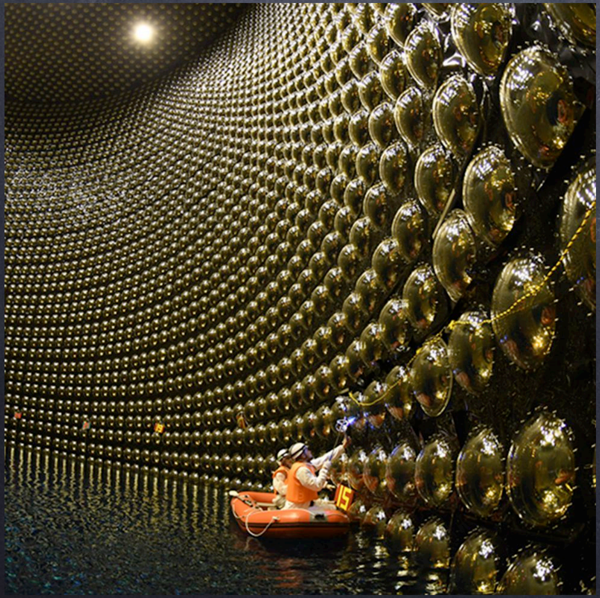
Sources of neutrinos:

- radioactive decays
- neutrinos produced in sun (from fusion)
- neutrinos produced in nuclear reactors (fission)
- neutrinos produced in accelerators

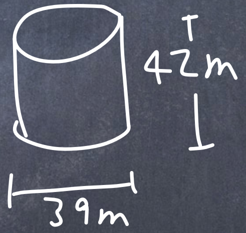
10^{11} neutrinos from the sun travel through each of your fingernails each second.

That's a lot!

Detecting neutrinos:



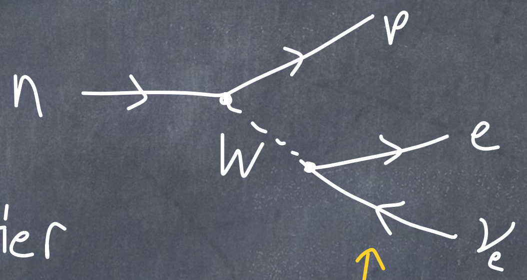
Super-Kamiokande: 50 kilotonnes of water



All of this water causes about 4000 neutrinos from sun to interact. Not so many!

weak nuclear force

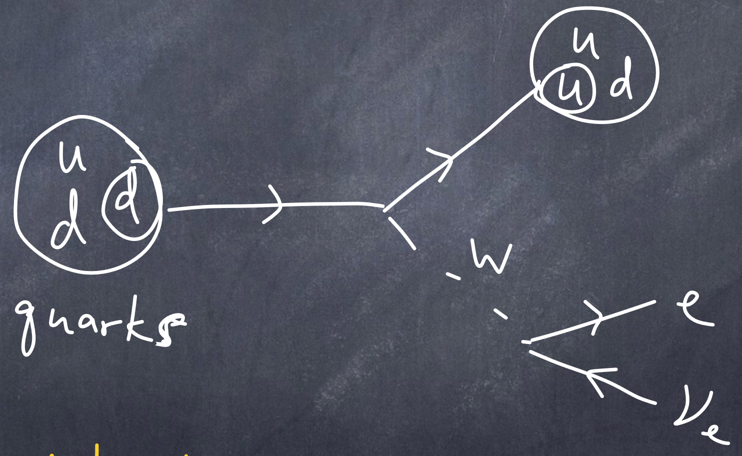
$$n \rightarrow p^+ + e^- + \bar{\nu}_e$$



moving backward in time ≡ anti-neutrino

W is a force carrier for the weak force.

$$d \rightarrow u + e^- + \bar{\nu}_e$$



fundamental interaction

relevant W interactions (affect matter)

remember

$Q(u) = +\frac{2}{3}$

$Q(d) = -\frac{1}{3}$

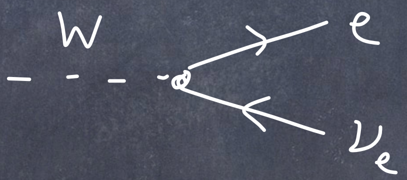
p → n

$u \rightarrow d + W^{+}$

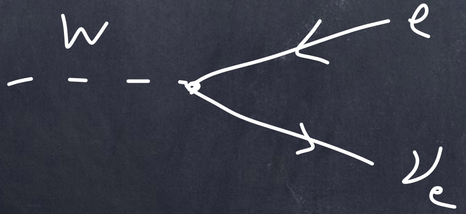
n → p

$d \rightarrow u + W^{-}$

produces
e^{-}
or
e^{+}



$W^{-} \rightarrow e^{-} + \bar{\nu}_e$



$W^{+} \rightarrow e^{+} + \nu_e$

$W^{\pm}$ is unstable and decays like this so a $p \rightarrow n + e^+ + \nu_e$

This interaction does not happen, however, because the neutron is heavier than the proton. The proton has been measured to be stable. For the proton to decay, a new force would be necessary.

Free neutrons decay into a proton $+ e^+ + \nu_e$ in ~15 minutes

Neutrinos were ~~posited~~ hypothesized because it was found that $n \rightarrow p + e^-$ doesn't conserve energy (so we need a neutrino also)

Some other weak nuclear decays:

positron emission $${}^{A}_{Z}E \rightarrow {}^{A}_{Z-1}E' + e^+ + {}^{0}_{0}\nu$$

electron capture $${}^{A}_{Z}E + e^- \rightarrow {}^{A}_{Z-1}E' + \nu$$

γ radiation (gamma)

Occurs when a nucleus makes a downward transition from one energy to a lower energy level (like in atoms). But nuclear transitions produce higher energy photons than atomic transitions (millions of times larger)

γ : used in medical applications (imaging) + radiation therapy.

γ process happens when a nuclei is left in an excited state following a α or β decay

"m" : excited state

$$ {}^{Am}_{Z}E \rightarrow {}^{A}_{Z}E' + {}^{0}_{0}\gamma $$

or also written

$$ {}^{A*}_{Z}E \rightarrow {}^{A}_{Z}E' + {}^{0}_{0}\gamma $$

Half-life of unstable nuclei:

$$N(t) = N_0 e^{-\lambda t}$$

λ: decay constant

N_0: initial number of radioactive particles

$N(t)$: # of particles remaining after some time, t

$$\tau_{\frac{1}{2}} = t_{\frac{1}{2}} = \text{half-life}$$

$$\tau_{\frac{1}{2}} = \frac{\ln(2)}{\lambda}$$

$$\text{activity} = \frac{\Delta N}{\Delta t} = -\lambda N$$

(−) means N is decreasing

This is explained more in next slides

Half-life principle:

The number of decays in a short period of time ΔN is proportional to the time interval Δt, and also to the number of particles, N.

Therefore, the formula is:

$$\boxed{\Delta N = -\lambda N \Delta t} \quad ①$$

where λ is the decay constant, which depends on the particular radioactive nuclide.

The solution to this formula is solved with calculus:

As $\Delta t \to 0$, we use dt, and $\Delta N \to dN$.

so $dN = -\lambda N dt$

Integrating, $\int \frac{dN}{N} = \int -\lambda dt \Rightarrow \ln N = -\lambda t + \text{constant}$

Exponentiating, $e^{\ln N} = e^{-\lambda t + \text{constant}} = e^{-\lambda t} \cdot e^{\text{constant}}$

And since $e^{\text{constant}} =$ another constant, then $\Rightarrow \boxed{N = Ce^{-\lambda t}}$

$\therefore$

Half-life

$$N(t) = N_0 e^{-\lambda t}$$

λ: decay constant

N_0: initial # of radioactive particles

$N(t)$: # of ptcls after time, t.

How long does it take to decay half of the initial amount of isotope?

$$\left.\begin{array}{l}\text{Initial \#}: N_0 \\ \text{Final \#}: \dfrac{N_0}{2}\end{array}\right\} \rightarrow \frac{\cancel{N_0}}{2} = \cancel{N_0}\, e^{-\lambda t}$$

(what is t)

$$\frac{1}{2} = e^{-\lambda t}$$

$$\log_e\left(\frac{1}{2}\right) = \ln\frac{1}{2} = \ln\left(e^{-\lambda t}\right) = -\lambda t$$

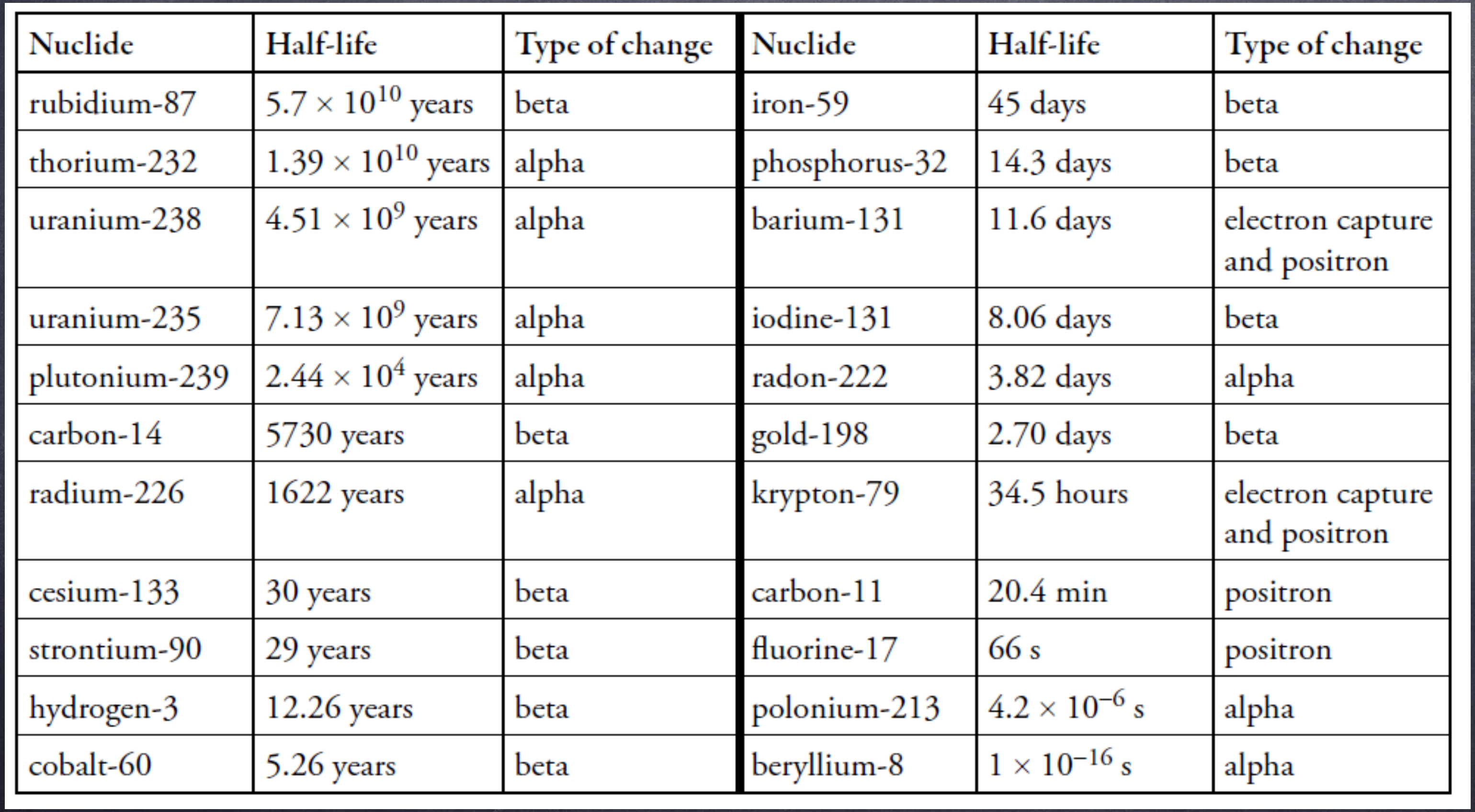
$$\tau_{1/2} = t_{1/2} = t = \frac{\ln(2)}{\lambda} = \frac{0.693}{\lambda}$$

