MMP I
Tutorial 5

HS 2017
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`http://www.physik.uzh.ch/en/teaching/PHY312/HS2017.html`

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Exercise 1: Ordinary differential equations (4 Pts.)

a) Solve the differential equation

$$y' = \frac{\cos x \cos y + \sin^2 x}{\sin x \sin y + \cos^2 y} \tag{1.1}$$

An implicit solution of the form $f(x, y(x)) = 0$ is sufficient.

b) A particle of mass m and charge q moving in a magnetic field $\vec{B}$ experiences a force

$$\vec{F} = q \cdot \vec{v} \times \vec{B}, \text{ with } \vec{v} = \dot{\vec{r}} \tag{1.2}$$

Solve the equation of the motion

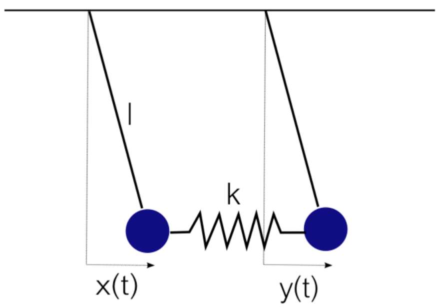
$$m\ddot{\vec{r}} = q \cdot \dot{\vec{r}} \times \vec{B} \tag{1.3}$$

for a constant $\vec{B}$, by choosing a convenient coordinate system which is at rest with respect to the magnetic field.

Exercise 2: Bound oscillators (4 Pts.)
The motion of two bound pendula of the same mass m and length l is described by the following equations (g is the gravitational acceleration, k is the spring constant):

$$\begin{aligned} m\ddot{x} &= -\alpha x - k(x - y) \\ m\ddot{y} &= -\alpha y - k(y - x), \end{aligned} \tag{2.1}$$

where $\alpha = mg/l$. The origin of the coordinate system is chosen so that $y = x = 0$ corresponds to the rest position. These equations are linearized and only valid for small oscillations. Find the fundamental system of solutions. Discuss the motion of the oscillators for the following initial conditions: $x(0) = y(0) = \dot{y}(0) = 0$, $\dot{x}(0) = 1$.



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Exercise 3: Systems of differential equations (4 Pts.)
Find the fundamental system of solutions for the following systems of linear differential equations:

a)
$$\begin{aligned} x' + x - y &= 0 \\ y' - 4x + y &= 0 \end{aligned} \tag{3.1}$$

b)
$$\begin{aligned} x' &= y \\ y' &= -2x + 2y \end{aligned} \tag{3.2}$$

c)
$$\begin{aligned} x' &= x - y + z \\ y' &= 2y \\ z' &= -2x + y - z \end{aligned} \tag{3.3}$$

d)
$$\begin{aligned} x' &= x + y + 2z \\ y' &= x + 2y + z \\ z' &= 2x + y + z \end{aligned} \tag{3.4}$$