

PHY 117 HS2026

Today:

Energy transformation / conservation

Non-conservative forces

Elastic and inelastic collisions

Momentum conservation

Relationship of momentum and force

Impulse = change in momentum

Week 3, Lecture 2
Sept. 30th, 2026
Prof. Ben Kilminster

Note: several "k"s:
I'll try to write
them differently

K : kinetic energy
units: $[N \cdot m] = [J]$

k : spring constant
units: $\frac{N}{m}$

so far: work done by spring:
is $W_s = -\frac{1}{2}k(\Delta x)^2$

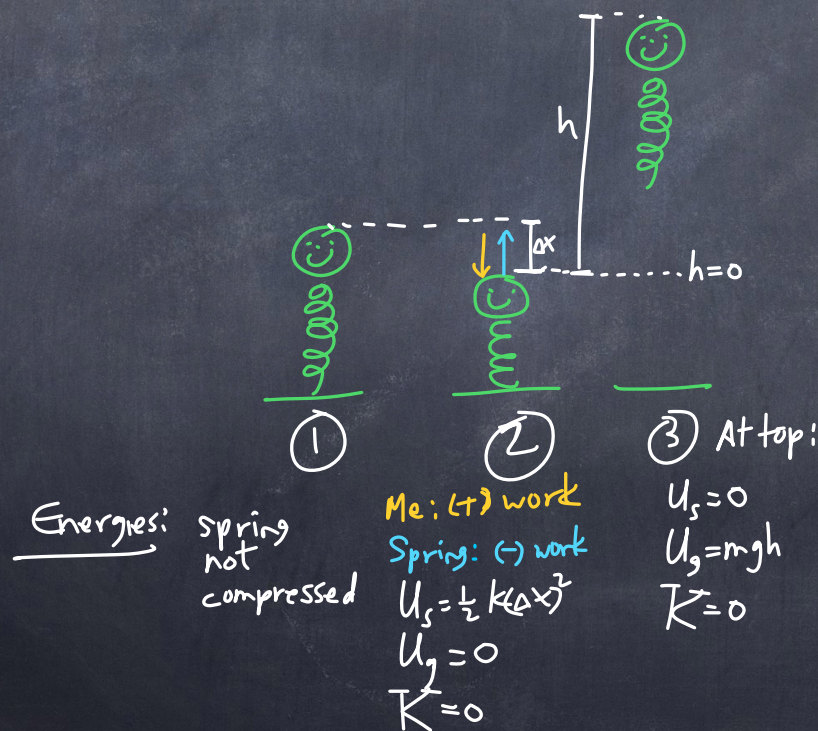
work done by me:
 $W_{me} = -W_s = +\frac{1}{2}k(\Delta x)^2$

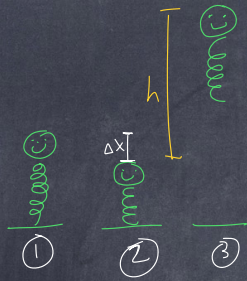
Negative work by the spring
gives it positive potential
energy

$$\Delta U_{\text{spring}} = -W_{\text{spring}} = \frac{1}{2}k(\Delta x)^2$$

Keep in mind $\begin{cases} \Delta U = U_f - U_i \\ U_f = \Delta U + U_i \end{cases}$

no initial stored
energy in spring





Apply energy conservation:

$$\text{Total energy } ② = \text{Total energy } ③$$

Non-conservative forces: $K + U$ is no longer conserved. Work depends on path taken.

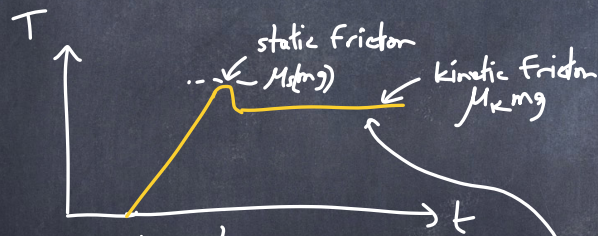
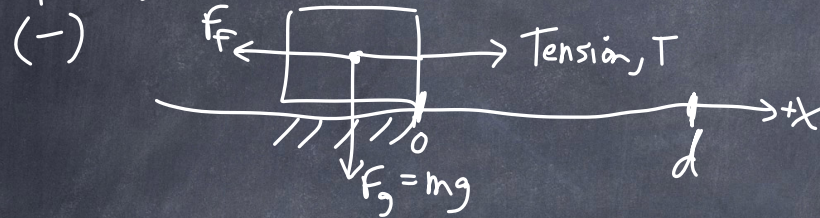
Generally, non-conservative forces convert mechanical energy into unusable forms such as heat.