half-life

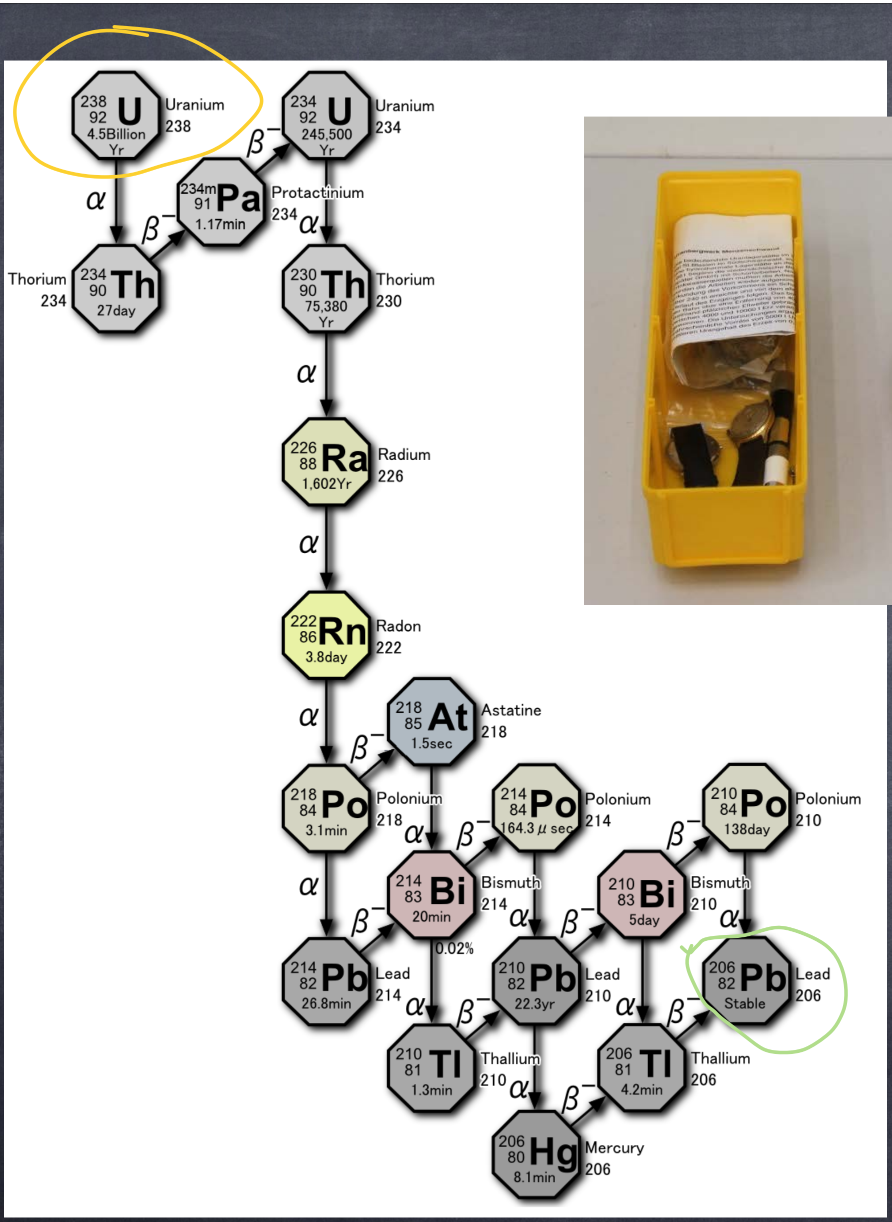
one half life : $N_0 \rightarrow N_0/2$

two half-lives : $N_0 \rightarrow N_0/4$

three ' ~ - - : $N_0 \rightarrow N_0/8$



Nuclide	Half-life	Type of change	Nuclide	Half-life	Type of change
rubidium-87	5.7×10^{10} years	beta	iron-59	45 days	beta
thorium-232	1.39×10^{10} years	alpha	phosphorus-32	14.3 days	beta
uranium-238	4.51×10^{9} years	alpha	barium-131	11.6 days	electron capture and positron
uranium-235	7.13×10^{9} years	alpha	iodine-131	8.06 days	beta
plutonium-239	2.44×10^{4} years	alpha	radon-222	3.82 days	alpha
carbon-14	5730 years	beta	gold-198	2.70 days	beta
radium-226	1622 years	alpha	krypton-79	34.5 hours	electron capture and positron
cesium-133	30 years	beta	carbon-11	20.4 min	positron
strontium-90	29 years	beta	fluorine-17	66 s	positron
hydrogen-3	12.26 years	beta	polonium-213	4.2×10^{-6} s	alpha
cobalt-60	5.26 years	beta	beryllium-8	1×10^{-16} s	alpha

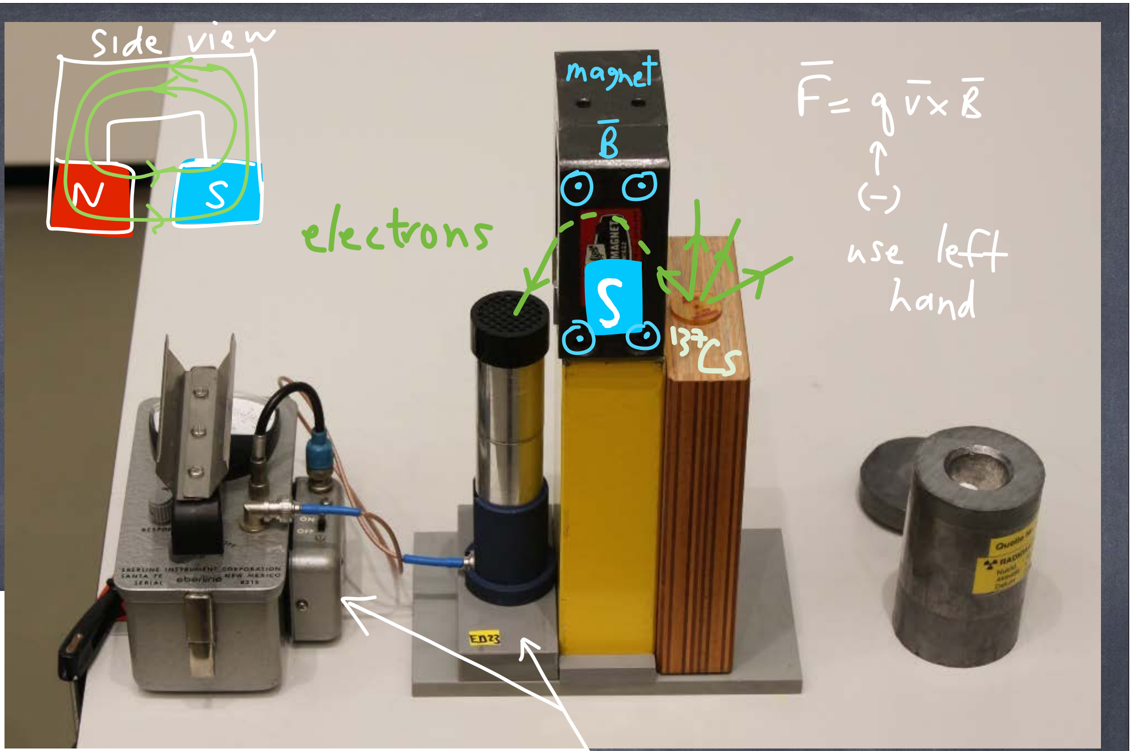


Experiment with uranium ore and radium. these decay producing α + β radiation.

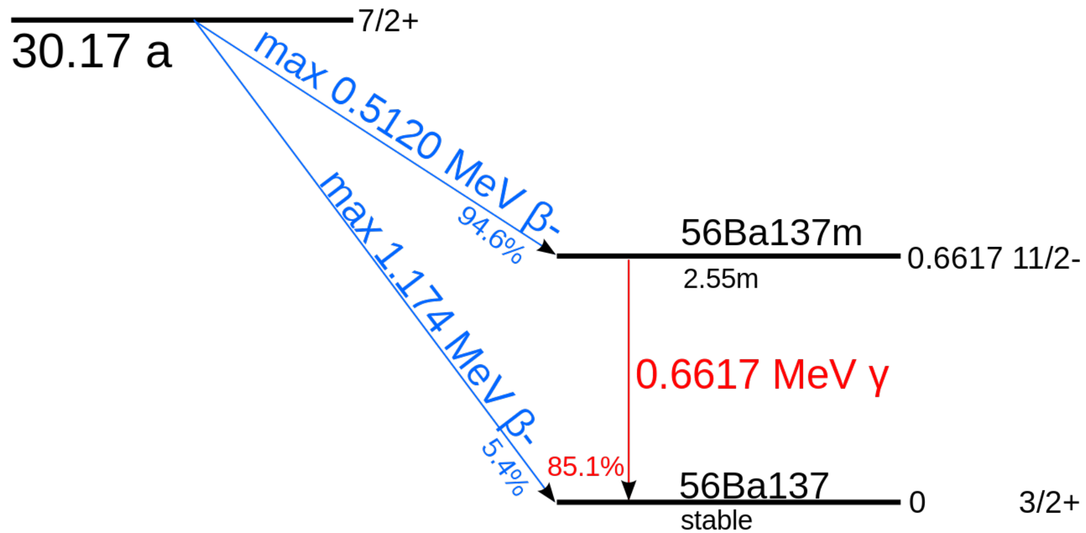
← ^{206}Pb is finally stable

Experiment with electrons from $^{137}_{55}Cs$.

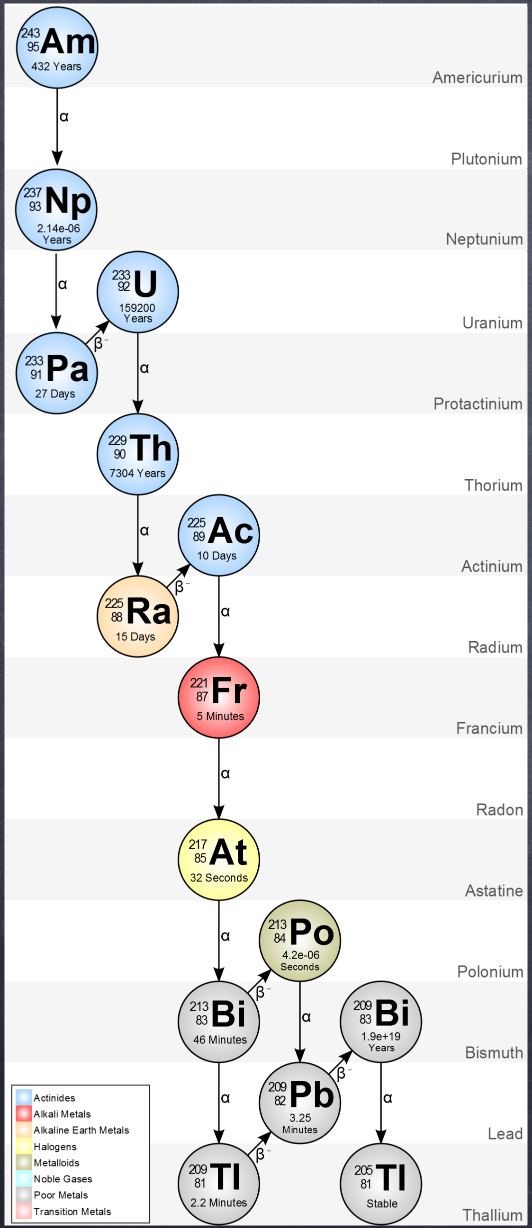
This is unstable and produces electrons + photons ($\beta + \gamma$)

