Applications of General Relativity

Sheet 8

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Exercise 1 [Non-monochromatic gravitational waves] (5 points)
In the lecture we considered monochromatic gravitational waves $h_{\mu\nu}(t,\boldsymbol{x}) = h_{\mu\nu}(\boldsymbol{x}) \exp[-i\omega t] + \text{c.c.}$. A general gravitational wave can be a superposition of different frequencies. Start from the second order Ricci tensor $R^{(2)}_{\mu\nu}$, Eq. (5.54) in the script, to derive the general expression for the energy-momentum tensor of a gravitational wave. First, impose the Lorentz gauge $\partial_\mu \gamma^{\mu\nu} = 0$ and the tracelessness $\gamma = \eta^{\mu\nu}\gamma_{\mu\nu} = 0$ condition, where $\gamma_{\mu\nu} = h_{\mu\nu} - \eta_{\mu\nu} h/2$. Then, perform the temporal average on $R^{(2)}_{\mu\nu}$ and, using Eq. (5.50) in the script, derive:

$$t^{\text{grav}}_{\mu\nu} = \frac{c^4}{32\pi G} \left\langle \partial_\mu \gamma^{\alpha\beta} \partial_\nu \gamma_{\alpha\beta} \right\rangle, \tag{1}$$

where $\langle \cdot \rangle$ denotes the temporal average. *Hint: note that spacetime derivatives ∂_μ in the temporal average can be integrated by parts, neglecting the boundary terms... Why?*

Exercise 2 [Plugging in the numbers] (7 points)

(i) In 2003, a double pulsar, a binary pulsar consisting of two neutron stars (each of mass $m \sim 1.4$ $\text{M}_\odot$, with an orbital period $t_0 \simeq 1.4$ h), was discovered. Estimate the timescale for coalescence via emission of gravitational radiation.

(ii) Suppose that two supermassive (10^9 $\text{M}_\odot$) black holes merge in a galaxy at a cosmological distance ($D \sim 1$ Gpc). Consider a detector built to detect the gravitational waves from such events.

  (a) Estimate the frequency range that the detector would have to operate at.

  (b) Estimate the strain sensitivity (smallest dimensionless amplitude h) that would be necessary to see mergers at these distances.

  (c) Estimate the duration of these events.

(iii) The LIGO gravitational-wave detector expects to detect gravitational waves produced by coalescing binaries at frequencies $\nu \sim 200$ Hz with dimensionless strains of $h \sim 10^{-21}$.

  (a) What is the flux of energy of such waves?

  (b) If the determined flux comes from $D \simeq 20$ Mpc, what would the luminosity of the source be?

  (c) How far away would the Sun have to be to produce the same flux in the electromagnetic radiation?