Examples: friction, air resistance, drag, deformation, breaking

Consider friction:

What is work done by force of friction?

f_f is opposite movement



spring stretching
 $F = k \Delta x \rightarrow W = \frac{1}{2} k (\Delta x)^2$

At constant velocity,

$$\sum F = 0$$

$$T - F_f = 0$$

$$T = F_f \text{ is constant}$$



Friction: friction does work,
 becomes heat. No longer
 available for U or K
 $U + K$ not conserved.

$$\vec{F}_f = -\mu F_N \hat{x} = -\mu mg \hat{x}$$

$$d\vec{x} = dx \hat{x}$$

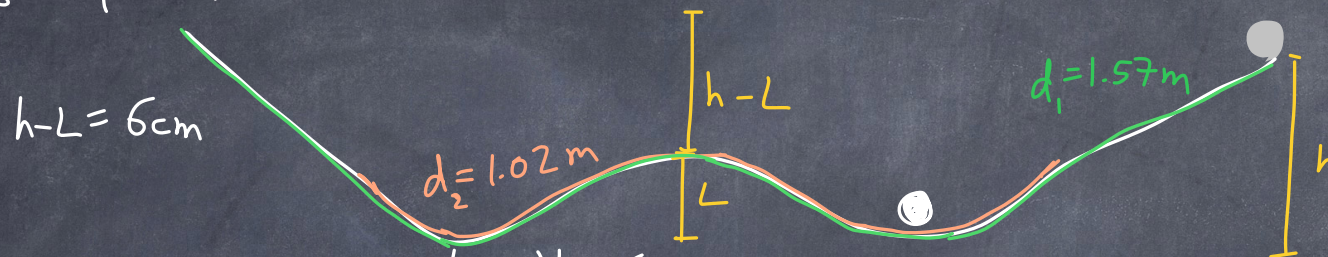
$$W_f = \int_{x_1}^{x_2} \vec{F}_f \cdot d\vec{x} = \int_0^d \vec{F}_f \cdot dx \hat{x} = \int_0^d -\mu mg \hat{x} \cdot \hat{x} dx$$

$$W_f = \int_0^d (-\mu mg) dx = -\mu mg x \Big|_0^d$$

$$W_f = -\mu mg d$$

work done
 by friction for
 the case of
 moving a distance, d .

How many times will a steel ball roll across this track before getting stuck?



d_1 : Full length of track

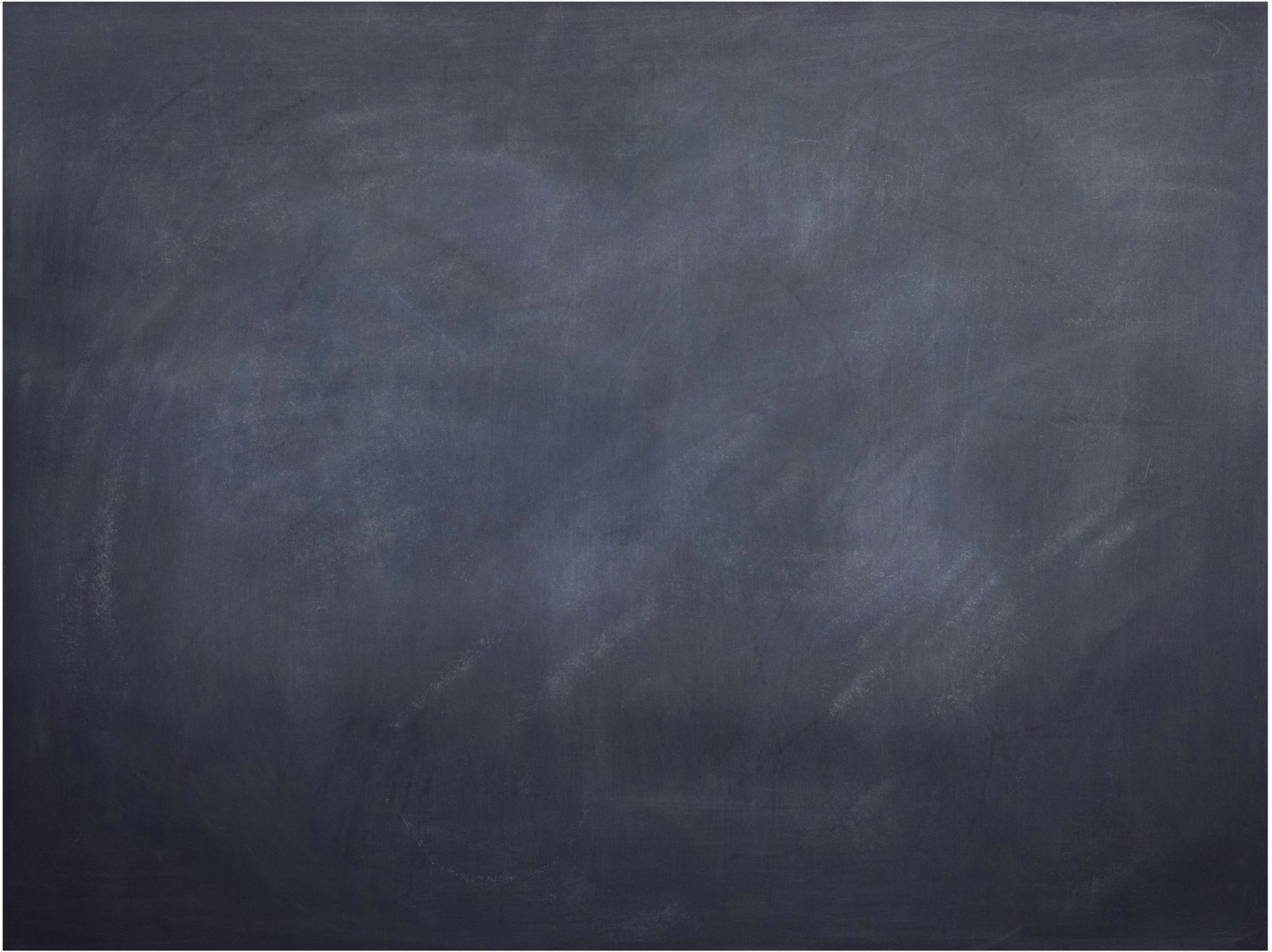
d_2 : length of track when ball gets stuck

This problem can be solved with energy conservation. we have K , U_g , and work done by friction, W_f . The ball will roll across the track until it loses gravitational potential energy through friction, and can no longer get over the bump. This happens when $U_g = mg(h-L)$ is lost due to the work of friction,

$$W_f = \mu_r mg D$$

D : total distance traveled.