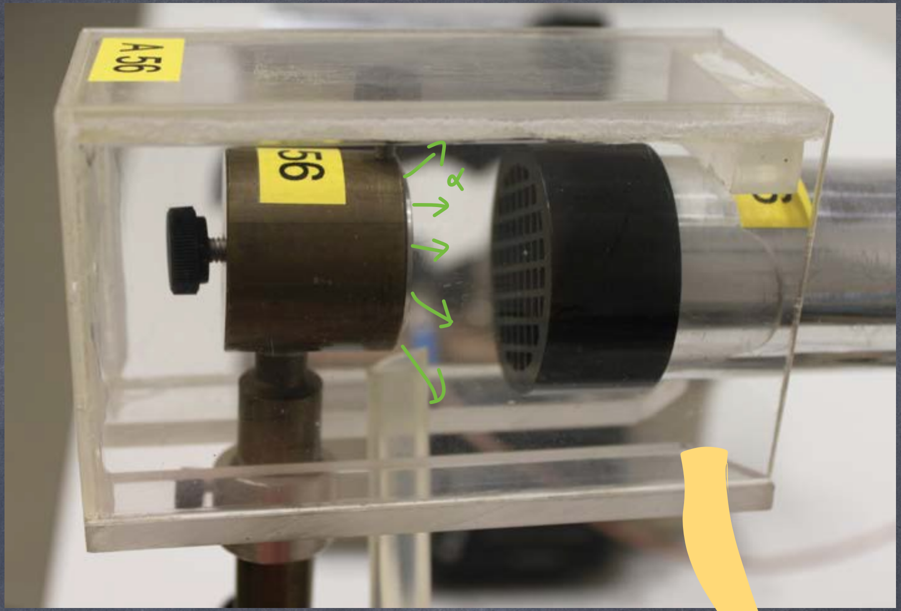


This is a Geiger-Müller counter



2.14e+6 years→





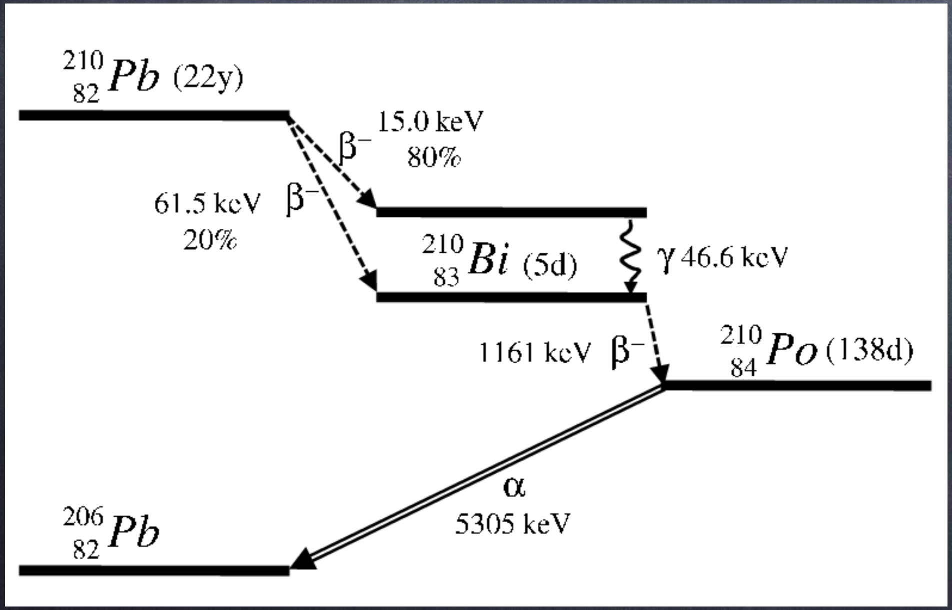
Experiment with ^{243}Am

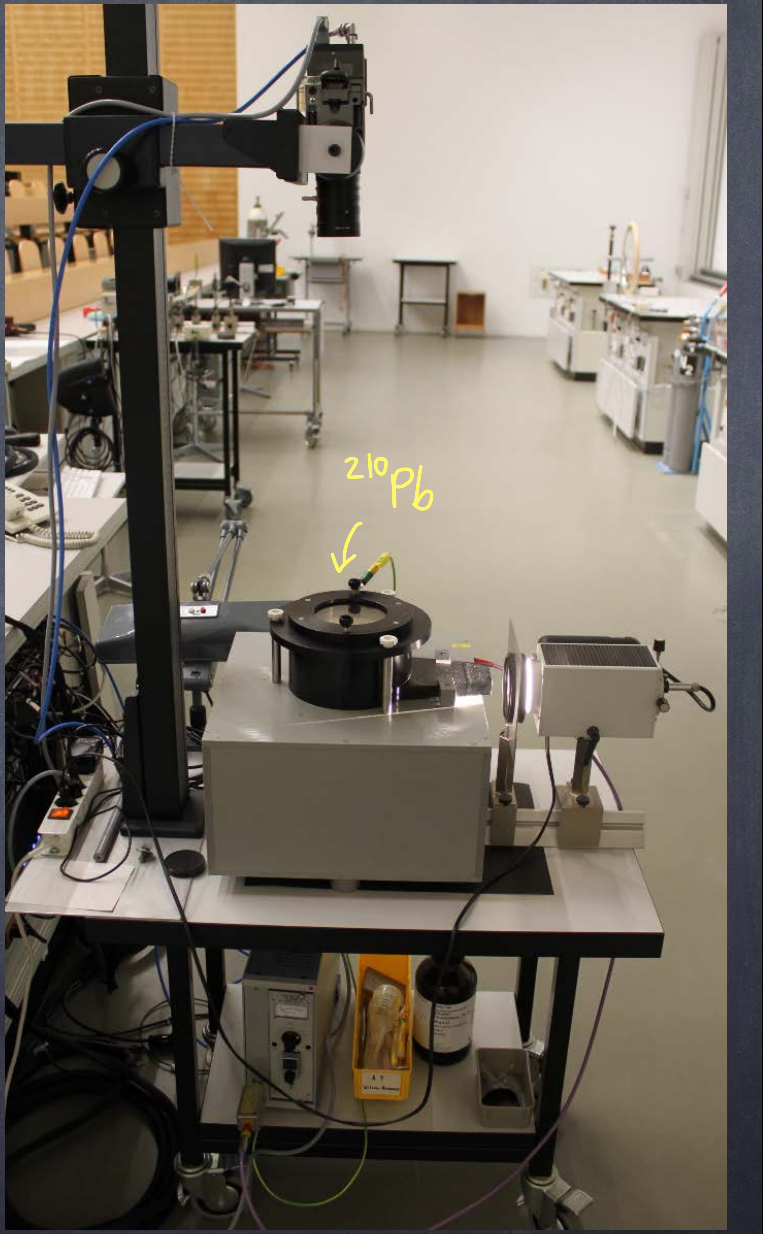
CO_2 or He

denser gas

less dense gas

Experiment : cloud chamber showing ^{210}Pb decaying





$\frac{\Delta N}{\Delta t}$: activity level

measured in $\frac{\text{disitegrations}}{\text{seconds}}$ = [Bq]

Becquerel

$1\ \text{Bq} = \frac{1\ \text{decay}}{\text{second}}$

or [Curie] = [Ci] $\quad 1\ \text{Ci} = 3.7\text{E}10\ \text{Bq}$

Applications : ^{14}C dating.

Living things are carbon-based.

^{14}C produced in the upper atmosphere by cosmic radiation.

Living things incorporate carbon (^{12}C, ^{13}C, ^{14}C) until they die.

One can measure the fraction of ^{14}C left in a sample from a (previously) living organism to see how much is left.

$\tau_{\frac{1}{2}}$ of ^{14}C is 5730 years

^{14}C dating works well for objects $< 60{,}000$ years old

60,000 years is 10 half-lives

$$\left(\frac{1}{2}\right)^{10} \sim 0.001$$

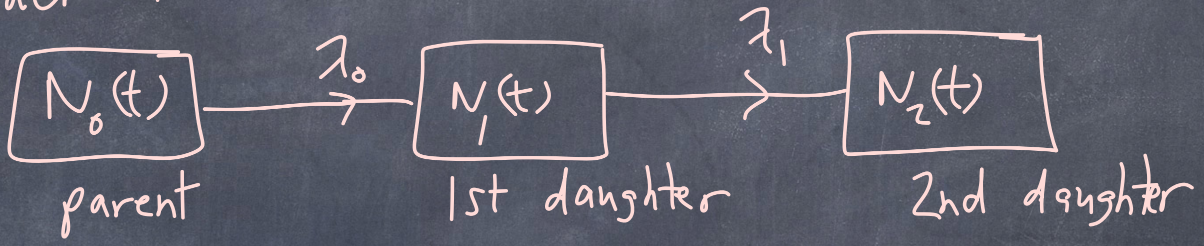
Geological dating uses ^{238}U

$\tau_{\frac{1}{2}} \sim 4.5 \in 9$ years

first fossils are 3 billion years old, found using ^{238}U dating

Some isotopes can reach equilibrium levels if they are produced by a long-lived parent and then decay themself.

Consider this case.



The number of 1st daughter particles increases due to the parent decays, but also decreases due to its own decay to 2nd daughters.

The number of N_1 particles can therefore reach an equilibrium.

The total change in ΔN in some time Δt

$$\boxed{\Delta N_1(t) = \left[N_0(t)\lambda_0 - N_1(t)\lambda_1\right]\Delta t}$$ ②

If λ_1 is much smaller (faster) than λ_0, then $N_0(t)$ and $N_1(t)$ are approximately constant in time.

So equation ② becomes

$$\Delta N_1(t) = 0 = \left[N_0\lambda_0 - N_1\lambda_1\right]\Delta t$$

Since $\Delta t \neq 0$, $N_0\lambda_0 = N_1\lambda_1$

and therefore, we can predict that the equilibrium number of N_1 daughters is $N_1 = \frac{\lambda_0}{\lambda_1} N_0$

→ The number of isotopes coming from long-lived parents is ~ constant and related to λ_0, λ_1, + N_0.

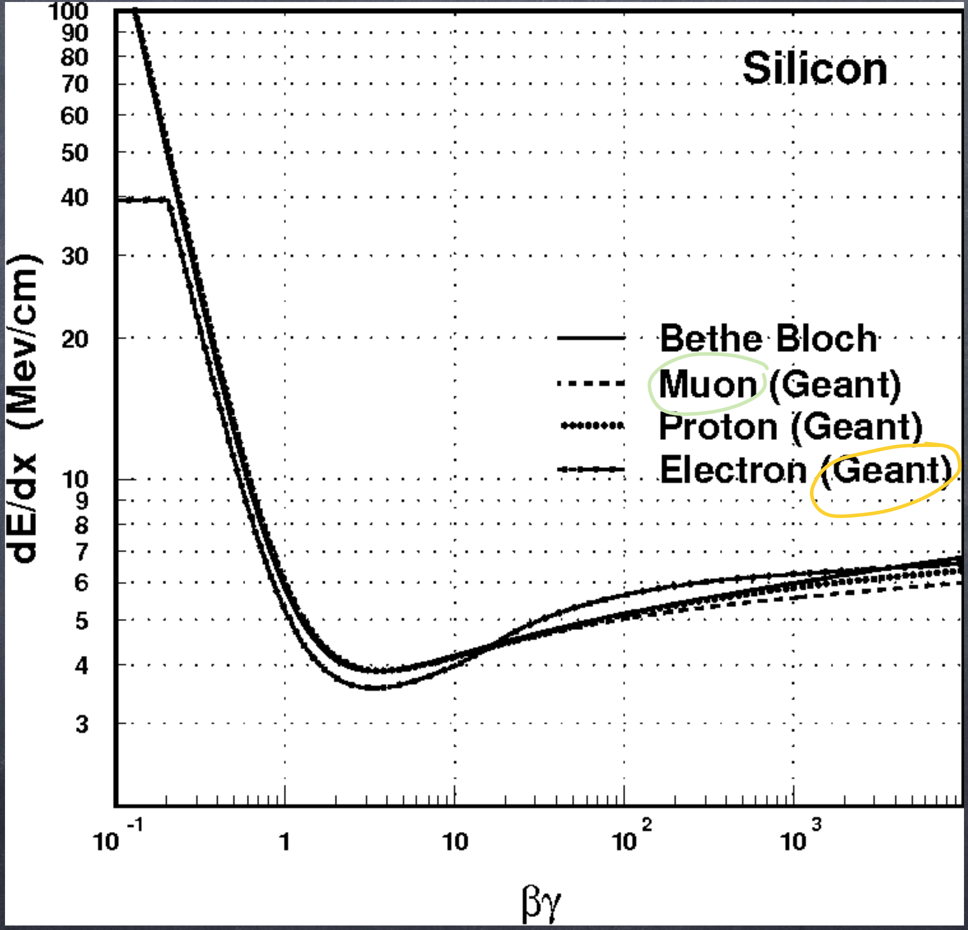
Radiation (here, charged particles) lose energy moving through materials.

The Bethe-Bloch Function describes this interaction

$$\frac{dE}{dx} \longrightarrow$$

energy loss per distance

Bethe-Bloch Function



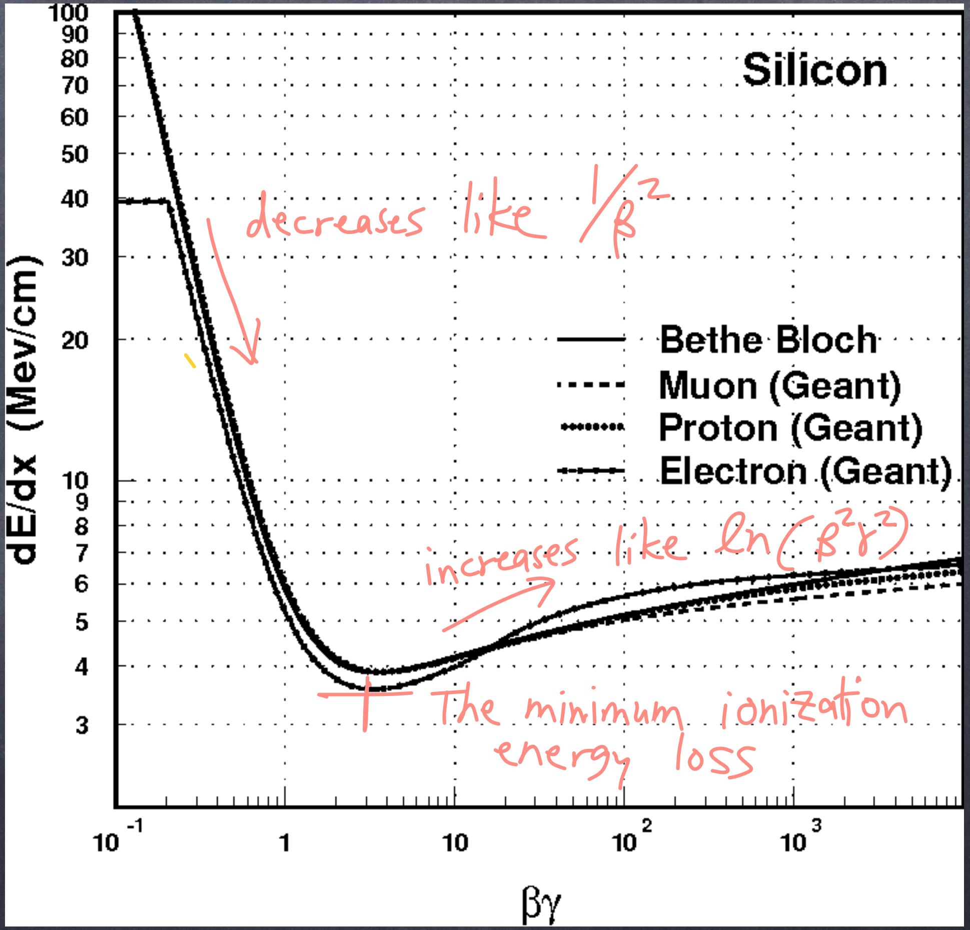
have the same shape curve

Muons come from cosmic rays. (10,000 go through your body every minute)

Geant is a simulation software for particle interactions

↑ related to velocity

Note that over a significant range of velocities (or $\beta\gamma$), dE/dx doesn't change much. (from $1 < \beta\gamma < 10^4$)



$\beta = \frac{v}{c}$ speed relative to light speed.

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

not photon, this γ is related to velocity (called the Lorentz Factor)

Note: dE/dx typically has units of $\frac{\text{MeV}}{\text{cm}}$ since it is energy deposited per length in a material.

