



Mechanics

Exercise Sheet 4

HS 2026
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<https://www.physik.uzh.ch/en/teaching/PHY311/HS2026.html>

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Exercise 1: Lagrangian system

A block of mass M is rigidly connected to a massless circular track of radius a on a frictionless horizontal table as shown in Fig. 1. A particle of mass m is confined to move without friction on the circular track which is vertical.

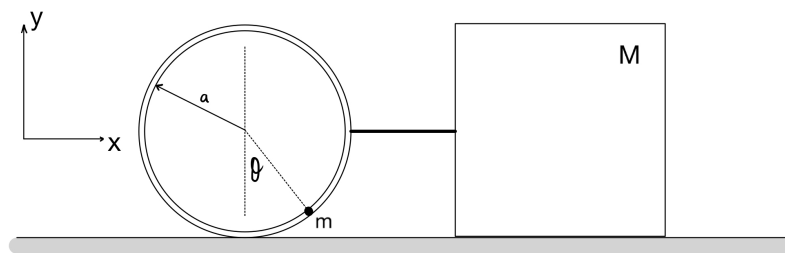


Figure 1: The system of a circular track connected to a block M .

- Set up the Lagrangian of the system, using θ as one coordinate.
- Find the equations of motion.
- In the limit of small angles, solve the equations of motion for θ as a function of time.

Exercise 2: Point mass on a cone

A point particle of mass m is moving under the influence of gravity and without friction on a cone surface of the form $z = \alpha\sqrt{(x^2 + y^2)}$.

- a) Find the Lagrangian of the system using cylindrical coordinates and express the constraint through a Lagrange multiplier. Afterwards, derive the equations of motion for the particle.
- b) Apply Noether's theorem to derive the conserved quantity associated to the Lagrangian in a). Compare your result with the Euler-Lagrange equations derived before.
- c) Show that for each fixed value of the conserved quantity, there exists a special solution to the Euler-Lagrange equations with constant radius of motion $r(t) = r_0 = \text{const.}$
- d) We will now investigate small perturbations to the above solution, i.e. nearly circular orbits. For this, expand the equations of motion around r_0 using $r(t) = r_0 + \delta r(t)$ with a small perturbation δr , where we have $\frac{\delta r}{r_0} \ll 1$. Now, find the solution for $\delta r(t)$.
Hint: Throughout your calculation only work up to first order in $\frac{\delta r}{r_0}$, meaning that you can set all terms of the form δr^k with $k \geq 2$ to zero.