Energie-Erhaltung

Arbeit: $W = \int \vec{F} \cdot d\vec{r}$ $\qquad dW = \vec{F} \cdot d\vec{r}$

($\vec{F}$: Kraft; $d\vec{r}$: Weg der Kraftrichtung)

kinetische Energie: $E_{kin} = \frac{m}{2}|\vec{v}|^2 = \frac{|\vec{p}|^2}{2m}$

$$dE_{kin} = m\vec{v} \cdot d\vec{v} = \frac{1}{m}\vec{p} \cdot d\vec{p}$$

$$dW = \vec{F} \cdot d\vec{r}$$

$$\shortparallel$$

$$dE_{kin} = m\vec{v} \cdot d\vec{v} = m\frac{d\vec{r}}{dt} \cdot d\vec{v} = m\frac{d\vec{v}}{dt} \cdot d\vec{r} = \frac{d\vec{p}}{dt} \cdot d\vec{r} = \vec{F} \cdot d\vec{r}$$

$$dE_{pot} = -dW = -dE_{kin}$$

$$dE_{kin} + dE_{pot} = 0 \implies d(E_{pot} + E_{kin}) = 0$$

$$\implies \boxed{E_{pot} + E_{kin} = \text{konst}}$$

Bremsweg

Ann: m, v, μ

(m: Masse; v: Geschwindigkeit; μ: Reibungskoeff.)

$E^{(t=0)}$ m 2 ΔE m 2

$E_{kin} = \frac{m}{2} v \quad \Rightarrow \Delta E_{kin} = \frac{m}{2} v$

$E_{kin}(t=\tau) = 0$

$W_R^{(t=0)} \stackrel{!}{=} 0$

$\Delta W_R = \mu m g d$

$W_R(t=\tau) = \mu m g \cdot d$

↑ Bremsweg

E-Erhaltung $\Rightarrow \Delta E_{kin} = \Delta W_R \qquad \frac{m}{2} v^2 = \mu m g d$

$$d = \frac{v^2}{2\mu g}$$

$m \frac{dv}{dt} = -\mu m g \qquad v(t=0) = v_0$

$dv = -\mu g\, dt \qquad v = -\mu g t + v_0$

$\frac{ds}{dt} = -\mu g t + v_0 \Rightarrow s = -\mu g \frac{t^2}{2} + v_0 t$

$v = 0 = -\mu g \tau + v_0 \Rightarrow \tau = \frac{v_0}{\mu g}$

$$s(\tau) = d = -\frac{\mu g}{2} \cdot \left(\frac{v_0}{\mu g}\right)^2 + \frac{v_0^2}{\mu g} = \frac{v_0^2}{2\mu g}$$

Hangjumper

$dW_F = kx \cdot dx$

..

$E_{Feder} = k\frac{x^2}{2}$ k: Federkonstante

x: Stauchung der Feder

Am Anfang

Lösen der Feder $\Rightarrow \Delta E_F = k\frac{x^2}{2} = \Delta E_{kin} = \frac{m}{2}v^2$

Hochfliegen $\Rightarrow \Delta E_{kin} = \frac{m}{2}v^2 = \Delta E_{pot} = mgh$

$$\Rightarrow mgh = \frac{kx^2}{2} \Rightarrow h = \frac{kx^2}{2mg} = \frac{8\cdot 10^2\,\mathrm{N/m}\cdot 13^2\cdot(10^{-2})^2\,\mathrm{m}^2}{8\,\mathrm{N}} = 1.7\,\mathrm{m}$$

$mg = 4\,\mathrm{N}$ $k = 800\,\mathrm{N/m}$ $x = 0.13\,\mathrm{m}$

Kraft aus potentieller Energie

$$dE_{pot} = -dW = -\vec{F}\cdot d\vec{r} = -F_x\,dx - F_y\,dy - F_z\,dz$$

in 1-dimension $F = -\frac{dE_{pot}}{dr}$

$$F_x = -\frac{\partial E_{pot}}{\partial x} \qquad F_y = -\frac{\partial E_{pot}}{\partial y} \qquad F_z = -\frac{\partial E_{pot}}{\partial z}$$

$$\vec{F} = -\begin{pmatrix} \partial E_{pot}/\partial x \\ \partial E_{pot}/\partial y \\ \partial E_{pot}/\partial z \end{pmatrix} = -\vec{\nabla}\cdot E_{pot}$$

$$\vec{\nabla} = \begin{pmatrix} \partial/\partial x \\ \partial/\partial y \\ \partial/\partial z \end{pmatrix}$$