However, often people divide it by the density of the material, so it has units of $\text{MeV}/\text{g}/\text{cm}^2 = \text{MeV} \cdot \text{g}^{-1} \cdot \text{cm}^2$

If you have a dE/dx in units of $\text{MeV} \cdot \text{g}^{-1} \cdot \text{cm}^2$, multiply by the density of the material to get an energy deposition per length.

dE/dx

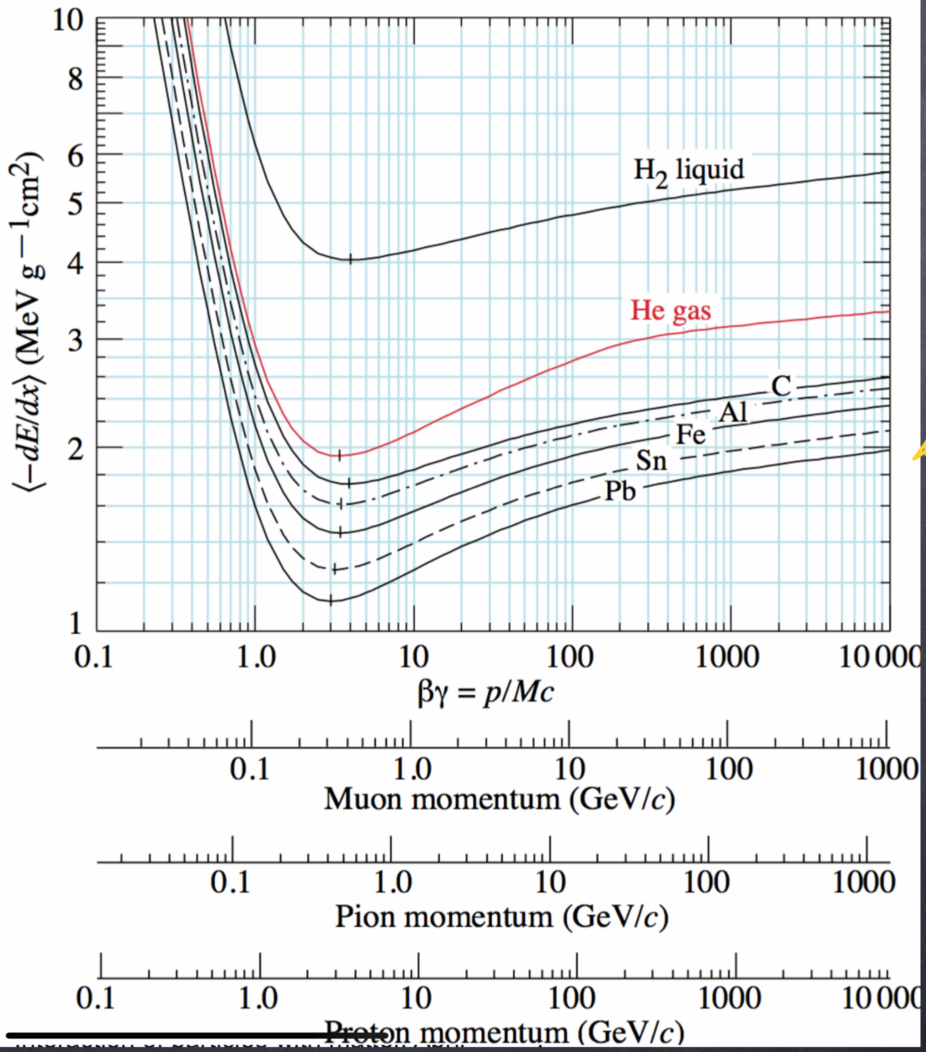
Particle Data Group:
pdg.lbl.gov/2016/reviews/rpp2016-rev-passage-particles-matter.pdf

Different detector materials

$$\frac{dE}{dx} \approx \frac{Z}{A}$$

(remember density!)

dE/dx depends on
βγ = p/(Mc)
→ at a given p, dE/dx is different for particles with different mass M



Here $\frac{dE}{dx}$ is divided by density of material

$$\left[\frac{\frac{MeV}{cm}}{\frac{g}{cm^3}}\right] = \left[\frac{MeV\,cm^2}{g}\right]$$

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Bethe-Bloch equation

Considering quantum mechanical effects:

$$-\left\langle \frac{dE}{dx} \right\rangle = K z^2 \frac{Z}{A} \frac{1}{\beta^2} \left[\frac{1}{2} \ln \frac{2 m_e c^2 \beta^2 \gamma^2 T_{\max}}{I^2} - \beta^2 - \frac{\delta(\beta\gamma)}{2} \right] [\cdot \rho]$$

density

$K = 4\pi N_A r_e^2 m_e c^2 = 0.307$ MeV g^{-1} cm^2

$T_{max} = 2 m_e c^2 \beta^2 \gamma^2 / (1 + 2\gamma m_e/M + (m_e/M)^2)$

[Max. energy transfer in single collision]

z : Charge of incident particle

M : Mass of incident particle

Z : Charge number of medium

A : Atomic mass of medium

I : Mean excitation energy of medium

δ : Density correction [transv. extension of electric field]

$N_A = 6.022 \cdot 10^{23}$

[Avogardo's number]

$r_e = e^2/4\pi\varepsilon_0 m_e c^2 = 2.8$ fm

[Classical electron radius]

$m_e = 511$ keV

[Electron mass]

$\beta = v/c$

[Velocity]

$\gamma = (1-\beta^2)^{-2}$

[Lorentz factor]

Validity:

$.05 < \beta\gamma < 500$

$M > m_\mu$

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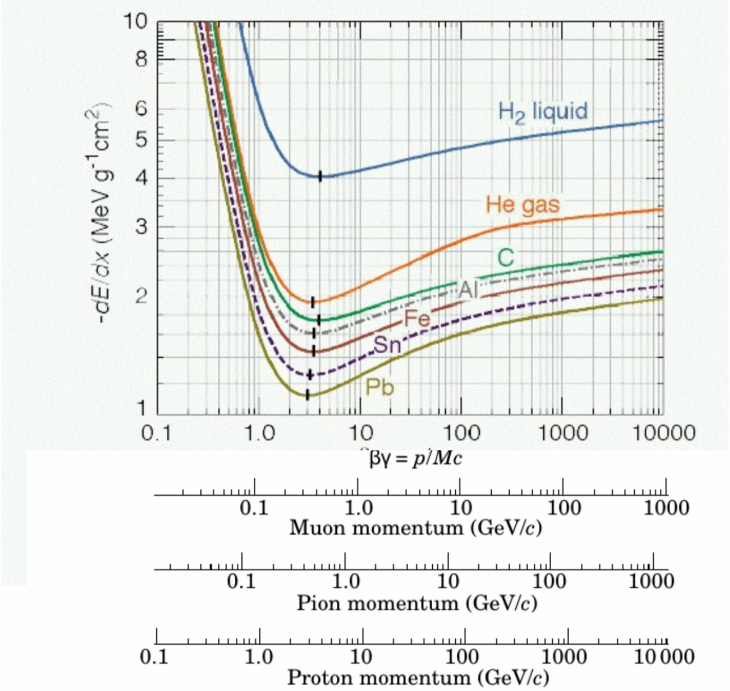
dE/dx_{min} different materials

$$\frac{-dE}{dX}=K z^2 \frac{Z}{A}\frac{1}{\beta^2}[\frac{1}{2}\ln\frac{2 m_e c^2 \beta^2 \gamma^2 T_{max}}{I^2} -\beta^2-\frac{\delta}{2}-\frac{C}{Z}]$$



PDG http://pdg.lbl.gov/pdg.html

dE/dX depends on A, Z of target material

$\beta\gamma \cong 3.5$ broad minimum

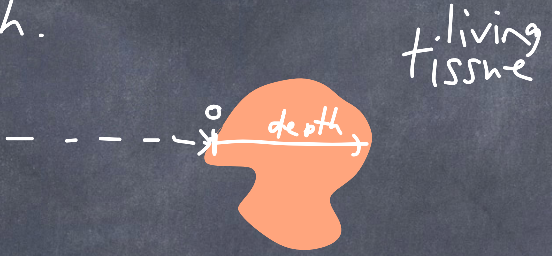
→ minimum ionising particles (MIP)

H_2 Z/A≅ 1 $dE/dX_{min} \cong 4$ MeV /(g/cm^2)

others Z/A≅ 0.5 $dE/dX_{min} \cong 2$ MeV /(g/cm^2)

$dE/dX_{min} \cong$ 1-1.7 MeV /(g/cm^2)
only weak material dependence

what is most important in understanding radiation damage to tissue is the dose delivered at each depth.



This depends on the type of radiation and its energy.

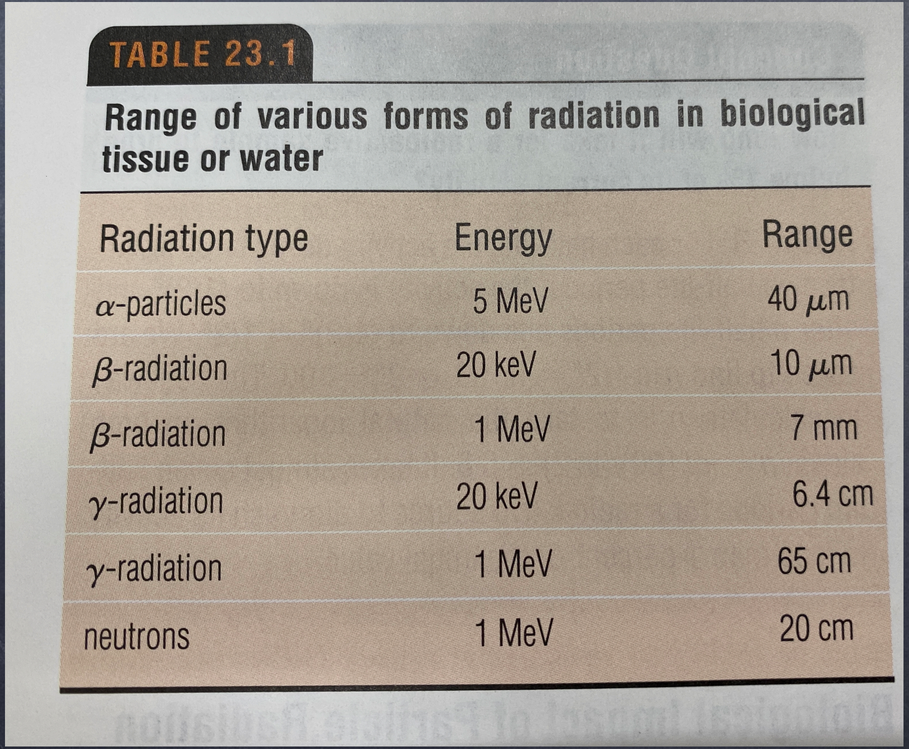


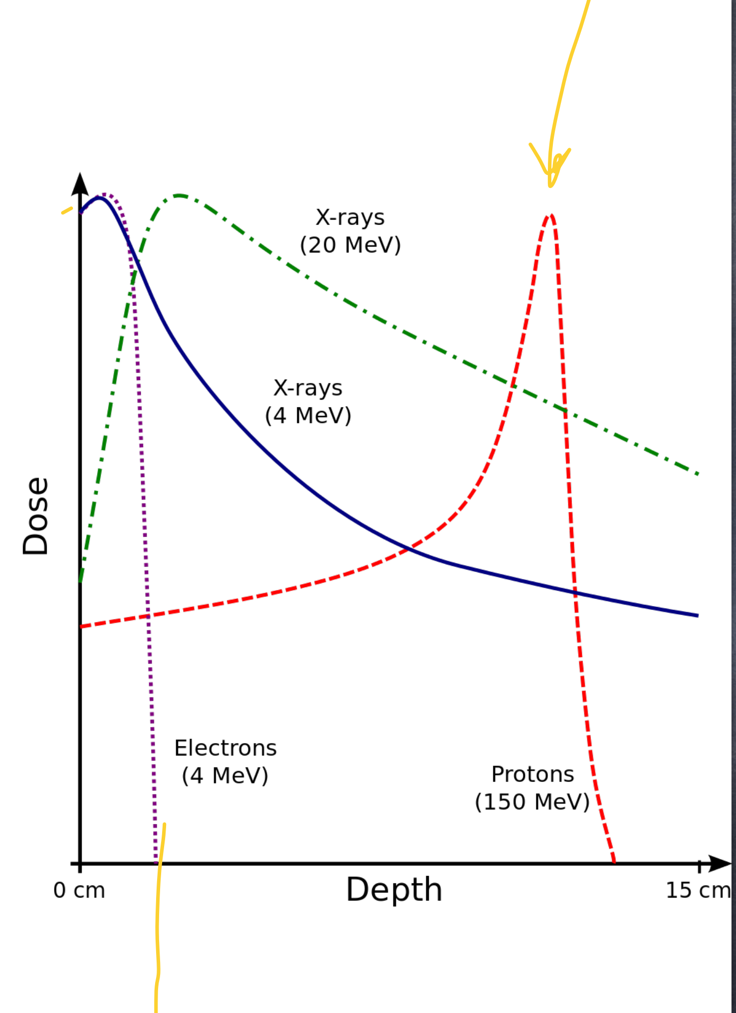
TABLE 23.1

Range of various forms of radiation in biological tissue or water

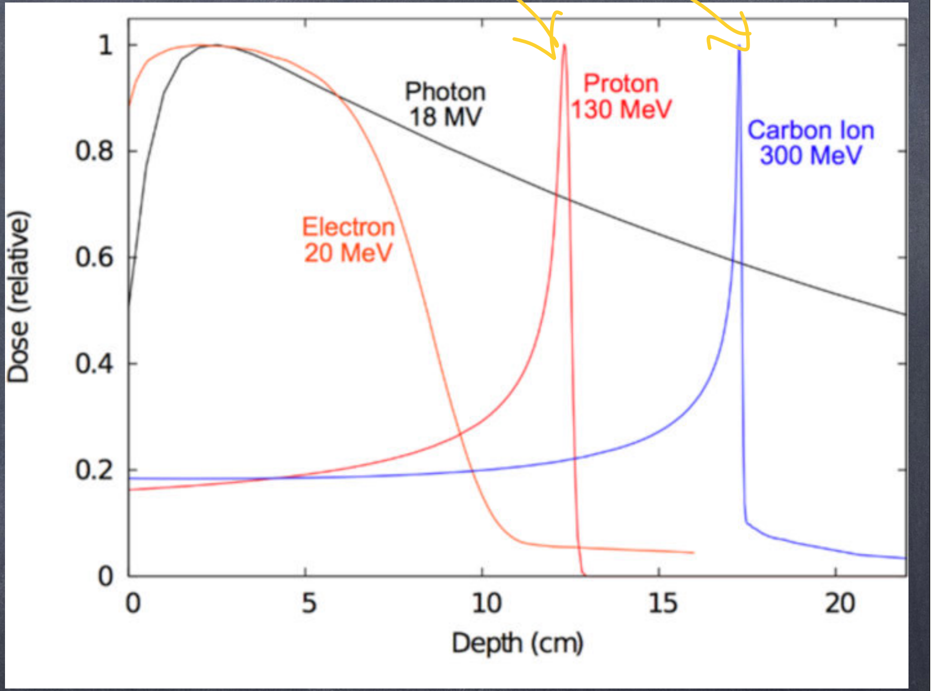
Radiation type	Energy	Range
α-particles	5 MeV	40 μm
β-radiation	20 keV	10 μm
β-radiation	1 MeV	7 mm
γ-radiation	20 keV	6.4 cm
γ-radiation	1 MeV	65 cm
neutrons	1 MeV	20 cm