μ_r : coefficient of friction for a steel ball rolling on an aluminum track = $\mu_r = 0.00076$



collisions : elastic + inelastic collisions

A collision is elastic if there is no work done by the forces of friction, ^{permanent} deformation, or sticking or breaking. Otherwise, it is inelastic.



In elastic collisions, $K + U = \text{constant}$.

If U doesn't change, then K must also stay the same: $K_{\text{initial}} = K_{\text{final}}$

So kinetic energy is conserved in elastic collisions

Elastic collisions: $K_{in} = K_{fin}$

In elastic collisions, momentum is also conserved.

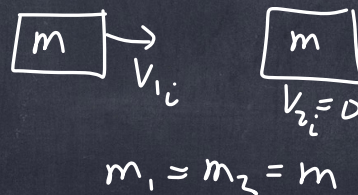
Momentum is defined as $\vec{p} = m\vec{v}$ (It is a vector)

Conservation of momentum means

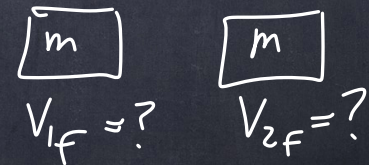
$$\vec{p}_{initial} = \vec{p}_{final} \quad \text{or} \quad \underbrace{\sum_i m_i \vec{v}_i}_{\text{total initial momentum}} = \underbrace{\sum_f m_f \vec{v}_f}_{\text{total final momentum}}$$



Experiment:



final?



conservation of momentum: $m \bar{V}_{1i} = m \bar{V}_{1f} + m \bar{V}_{2f}$ ①

conservation of K : $\frac{1}{2} m V_{1i}^2 = \frac{1}{2} m V_{1f}^2 + \frac{1}{2} m V_{2f}^2$ ②

Square ①: $V_{1i}^2 = (V_{1f} + V_{2f})^2 = V_{1f}^2 + V_{2f}^2 + 2V_{1f}V_{2f}$ ③ 2 equations, 2 unknowns

Two formulas for V_{1i}^2 : $V_{1i}^2 = V_{1f}^2 + V_{2f}^2 + 2V_{1f}V_{2f} = V_{1f}^2 + V_{2f}^2$

This means $V_{1f}V_{2f} = 0$, so either $V_{1f} = 0$ or $V_{2f} = 0$

Two possible solutions: $V_{1f} = 0 \Rightarrow$ ① $\bar{V}_{2f} = \bar{V}_{1i}$ (a)

or $V_{2f} = 0 \Rightarrow$ $\bar{V}_{1f} = \bar{V}_{1i}$ (b)

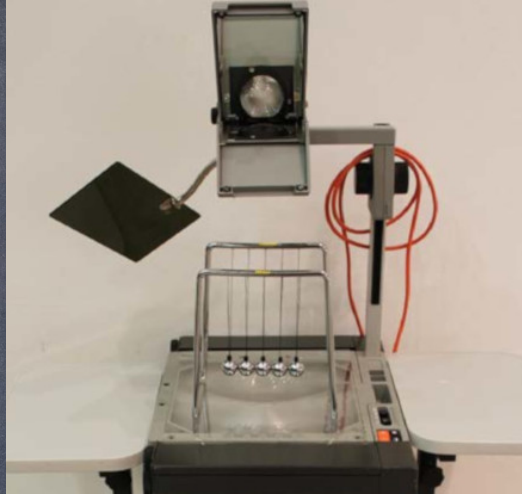
Since car 1 cannot travel through car 2 without car 2 moving, only solution (a) is physical.

So we predict solution (a):

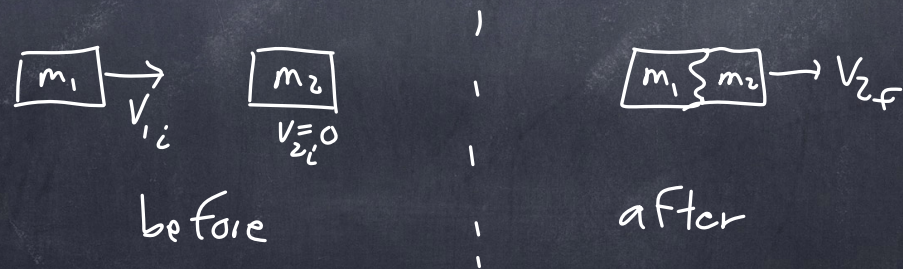
car 1 stops

car 2 continues with V_{1i}

Elastic collisions:
momentum + Kinetic energy both conserved



Inelastic collisions:
momentum conserved, kinetic energy not conserved.



