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Applications of General Relativity

Sheet 1

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Prof. P. Jetzer

M. Haney, S. Tiwari, M. Ebersold
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Exercise 1 [The cosmological constant in the black hole context]

a) Show that the Einstein Field Equations with cosmological constant

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} - \Lambda g_{\mu\nu} = \kappa T_{\mu\nu}\,, \tag{1}$$

reduce to

$$R_{\mu\nu} + \Lambda g_{\mu\nu} = 0\,, \tag{2}$$

in vacuum.

b) Assume spherical symmetry, and solve the equations to find the black hole solution.

c) Given a photon passing the black hole, show if and how the formula for the deflection of light is modified. Do not assume a weak field.

Exercise 2 [Age and size of the universe]

Calculate age and size of a flat ΛCDM universe. In order to find the size, consider the comoving distance covered by a photon traveling on a radial geodesic from the big bang to the present time. The Planck 2013 best fit density parameters are $\Omega_{\mathrm{m},0} = 0.30$, $\Omega_{\Lambda,0} = 0.70$ and the Hubble constant is measured to be $H_0 = 68\ \mathrm{km\,s^{-1}Mpc^{-1}}$ ($1\,\mathrm{Mpc} = 3.085 \times 10^{19}\,\mathrm{km}$). For simplicity, the expansion is normalized such that present time corresponds to $a_0 = 1$.
Hint: Show that the first Friedmann equation can be written as

$$H(a) = H_0\sqrt{\Omega_{\mathrm{m},0}a^{-3} + \Omega_{\Lambda,0}}\,. \tag{3}$$