Range: ← how far the radiation can travel before stopping

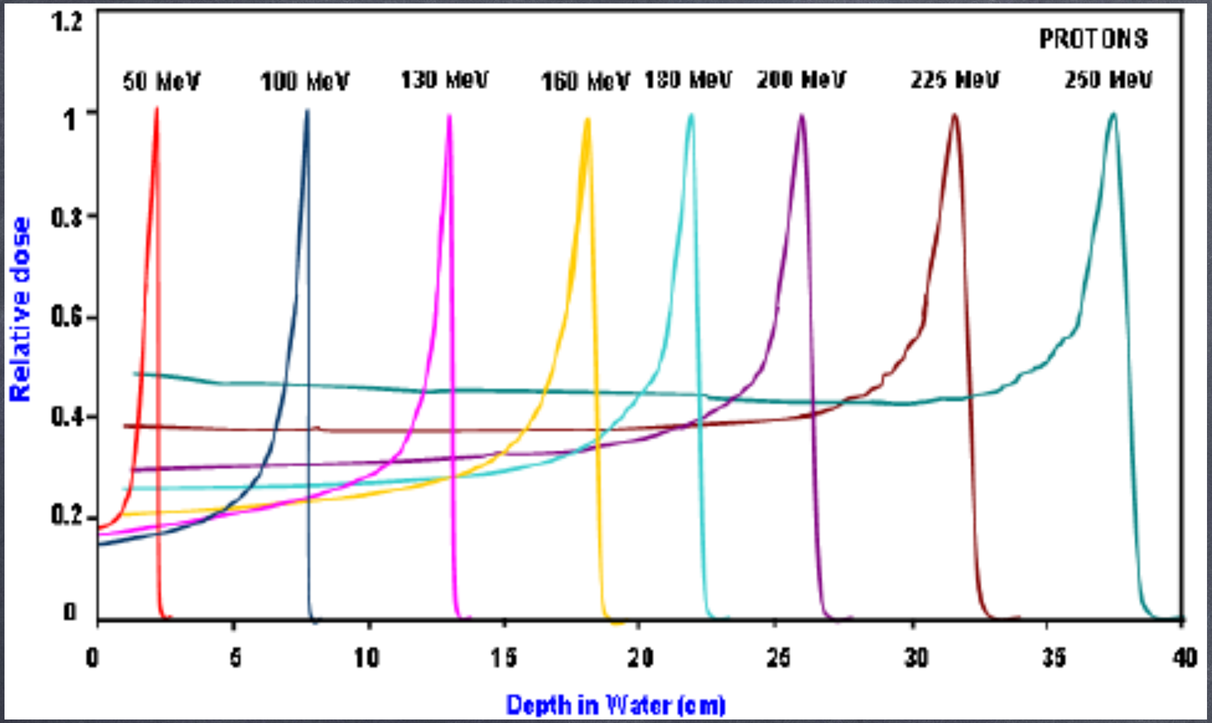
Bragg peak: shows that most of the energy is deposited when the proton or ion stops.



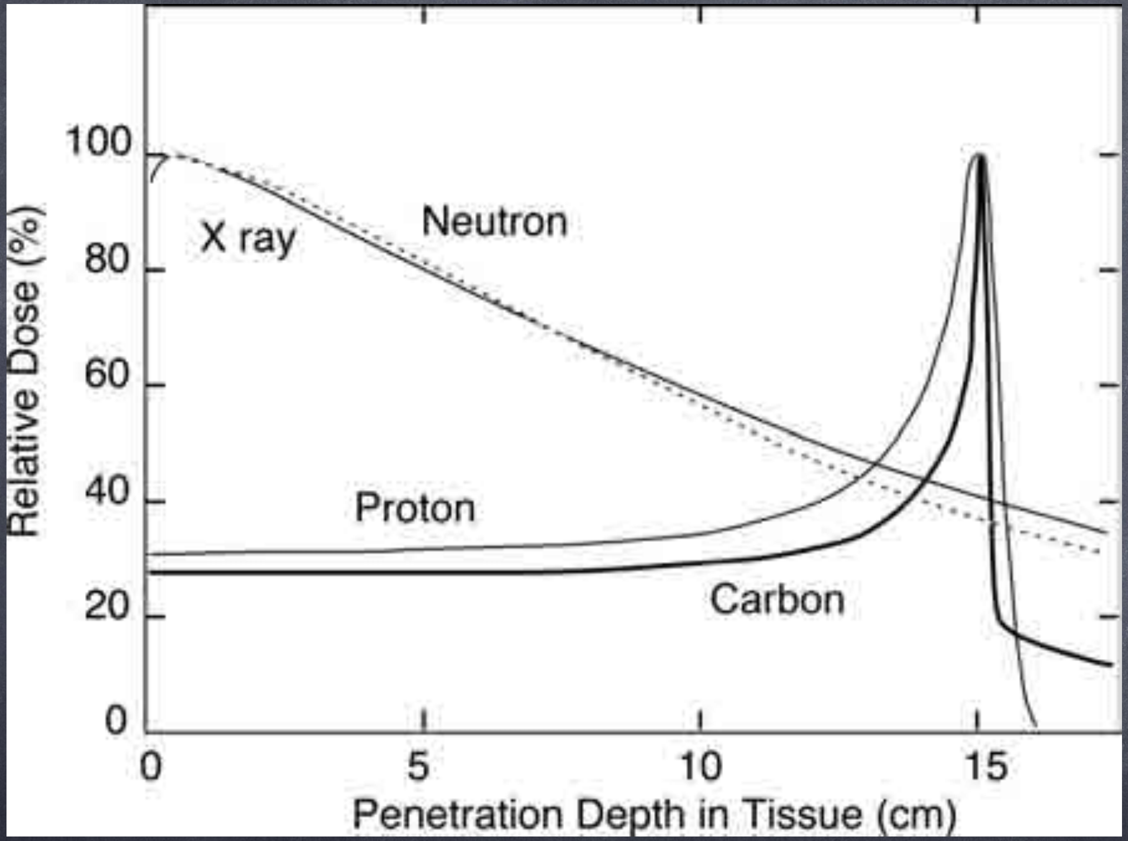
few cm



proton dose vs. depth for different energies.



we can tune the energy of protons to deposit energy at a certain depth.


Relative Dose (%)
100
80
60
40
20
0
0
5
10
15
Penetration Depth in Tissue (cm)
X ray
Neutron
Proton
Carbon

Radiopharmaceuticals

Incorporation of radioactive isotopes into cells or in organs of body allows radioactive tracing or radiolabeling of a particular molecule as it passes through an organism.

For instance, hydrogen ^1_1H and ^3_1H (tritium) have the same chemistry, but tritium is radioactive ($\tau_{1/2} = 12$ years): $^3_1H \rightarrow {}^3_2He + e^- + \bar{\nu}_e$

Since its lifetime is long, you wouldn't necessarily put it in a human body, but it could be used as a radiolabel for investigating hydrogen molecules in a sample. Also, electrons don't travel very far...

So other radioactive nuclides are often used.

(See next page)

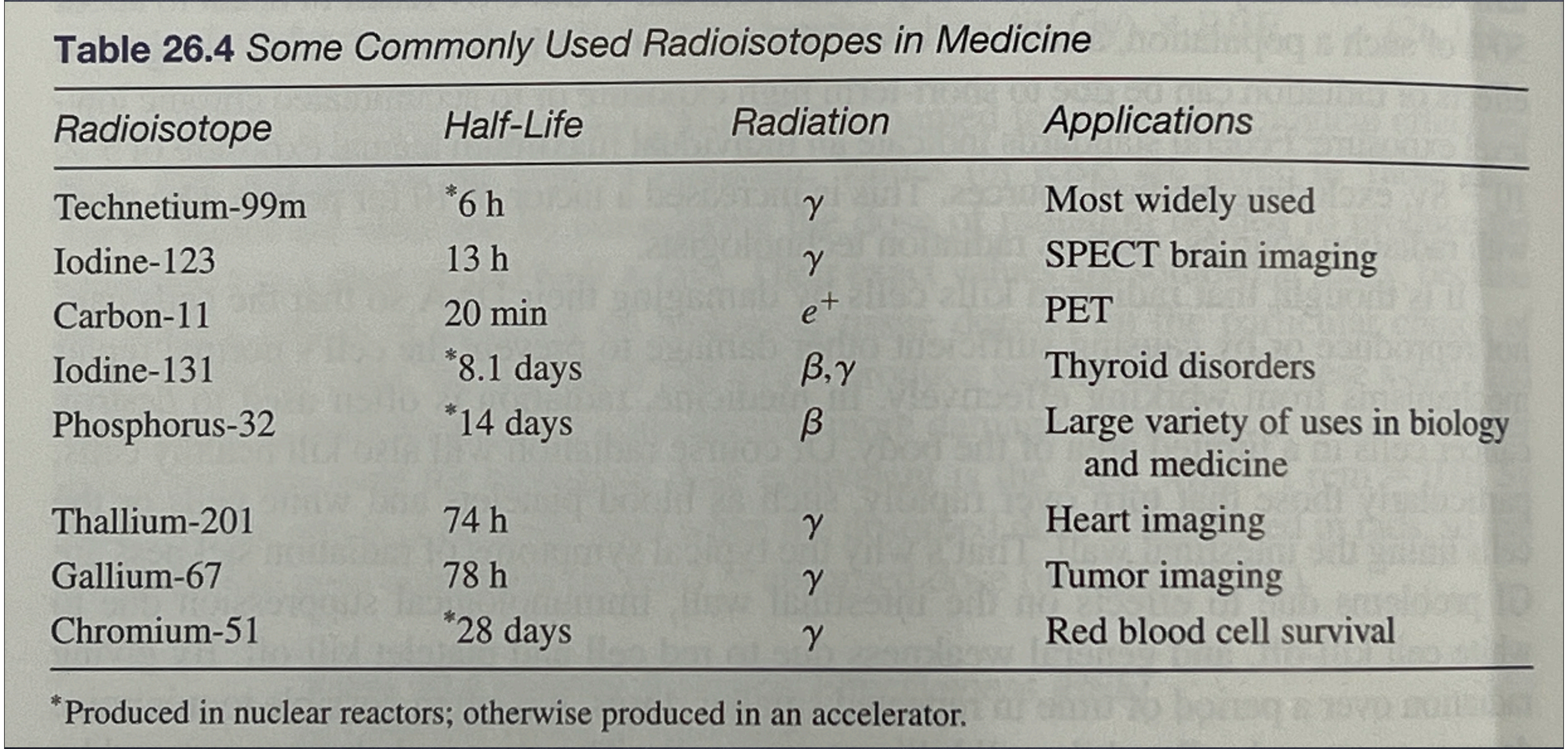
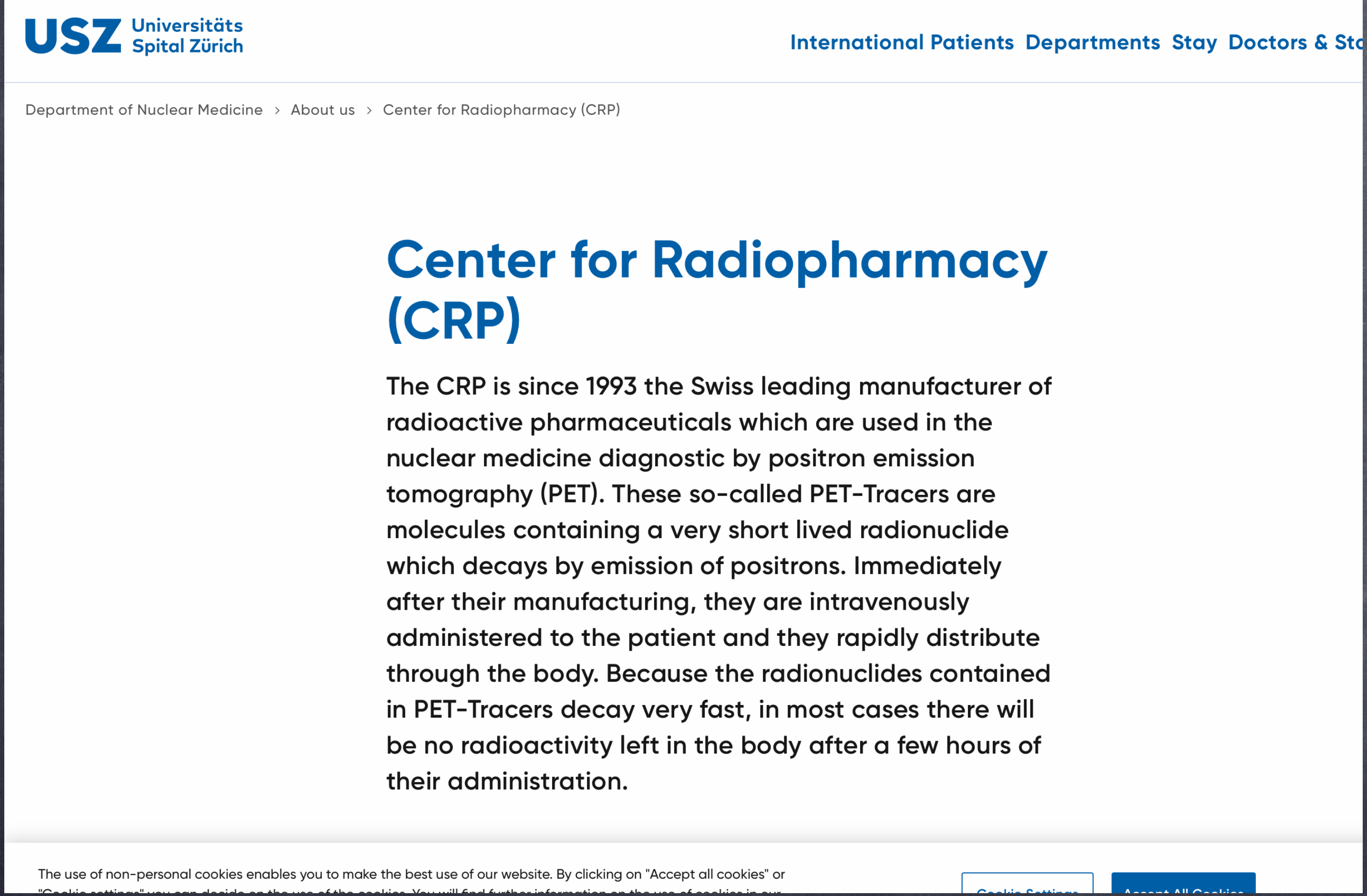