Relation of momentum + force:

$$\vec{p} = m\vec{v}$$

$$\frac{d\vec{p}}{dt} = m \frac{d\vec{v}}{dt} + \vec{v} \frac{dm}{dt}$$

If m doesn't change, this is

$$\frac{d\vec{p}}{dt} = m \frac{d\vec{v}}{dt} = m\vec{a} = \vec{F}_{\text{net}}$$

$$\text{so } \frac{d\vec{p}}{dt} = \vec{F}_{\text{net}}$$

$\Rightarrow$

A net force will change an object's momentum.

Change in momentum:

$$\text{so } \vec{F} = \frac{d\vec{p}}{dt}$$

$\Rightarrow$

$$d\vec{p} = \vec{F}_{\text{net}} dt \Rightarrow \Delta p = \int_{t_1}^{t_2} \vec{F}_{\text{net}} dt$$

where $\Delta p = \text{change in momentum}$
 $= \vec{p}_2 - \vec{p}_1$

If $\vec{F}_{\text{net}} = 0$, then $\Delta p = 0 \Rightarrow$ momentum is conserved in a system if no external force.

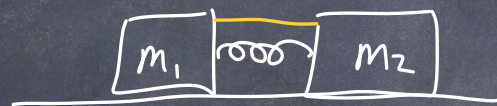
general
rule:

$$f(x) = g(x) h(x)$$

$$f'(x) = g(x) h'(x) + g'(x) h(x)$$

momentum conservation examples:

①



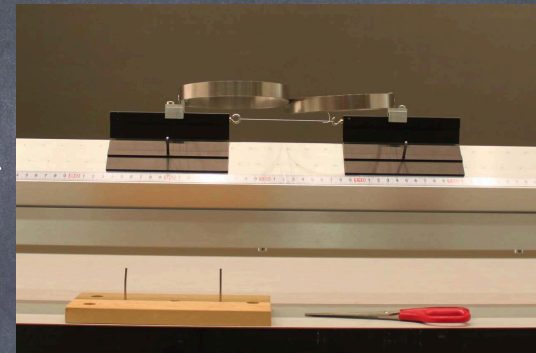
initial state:
 $\sum \vec{p}_i = 0$

final momentum = 0

$$0 = m_1 V_1 + m_2 V_2$$

$$m_1 V_1 = -m_2 V_2$$

$$\text{If } m_1 = m_2 \Rightarrow V_1 = -V_2$$



②



$m_2 > m_1$

$$p_i = 0 = p_f$$

$$0 = m_1 V_1 + m_2 V_2$$

$$V_1 = -V_2 \left(\frac{m_2}{m_1} \right)$$

If $m_2 > m_1$, then
 $|V_1| > |V_2|$

All real objects deform during a collision.
 The question is whether the energy stored during that deformation comes back as kinetic energy.



Here, the object is elastic: (deforms but returns to original shape)



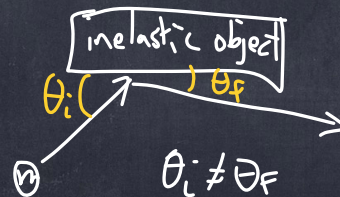
$$\theta_i = \theta_f$$

$$x: \overset{\text{initial}}{p_{ix}} = \overset{\text{final}}{p_{fx}}$$

$$y: p_{iy} = -p_{fy}$$

$$|\vec{p}_i| = |\vec{p}_f|$$

Here, the object deforms permanently:



$$\theta_i \neq \theta_f$$

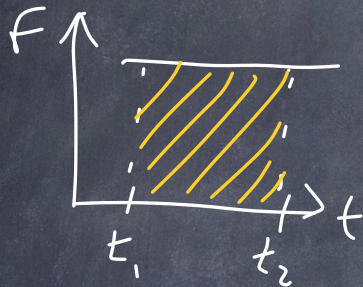
$$|\vec{p}_i| \neq |\vec{p}_f|$$

Let's look at forces changing with time.

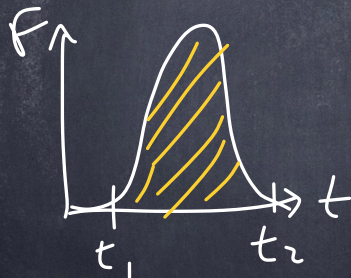
$$\vec{F} = \frac{d\vec{p}}{dt}, \quad d\vec{p} = \vec{F} dt$$

If the force is constant, then $\Delta p = \overset{\substack{\uparrow \\ \text{constant} \\ \text{force}}}{F} \overset{\substack{\uparrow \\ \text{amount} \\ \text{of time} \\ \text{of force}}}{\Delta t}$

