Projects 2022 for bachelor/master students

Condensed Matter Theory Group, Prof. Titus Neupert

BEC-BCS crossover in quantum Hall bilayers (with Glenn Wagner)

The quantum Hall effect hosts a quantum fluid of strongly correlated electrons. In a quantum Hall bilayer, where two such fluids are coupled together, the competition between interlayer and intralayer Coulomb interactions gives rise to transitions between different states: When the layers are close together, the system is a quantum Hall ferromagnet and when the layers are far apart, they form two decoupled composite Fermi liquids. In a recent study using exact diagonalization, we found evidence of a BEC-BCS crossover between these states: There is an instability of the composite Fermi liquid towards forming interlayer excitons (much like the instability of a Fermi liquid towards superconductivity) and in the limit where the excitons are tightly bound, they describe a quantum Hall ferromagnet. The aim of this project will be to investigate this proposal using variational Monte-Carlo simulations and compare this to the predictions from the BCS theory of superconductivity. This project will involve numerical calculations for which knowledge of python will be useful.

Non-equilibrium physics of Luttinger liquids (with Bastien Lapierre)

Luttinger liquids are effective theories of interacting electrons in a one-dimensional quantum wire, which can be solved exactly following the bosonization procedure and provide low-energy descriptions of certain physical systems such as ultracold atoms, charges in nanowires, superconducting circuits, and critical spin chains. There has been recent studies on periodically driven Luttinger liquids [PRL **126**, 243401 (2021)], where the drive is implemented by varying periodically in time the Luttinger parameter, corresponding to modulating in time the interaction strength of the theory. We are interested in studying this time-dependent setup and its various generalizations in more detail, in particular by studying the structure of the excitations in this model, computing other physical quantities of interest such as entanglement entropy and energy density and/or extending it to quasi-periodic drives or to spatially inhomogeneous Luttinger liquids. The project will involve analytic calculations using methods from quantum field theory, as well as simple numerical calculations, for which basic knowledge of mathematica/python would be useful.

Chiral p-wave superconducvity on a buckyball (with Martina Soldini)

In conventional superconductivity, Cooper pairs are characterised by s-wave and spin-singlet wavefunctions, while unconventional superconductors are not subjected to these constraints. There, Cooper pairs can have higher order angular momentum and different spin states. An interesting type of unconventional pairing is realised in chiral p-wave superconductors, where the wave function of Cooper pairs carries a finite angular momentum and has a net chirality. This leads to intrinsic magnetism in Cooper pairs, which competes with the tendency of superconductors to expel magnetic field from their bulk -a property known as Meissner effect-. The coexistence of two natures, magnetism and Meissner effect, leads to a rich phenomenology of the chiral p-wave superconducting phase. In previous works, it was predicted that bending a chiral p-wave superconductor leads to the spontaneous emergence of magnetic field, arising from the curvature of the lattice. Another prediction concerns the presence of defects in the lattice, as fractional magnetic flux may be trapped at the core of these defects. With this project we would further explore these phenomena, for instance by studying this model on a buckyball (similar to a football ball), where curvature and lattice defects coexists. This may involve a phenomenological study of a chiral p-wave superconductor on a buckyball at first, and then the numerical solution of a discrete lattice model on this geometry. This project requires analytical as well as numerical skills, some knowledge of Python and/or Mathematica may be useful.

Vortex bound states in topological surface states (with Songbo Zhang)

Topological surface states have been found in ion-based superconductors such as $\mathrm{FeTe}_x\mathrm{Se}_{1-x}$, LiFeAs and $\mathrm{CaKFe}_4\mathrm{As}_4$ [see e.g., Nature Physics **15**, 41 (2019)]. It is well known that ion-based superconductors host an extended s-wave pairing order parameter. In the last few years, vortex bound states (oftentimes at zero bias voltage) in the presence of perpendicular magnetic fields have also been extensively observed in experiments [see e.g., Science **362**, 333 (2018); Science **367**, 189 (2020); Nature materials **18**, 811 (2019)]. However, so far these bound states are always explained with conventional s-wave pairing for simplicity. To better understand their behaviors, it may be necessary to consider the full form of the pairing. Therefore, this project will perform the following investigation:

i) first calculate the vortex bound states of 2D gapless Dirac surface states, described by $H = v(k_x s_x + k_y s_y)$, with an extended s-wave pairing $\Delta = \Delta_0 - \Delta_2(k_x^2 + k_y^2)$. We can do the calculation in a self-consistent Bogoliubov–de Gennes formalism [PRL **80**, 4763 (1998); PRL **84**, 554 (2000)]. It will be interesting to see, for example, how the chemical potential μ affect the vortex bound states. From the form of the pairing, we see that the pairing gap vanish when $\mu = v\sqrt{\Delta_0/\Delta_2}$. Thus, when changing μ across this critical value, we may see the change of the vorticity of the vortex bound states.

ii) we would then generate to study more complicated models, for instance, with adding a Dirac mass, third dimension, spectrum asymmetry terms (e.g., quadratic or warping corrections), or more orbitals. For complicated systems, we may appeal to alternative approaches such as tight-binding calculations.

Exceptional topological insulator (with Micheal Denner)

Non-Hermitian topology deals with systems that interact with the environment, and therefore lose energy. This can lead to completely new topological phases, which are absent in Hermitian systems. One example is exceptional topological insulators (ETI), three-dimensional systems that host exotic surface states. Mathematically, they have a band structure with a so-called exceptional point, which can only exist because of the 3D topological bulk embedding. As non-Hermitian systems do not conserve energy, their realization in nature proves to be tricky. Therefore, we have to rely on metamaterials to realize these phases. One particularly suited platform are electrical circuits, which mimic a crystal lattice: hoppings between sites are implemented by capacitors and inductors, whereas non-Hermitian terms are represented by resistors.

This project will revolve around measuring the unique signature of this circuit. Its modular design allows studying three models, by which you will not only learn about Hermitian topological phases but also about their non-Hermitian counterparts. Specifically, we want to study their bulk-boundary correspondence, i.e. we want to see how the bandstructure under periodic boundary conditions gives rise to surface states under open boundary conditions. The measurement setup has already been built and partly automized at University of Würzburg, so measuring the circuit would also entail a stay there.

Understanding the exotic surface states of an ETI is a central open research question, which could provide useful information not only for the realization of other non-Hermitian phases but also pave the road for a future material discovery. The key tasks of this project are to

- Gain an understanding of non-Hermitian topological phases, specifically the ETI
- Measure the bulk and surface spectrum of a Weyl semimetal, the ETI, and a trivial non-Hermitian model
- Analyze the obtained data to extract the topological signatures
- Investigate the dynamical properties of the surface states of each of these systems