Table 26.4 *Some Commonly Used Radioisotopes in Medicine*

Radioisotope	*Half-Life*	*Radiation*	*Applications*
Technetium-99m	*6 h	γ	Most widely used
Iodine-123	13 h	γ	SPECT brain imaging
Carbon-11	20 min	e^+	PET
Iodine-131	*8.1 days	β,γ	Thyroid disorders
Phosphorus-32	*14 days	β	Large variety of uses in biology and medicine
Thallium-201	74 h	γ	Heart imaging
Gallium-67	78 h	γ	Tumor imaging
Chromium-51	*28 days	γ	Red blood cell survival

*Produced in nuclear reactors; otherwise produced in an accelerator.



Center for Radiopharmacy (CRP)

The CRP is since 1993 the Swiss leading manufacturer of radioactive pharmaceuticals which are used in the nuclear medicine diagnostic by positron emission tomography (PET). These so-called PET-Tracers are molecules containing a very short lived radionuclide which decays by emission of positrons. Immediately after their manufacturing, they are intravenously administered to the patient and they rapidly distribute through the body. Because the radionuclides contained in PET-Tracers decay very fast, in most cases there will be no radioactivity left in the body after a few hours of their administration.

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- ^{18}F-Fluoroglucose ZRP: Imaging of regional glucose consumption in cardiology, neurology and oncology
- ^{18}F-Fluorocholine ZRP: imaging of prostate or parathyroid cancer
- ^{18}F-Fluoethyltyrosine ZRP: Imaging of amino acid metabolism in the diagnosis of brain tumors
- ^{18}F-Sodium fluoride ZRP: Imaging of bone to detect abnormally altered bone formation activity
- ^{18}F-Vizamyl: Imaging of the density of neuritic β-amyloid plaques in the brain of adult patients with cognitive impairment who are being evaluated for Alzheimer's disease

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- ^{13}N-ammonia: Imaging of myocardial perfusion
- ^{15}O-Water: Imaging of regional brain perfusion
- ^{68}Ga-DOTATATE ZRP: Imaging of neuroendocrine tumors with somatostatin receptors
- ^{68}Ga-PSMA 11: Imaging of prostate cancer
- ^{18}F-PSMA 1007 ZRP: Imaging of prostate cancer

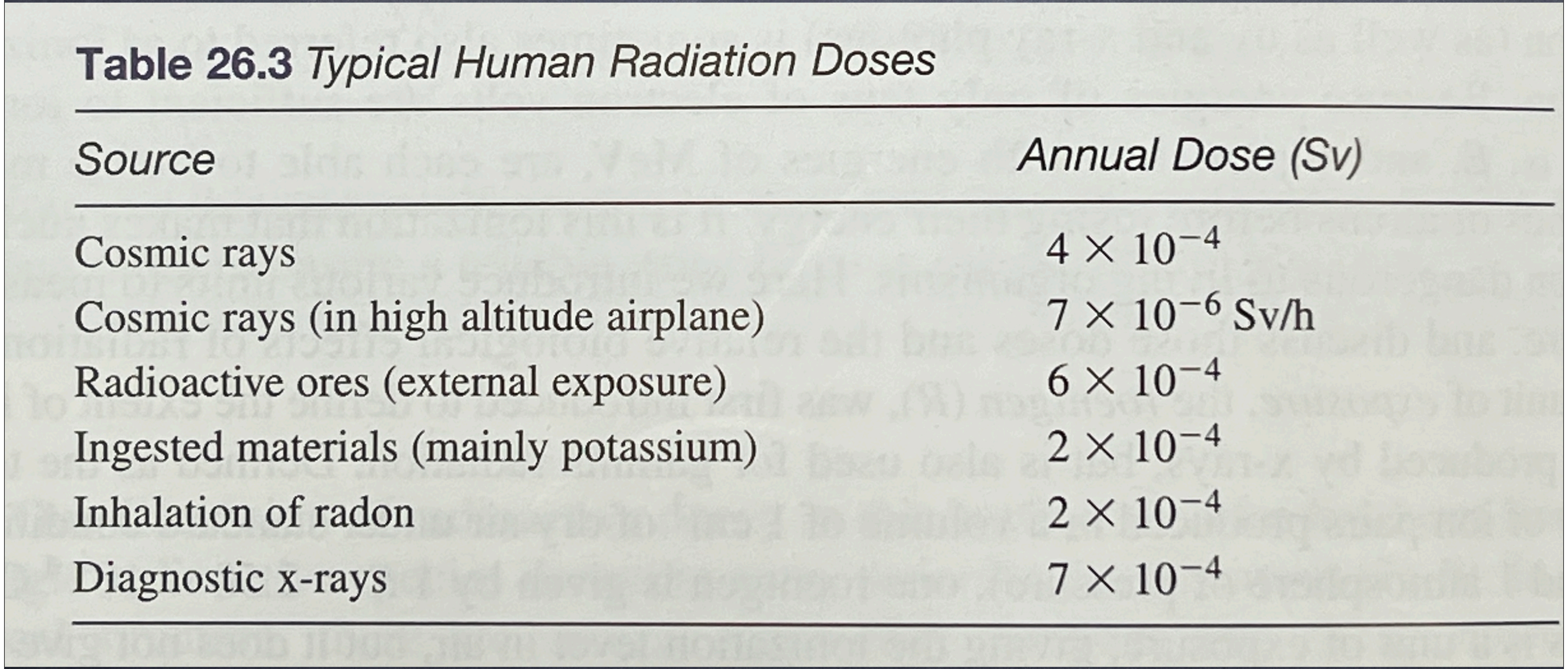


Table 26.3 *Typical Human Radiation Doses*

Source	*Annual Dose (Sv)*
Cosmic rays	4×10^{-4}
Cosmic rays (in high altitude airplane)	7×10^{-6} Sv/h
Radioactive ores (external exposure)	6×10^{-4}
Ingested materials (mainly potassium)	2×10^{-4}
Inhalation of radon	2×10^{-4}
Diagnostic x-rays	7×10^{-4}

A Sv is equivalent dose of 1 J/kg (see previous notes)

Dose in Sieverts : biological dose =
absorbed dose * factor
[Gy]

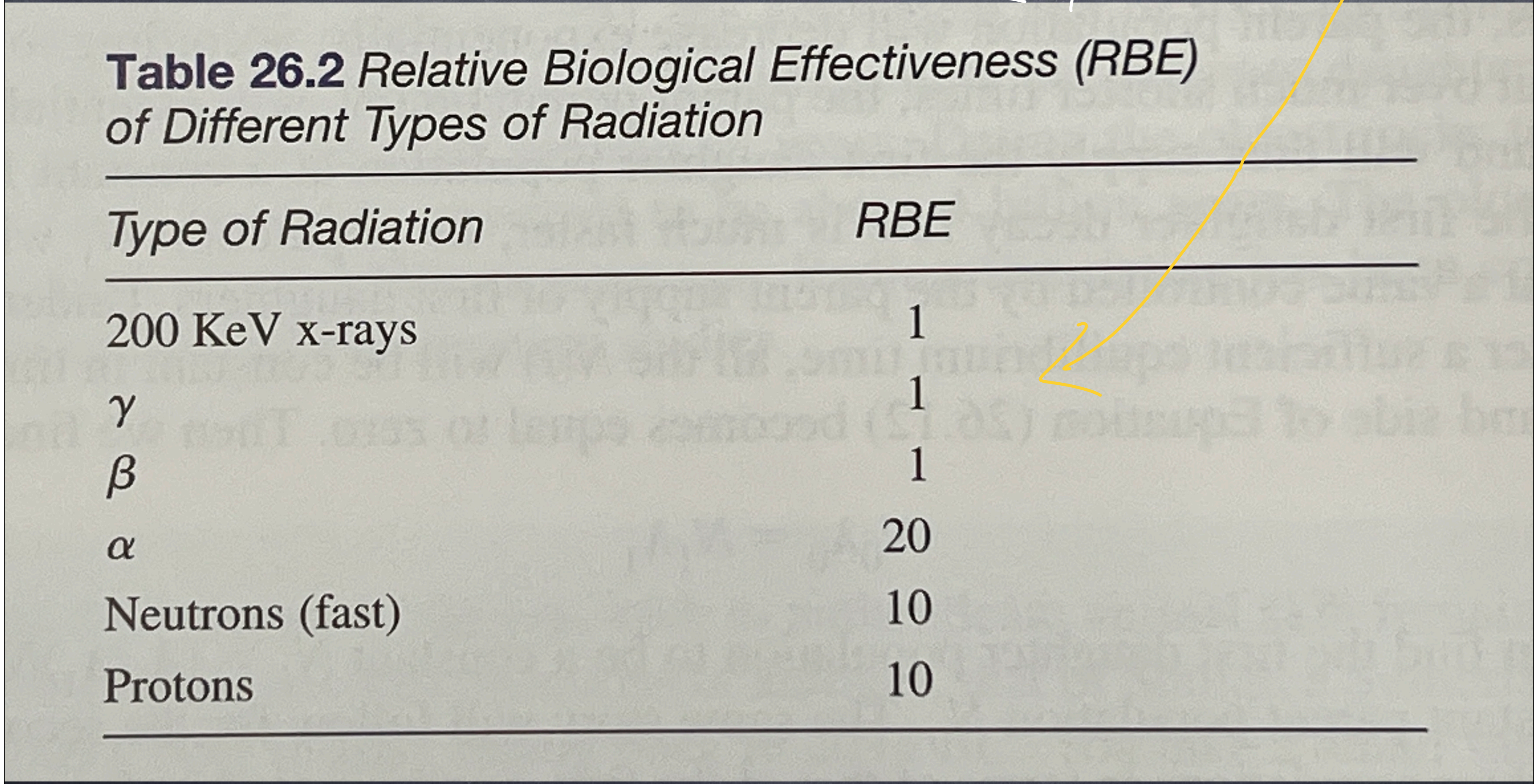
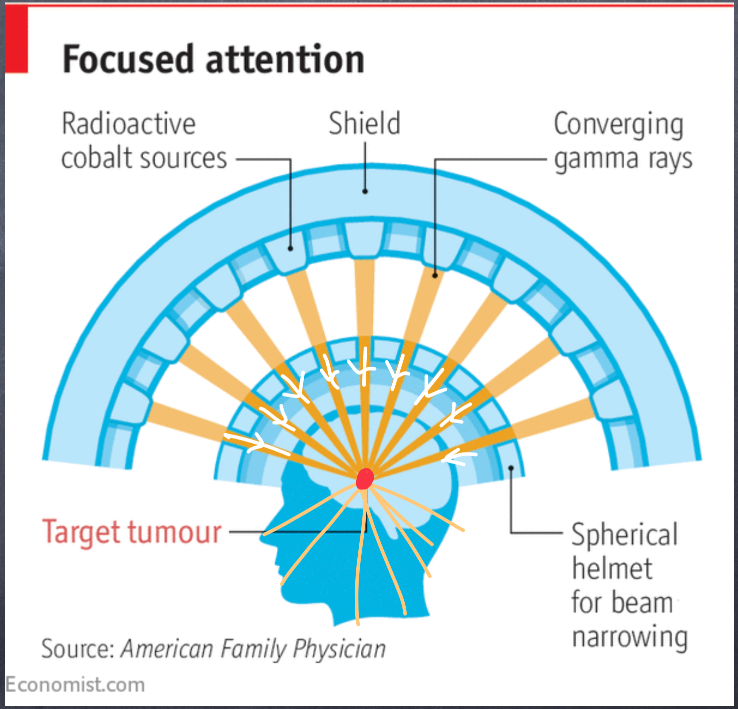