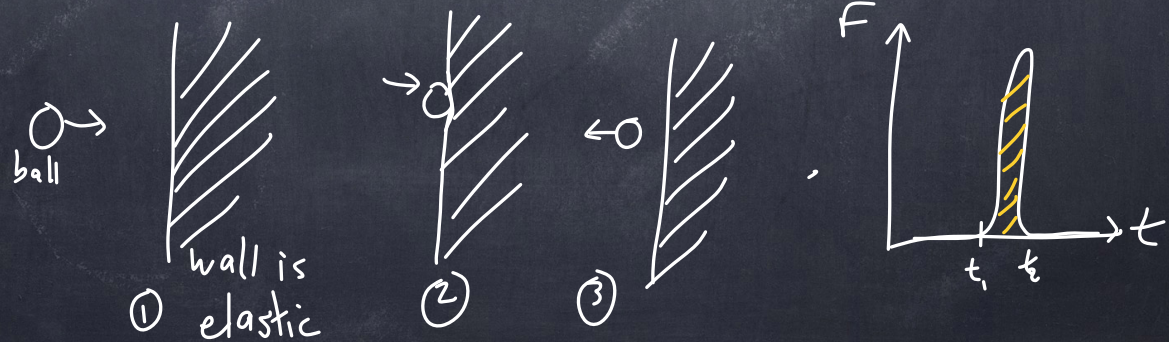
change in momentum is
the area of F vs. t from t_1 to t_2

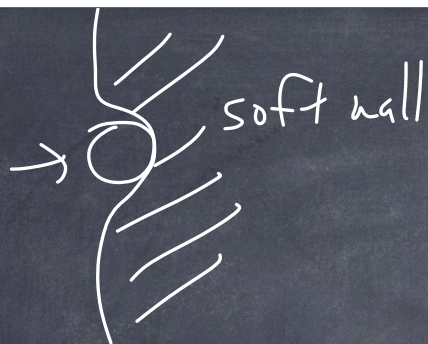


If the force is not constant, then



then $dp = \int_{t_1}^{t_2} F(t) dt =$ area under the curve of F vs. t from t_1 to t_2





In this case, Δt is longer,
so the contact is longer



$$\Delta p = \int F(t) dt = \text{Impulse} = I$$

we can approximate the average force:



Time average of force = F_{AV}
is equivalent to a constant
force over time

$$(F_{AV}) \Delta t = \int_{t_1}^{t_2} F(t) dt$$

Note: $\vec{p}$ is momentum

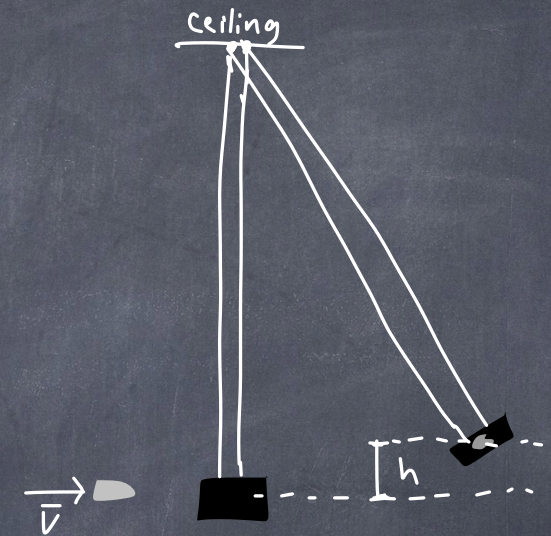
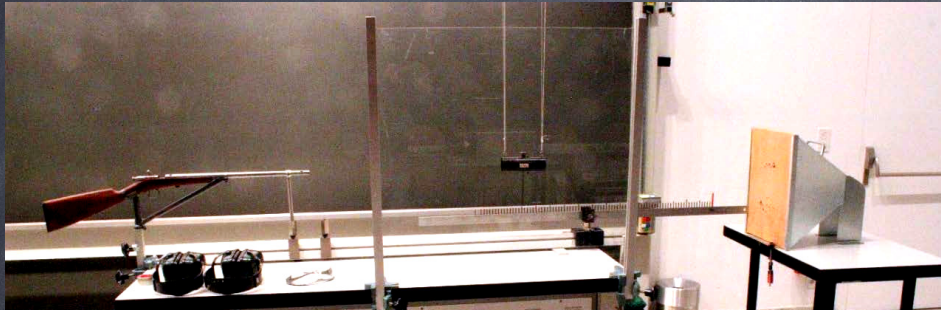
$$\Delta \vec{p} = \vec{p}_f - \vec{p}_i = \text{"Impulse"} = \vec{I} = \text{the change in momentum.}$$

Can be confusing in german.

we can minimize the average force in a collision if Δt is long or if the objects deform or break (inelastic collision).

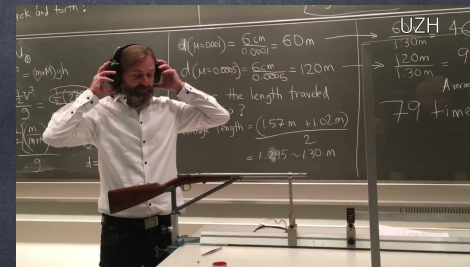


Ballistic pendulum experiment



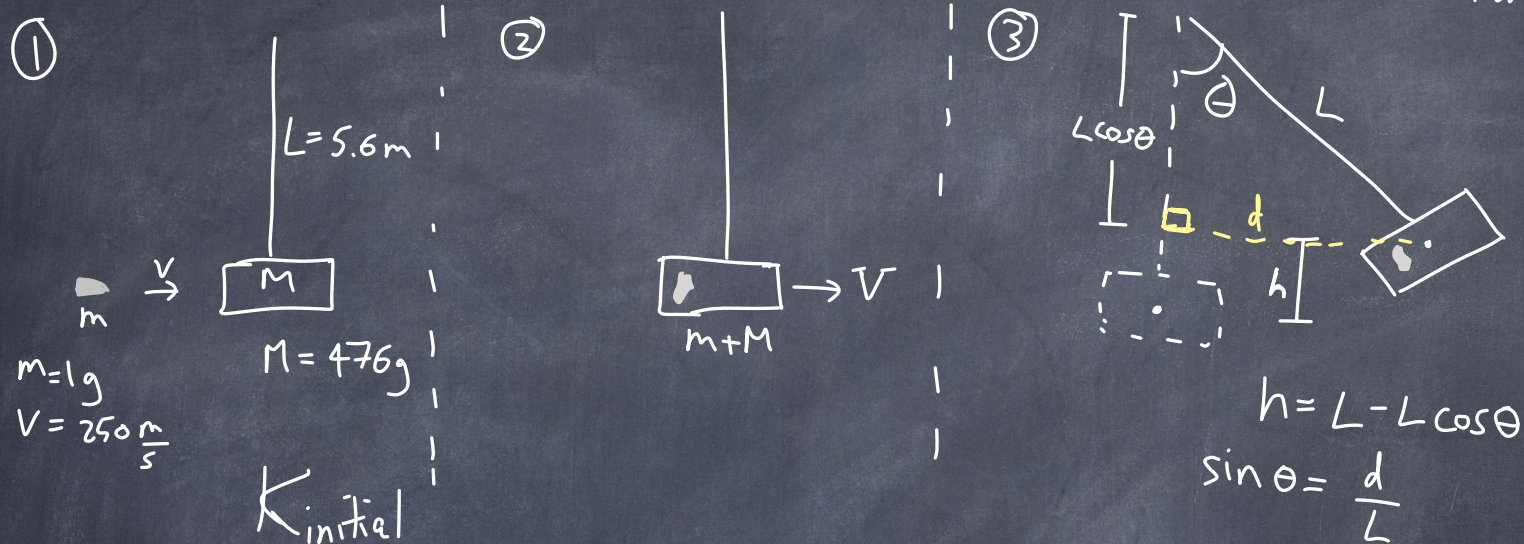
How high will the block go?

video posted



<https://www.youtube.com/watch?v=v2s6Nxc6Dr8>

Ballistic Pendulum - Momentum conservation in inelastic collision.



Between ① + ②, momentum is conserved. Inelastic collision
 Kinetic energy is not conserved.
 $mv = (m+M)V$ V : velocity of block + bullet.

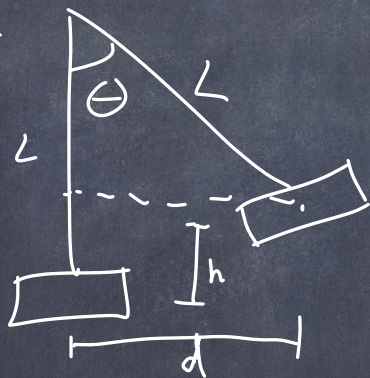
Between ② + ③, this is elastic.
 $K_{\text{②}} = U_{\text{③}}$

$$\underbrace{\frac{1}{2}(\cancel{m+M})V^2}_{K_2} = \underbrace{(\cancel{m+M})gh}_{U_3}$$

solve for $h = \frac{\frac{1}{2}V^2}{g} = \frac{\frac{1}{2}\left[\frac{m}{m+M}V\right]^2}{g} =$

predicted h
0.014 m

$L = 5.6 \text{ m}$



we measure d .

$$h = L - L \cos \theta$$

$$\sin \theta = \frac{d}{L} \Rightarrow \theta = \sin^{-1}\left(\frac{d}{L}\right)$$

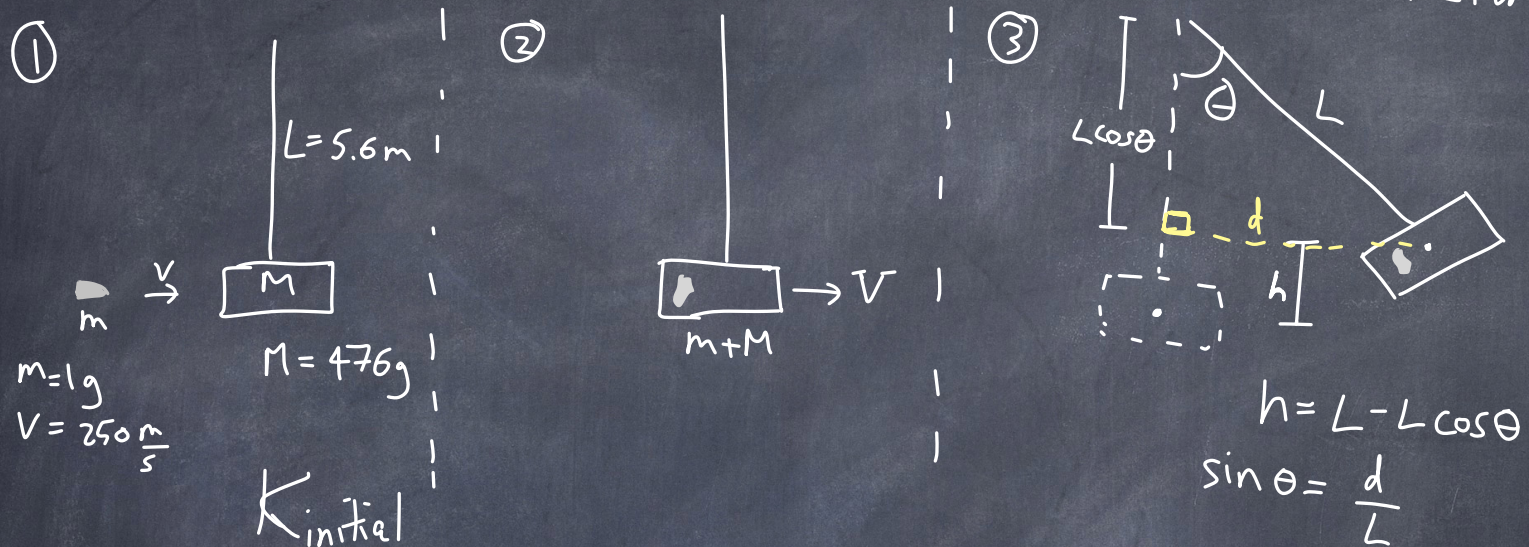
we can measure d , figure out θ , then figure out h .