Table 26.2 *Relative Biological Effectiveness (RBE) of Different Types of Radiation*

Type of Radiation	*RBE*
200 KeV x-rays	1
γ	1
β	1
α	20
Neutrons (fast)	10
Protons	10

Dose depends on type of radiation.
α: largest because it interacts so quickly
β: also quite large

Gamma therapy
radiation



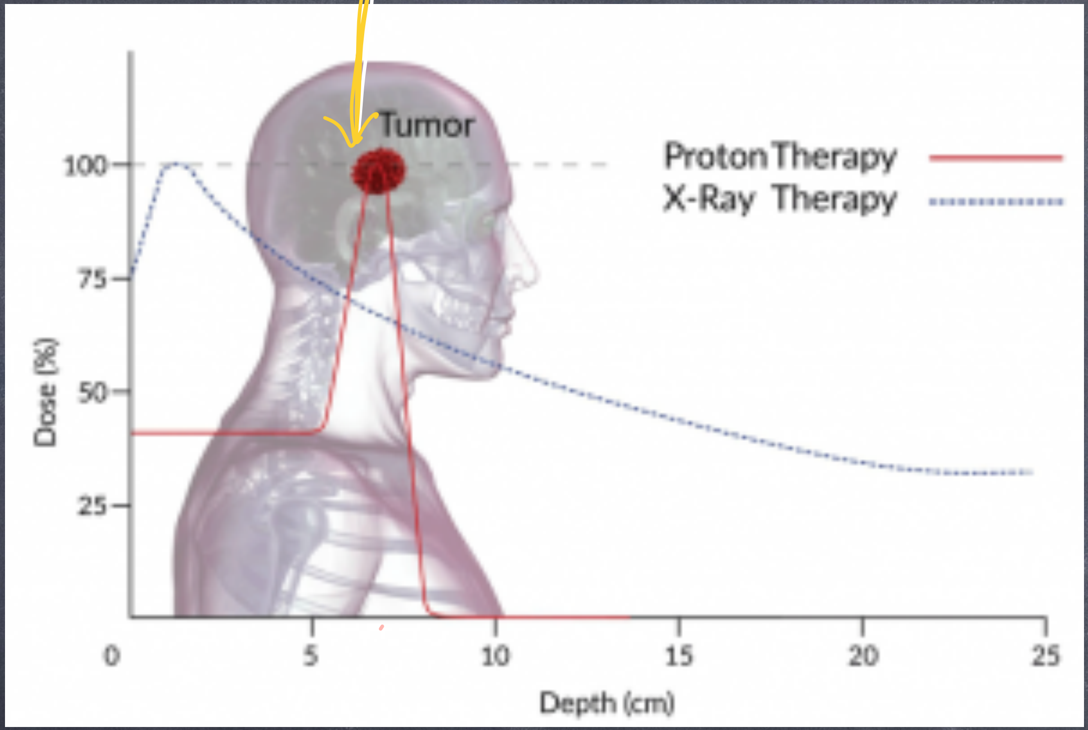
Focused attention

Source: *American Family Physician*

Economist.com

By attacking tumor from many angles, the intersection gets more dose, while heathy tissue damage is minimized.

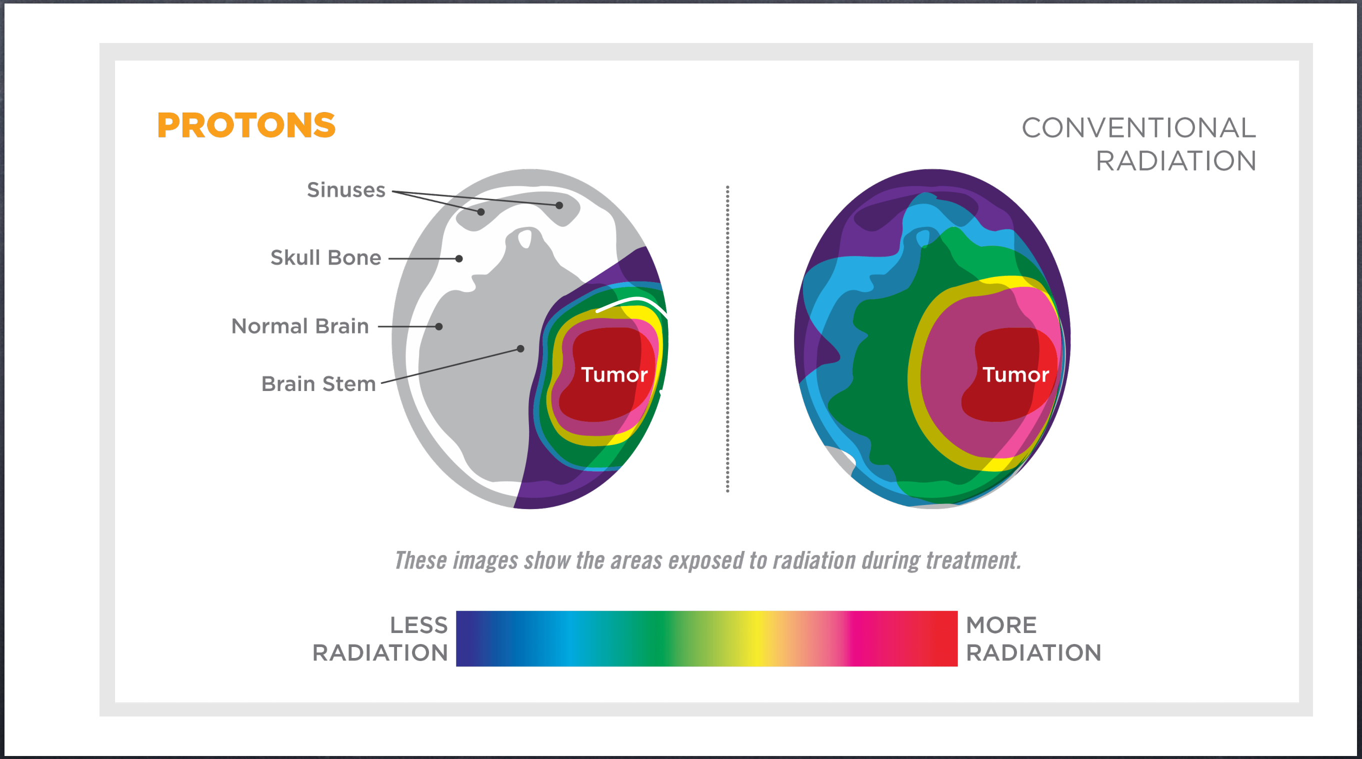
proton can target just the tumor, and destroy less tissue before & after the tumor



↑ Bragg peak tuned to a depth of 7 cm

comparison

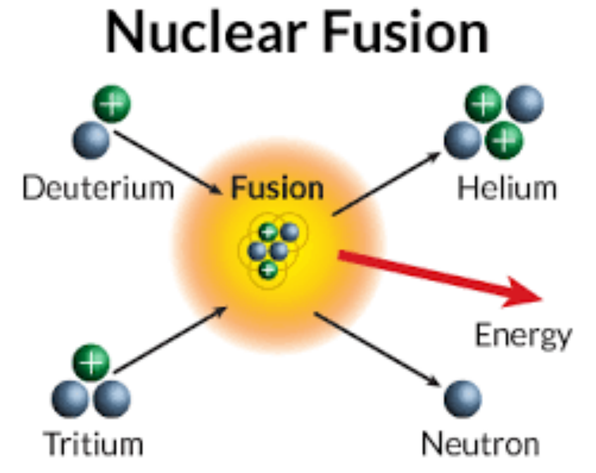
proton therapy gamma therapy



These images show the areas exposed to radiation during treatment.

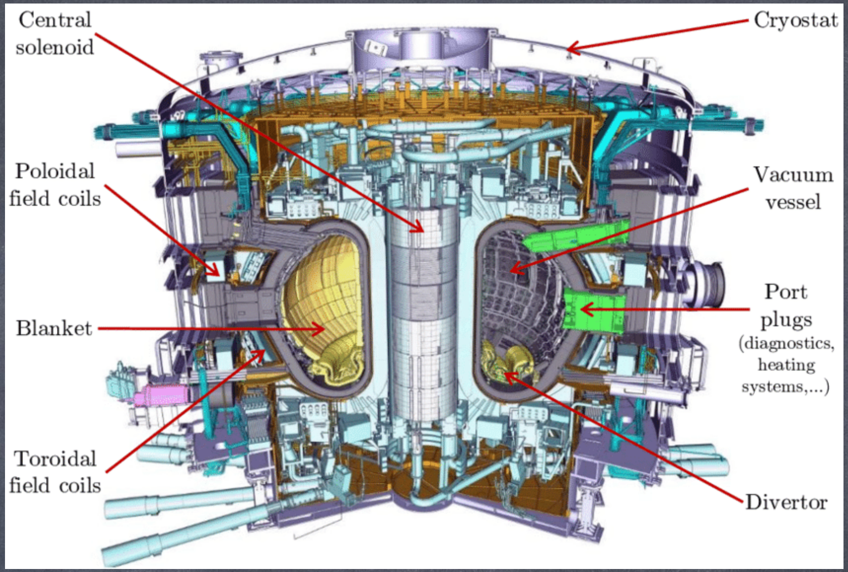
proton therapy facilities.

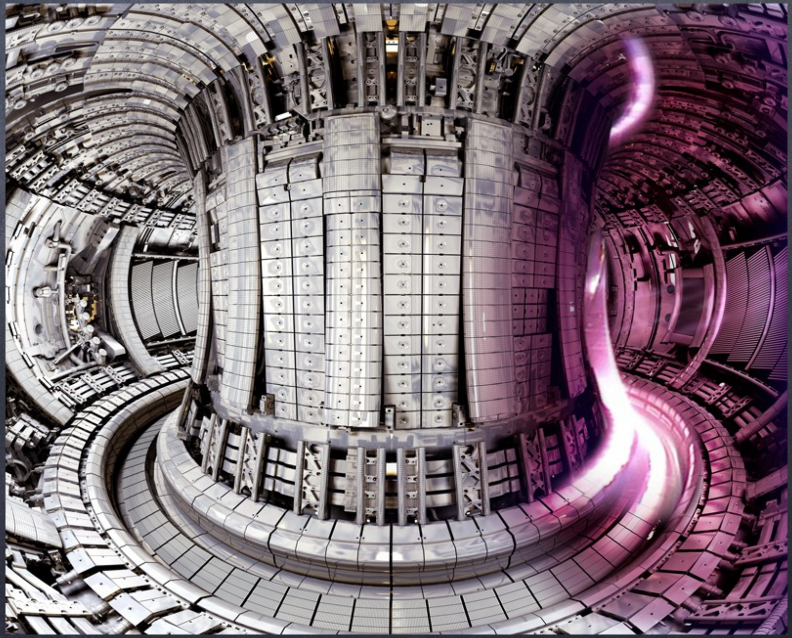
PSI :



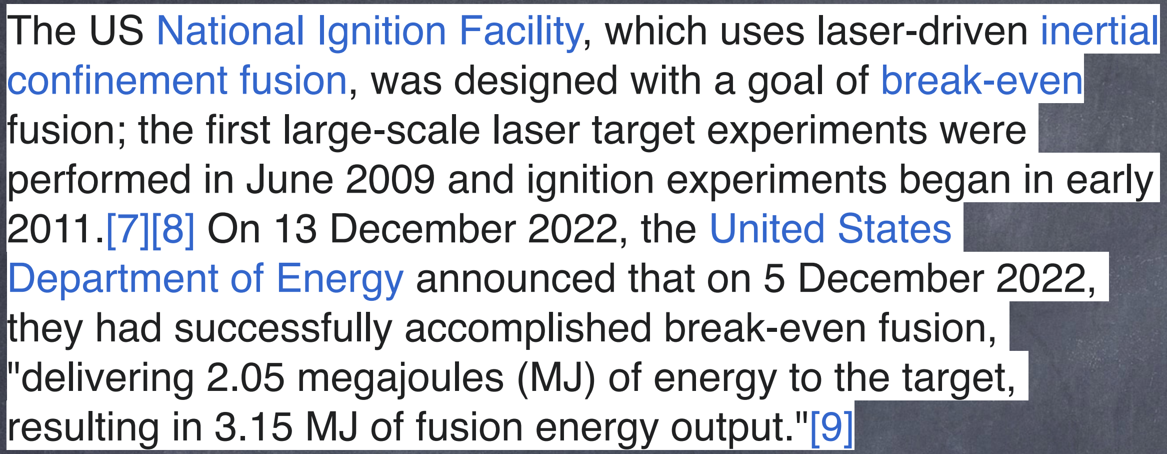
ITER

(Fusion not possible yet)