measure $d = 0.4 \text{ m}$, $\theta = \sin^{-1}\left(\frac{0.4 \text{ m}}{5.6 \text{ m}}\right) = 0.07 \text{ radians}$

and calculate $h = L - L \cos(0.07 \text{ rad}) = 5.6 \text{ m} - 5.585$

measured $h = 0.014 \text{ m}$

Ballistic Pendulum - Momentum conservation in inelastic collision.



Note:

If kinetic energy was conserved, then

$$K_1 = U_3$$

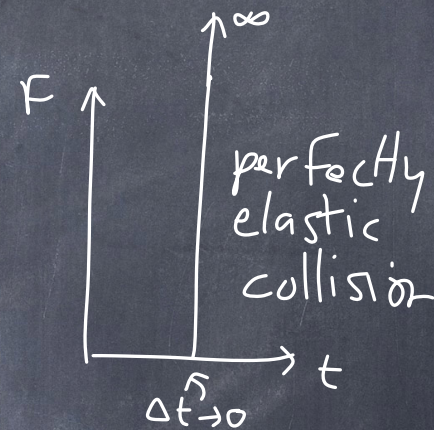
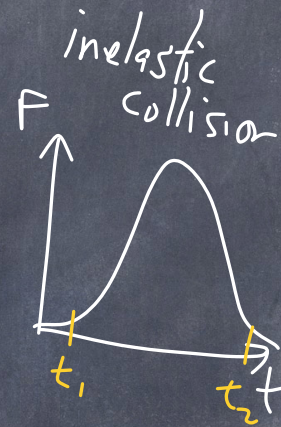
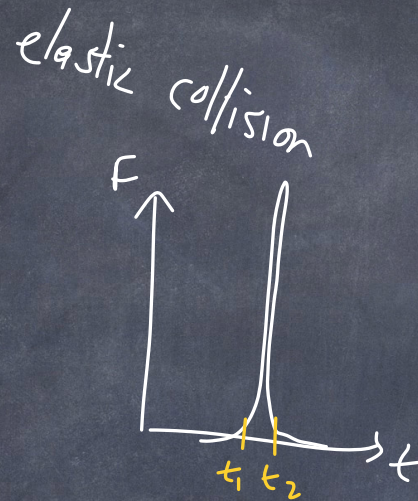
$$\frac{1}{2}mv^2 = mgh \rightarrow h = \frac{v^2}{2g}$$

$h = \underline{\underline{6.7m}}$ ← wrong!
(collision is inelastic)

Questions from students:

- 1) what is the difference between elastic & inelastic collisions in terms of f vs. t ?
- 2) If momentum changes directions, wouldn't the average force always be the same if the momentum change was the same?
- 3) How can something change directions?
Isn't momentum not conserved if this happens?
- 4) What if the problem is 2-dimensional?
How is momentum conserved?

1) what is the difference between elastic & inelastic collisions in terms of F vs. t ?

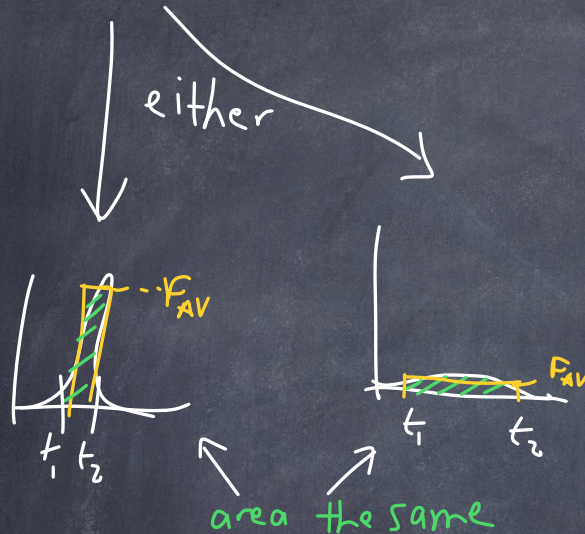


for perfectly elastic collisions,
 $\Delta t \rightarrow 0$, $F \rightarrow \infty$
 (these don't happen in the real world.)
 All interactions have some Δt

$$\Delta p = I = F_{AV} \Delta t$$

2) If momentum changes directions, isn't F_{AV} always the same?

The average force depends on Δt .

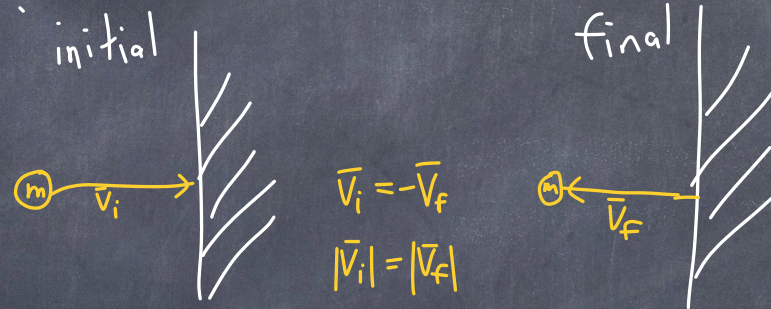


Notice, we can have the same $\Delta p = p_2 - p_1$, in cases where Δt is small and F_{AV} is large, or when Δt is large and F_{AV} is small.

In both cases Δp is the same
(area is the same)

How can something change directions?
Isn't momentum not conserved if this happens?

Consider this case:



The momentum changes from mv_i to mv_f

$$\bar{p}_f = -\bar{p}_i \quad \text{①}$$

What happened?

$$\Delta \bar{p} = \bar{p}_2 - \bar{p}_1 = p_f - p_i = -\bar{p}_i - \bar{p}_i = -2\bar{p}_i$$

The momentum changed.

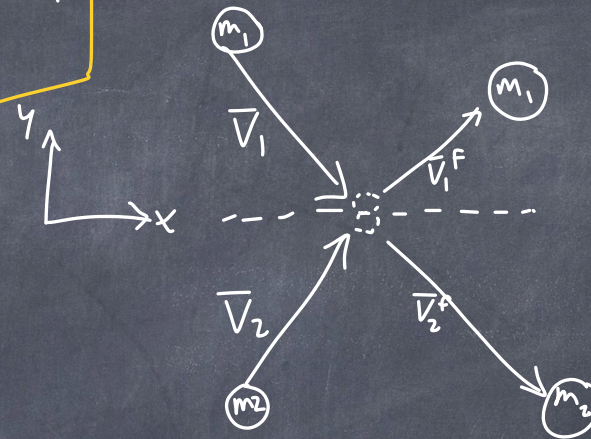
How? The wall provides a force F for a time Δt

$$\Delta p = F_{AV} \Delta t \Rightarrow \bar{F}_{AV} = \frac{-2\bar{p}_i}{\Delta t}$$

If we know Δt , we can calculate F_{AV}

④ What if the problem is 2-dimensional?
How is momentum conserved?

consider this inelastic collision



Momentum will be conserved in all directions:

Therefore, $\sum \vec{p}_i = \sum \vec{p}_f$

$$\sum p_{xi} = \sum p_{xf}$$

$$\sum p_{yi} = \sum p_{yf}$$

we have these equations.

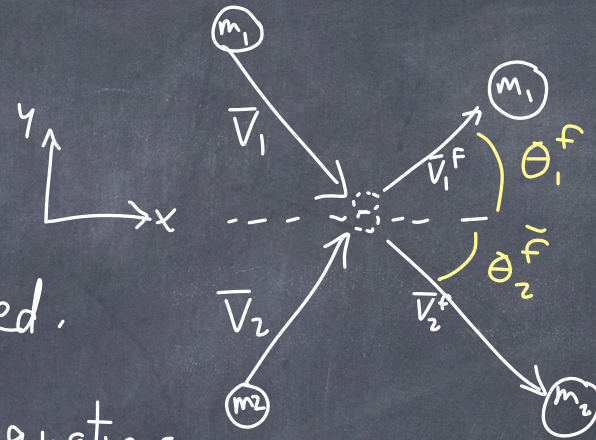
$$\begin{cases} m_1 v_{1x} + m_2 v_{2x} = m_1 v_{1x}^f + m_2 v_{2x}^f \\ m_1 v_{1y} + m_2 v_{2y} = m_1 v_{1y}^f + m_2 v_{2y}^f \end{cases}$$

If we know the masses and initial velocities,
we would have 4 unknowns and 2 equations.
So we need more information to solve this.

④ Continued...

If the collision in 2D were also elastic, then K would be conserved.

Then we would have 3 equations and 4 unknowns:



Kinetic energy conservation: $\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_1v_1^f{}^2 + \frac{1}{2}m_2v_2^f{}^2$

Momentum conservation in x : $m_1v_{1x} + m_2v_{2x} = m_1v_{1x}^f + m_2v_{2x}^f$

Momentum conservation in y : $m_1v_{1y} + m_2v_{2y} = m_1v_{1y}^f + m_2v_{2y}^f$

Knowing one of the angles, θ_1^f or θ_2^f , or one final velocity component ($v_{1x}^f, v_{2x}^f, v_{1y}^f, v_{2y}^f$) would be enough information to solve the problem.