New this year:

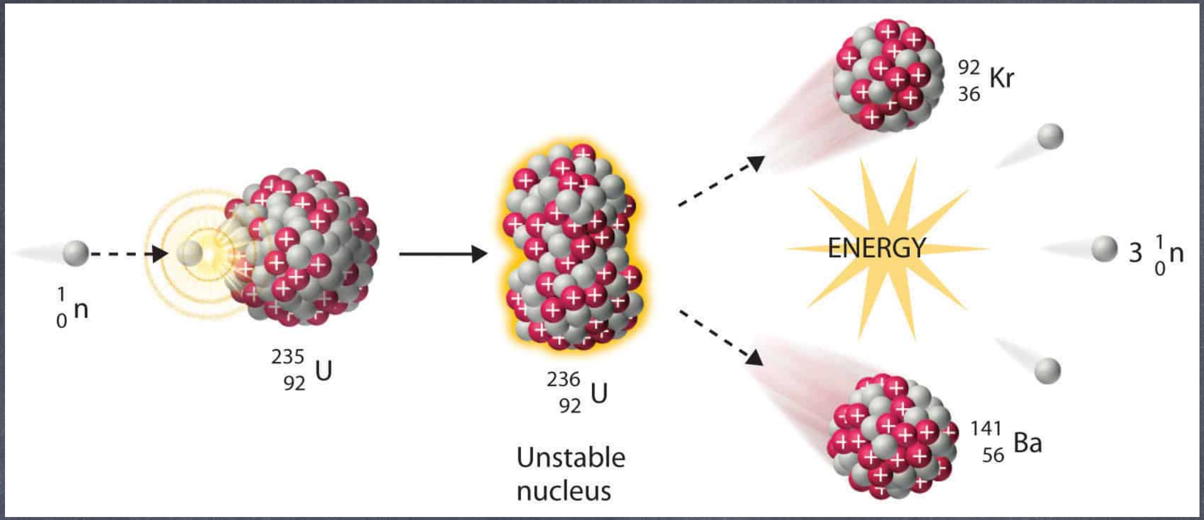


The US National Ignition Facility, which uses laser-driven inertial confinement fusion, was designed with a goal of break-even fusion; the first large-scale laser target experiments were performed in June 2009 and ignition experiments began in early 2011.[7][8] On 13 December 2022, the United States Department of Energy announced that on 5 December 2022, they had successfully accomplished break-even fusion, "delivering 2.05 megajoules (MJ) of energy to the target, resulting in 3.15 MJ of fusion energy output."[9]

Still a long time to go before commercial reactors possible, but this is a huge milestone.

nuclear fission

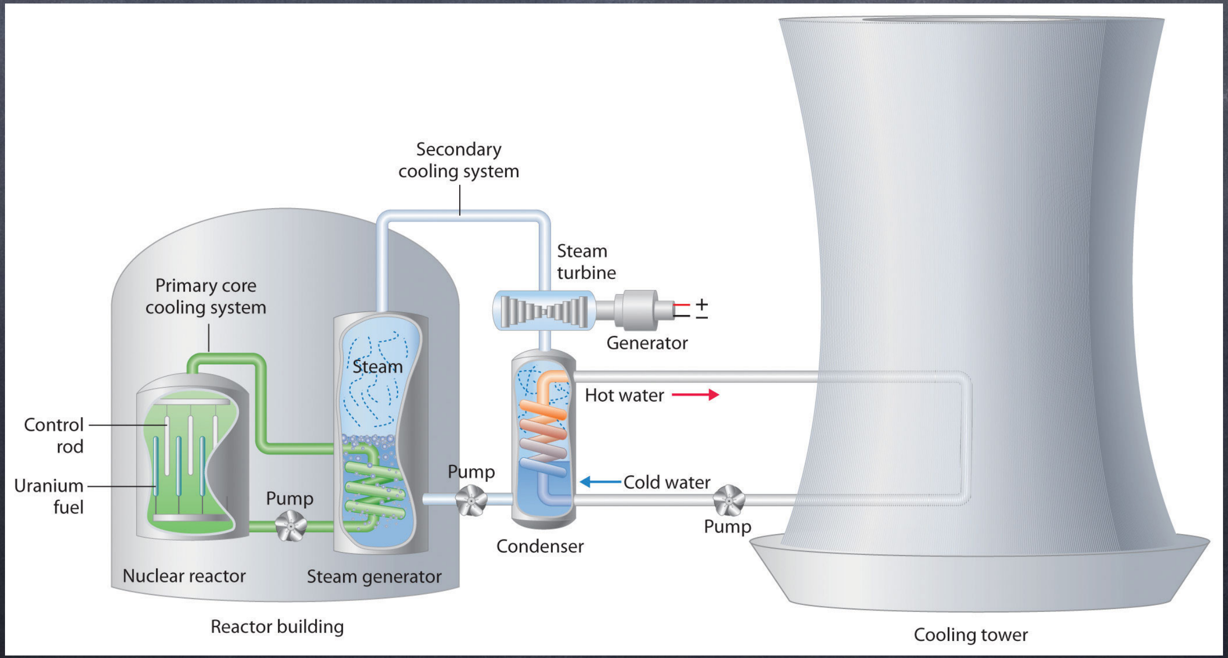
one neutron starts the reaction



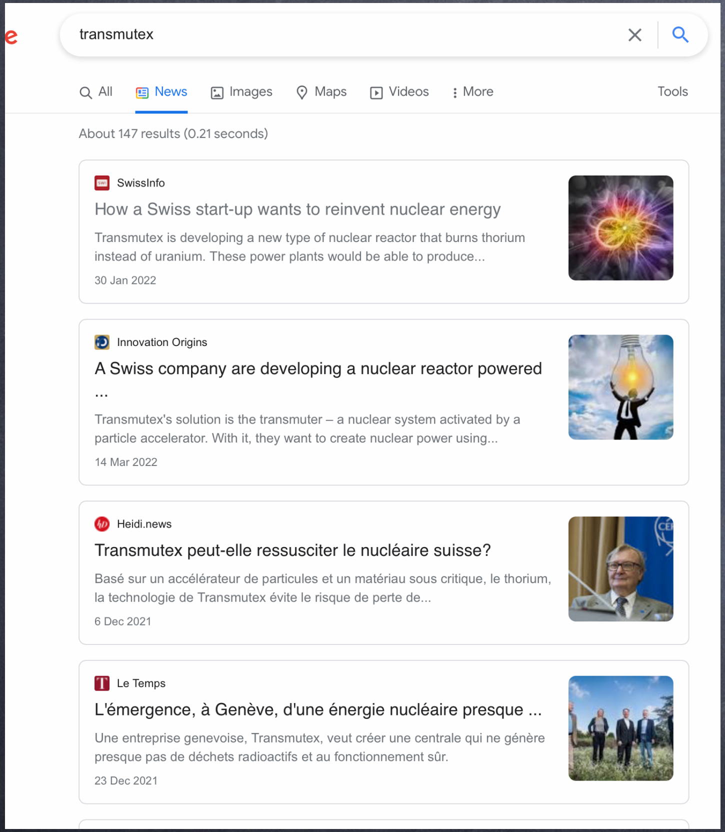
3 neutrons are produced.

A chain reaction starts.

This gets hot! (needs cooling)



subcritical nuclear fission reactor
can use nuclear waste as a fuel (Thorium)



transmutex

About 147 results (0.21 seconds)

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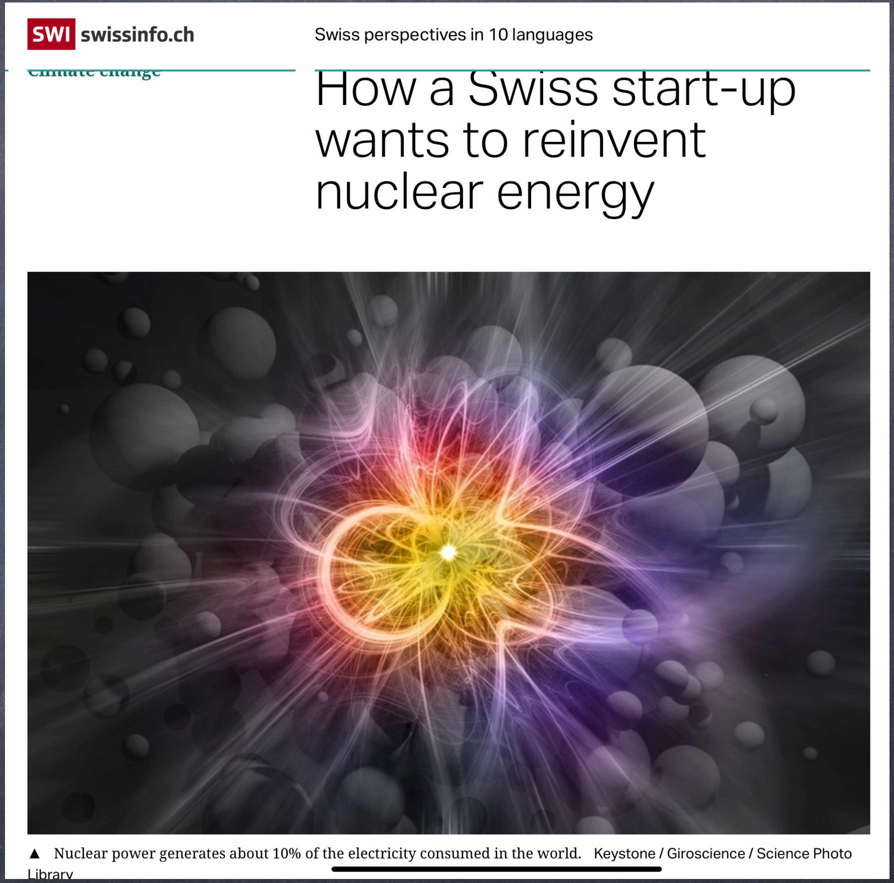


SWI swissinfo.ch   Swiss perspectives in 10 languages

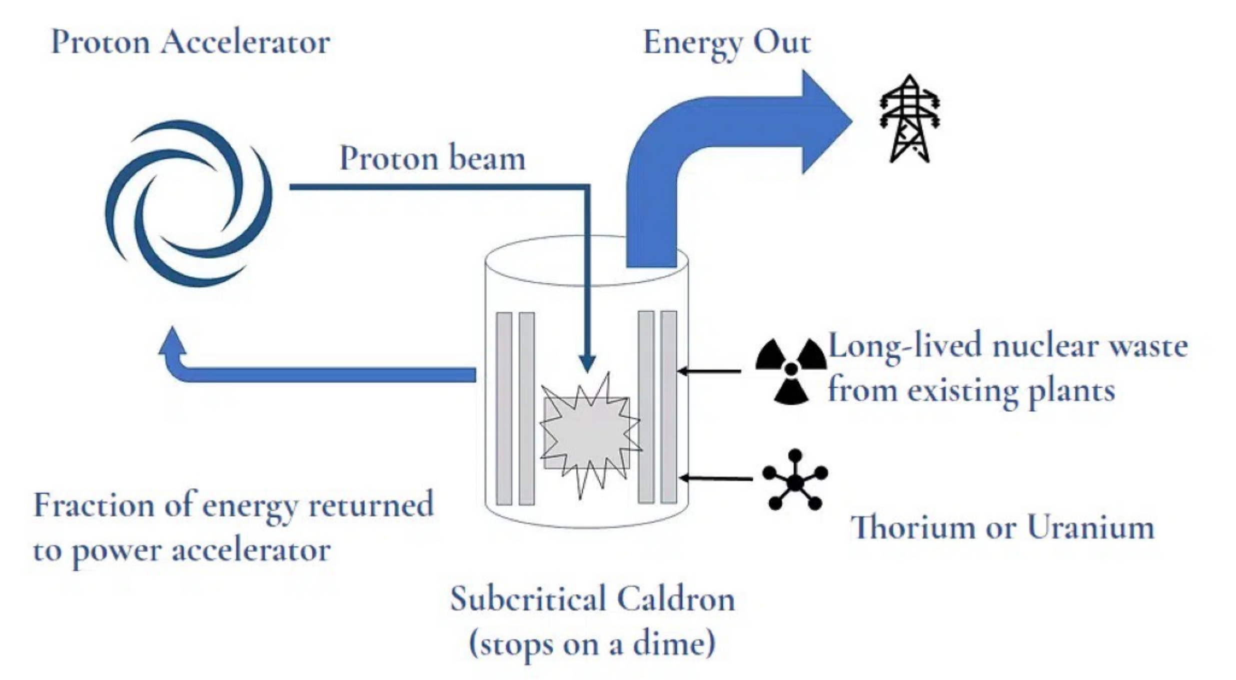
Climate change

How a Swiss start-up wants to reinvent nuclear energy

▲ Nuclear power generates about 10% of the electricity consumed in the world. Keystone / Giroscience / Science Photo Library

No risk of meltdown because fission only happens when beam of particles is on.
Transmutex aims to produce a proof of concept in ~25 years.

principle of subcritical nuclear fusion reactor
proton beam keeps chain reaction going.
↓



No risk of
"meltdown"
(uncontrolled
reactions)

SAFE

- burns radioactive waste
- thorium is abundant

Need to support R+D for these safe nuclear reactors since they will take decades to develop.