Data Analysis - 2020

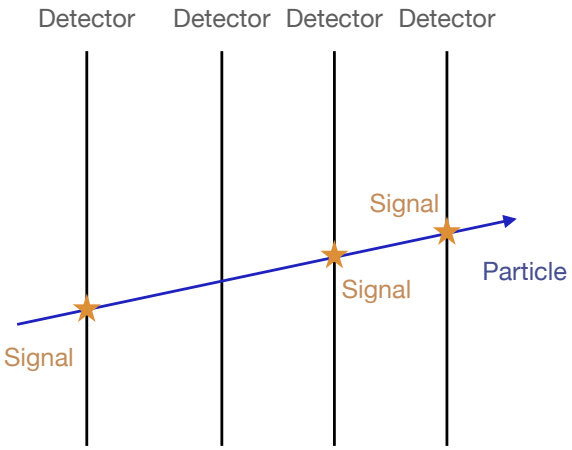
Exercise sheet no 2: 29. September 2019

Useful commands:

- `help(scipy.stats)`

Exercise 1: Particle detection efficiency (6 Points)

You are designing an experiment to detect particles as shown in the figure. The probability to have a signal in each detector is 85%. Your goal is to decide how many detectors to build.



(a) Write a python script to plot the probability distribution for the number of signals if the particle travels through four detectors.

(b) In order to detect particles, you must have signals in at least three detectors. How many detectors need to be built to ensure that the particle detection efficiency is above 99%?

(c) When you run your experiment, you detect 987 particles from a total of 1000 produced. Choose the relevant probability distribution and use it to calculate the statistical variance for the number of particles detected.

(d) Using this variance, calculate the value and the standard deviation (uncertainty) of the efficiency.

Exercise 2: Uniform distribution (4 Points)

The *uniform probability distribution* $p(x|a,b)$ is constant in the interval $a < x < b$ and zero elsewhere.

(a) What is the value of the PDF in the interval $a < x < b$? Why? (1 point)

(b) Calculate the mean of the distribution. (1 point)

(c) Calculate the variance of the distribution. (1 point)

(d) Calculate the cumulative distribution function for $x < a$, $a < x < b$ and $x > b$. (1 point)

Exercise 3: PDFs and CDFs (2 Points)

To build a fence around your house you order 100 wood boards from a local carpenter. Being a fellow physicist, the carpenter reports that the heights of the wood boards are well described by a gaussian distribution with $\mu = 1\text{m}$ and $\sigma = 1\text{cm}$. Using the properties of the Gaussian distribution, answer the following:

(a) What is the probability to observe a wood board with a height within $[0.97, 1.03]\,\text{m}$? (0.5 points)

(b) What is the probability to observe a wood board with a height within $[0.99, 1.00]\,\text{m}$? (0.5 points)

(c) What is the probability to observe a wood board with a height within $[0.95, 1.05]\,\text{m}$? (0.5 points)

(d) What is the probability to observe a wood board with a height less than $1.015\,\text{m}$? (0.5 points)

Exercise 4: Approximation of the binomial distribution (8 Points)

The Z-boson decays with a probability of 82% into charged particles and with about 18% probability into neutrinos, which cannot be detected in regular particle detectors. In some experiment, 500 Z-bosons were produced during a running time of 125 hours.

(a) Write a Python script that uses a binomial distribution to calculate the probability for 390 or more Z-bosons to be detected using charged particles. (2 points)

(b) Knowing the expected value and its standard deviation, use a gaussian approximation of the binomial distribution. Write a Python script that determines the same as in a). Plot both the original distribution and the approximation together. How good is this approximation? (2 points)

(c) Now make a Poisson approximation. Write a Python script to determine the same as in a). Plot both the original distribution and its approximation together. How good is this approximation? (2 points)

(d) Write a Python script to determine the probability that at least one Z-boson was created, but could not be observed because it decayed to neutrinos, during the first hour of running this experiment. You may assume that the rate of Z-bosons being produced is constant.
Use both the binomial distribution and its Poisson approximation to determine this and plot both distributions together. How good is the approximation this time? Why is it different to c)? (2 points)

Deadline for submission: Friday, 9th October 2019 14:00
Form: Please submit your solutions to da@physik.uzh.ch. The solution should be a single python script with the answers (and justification) to each question as print outs and plots saved when running the script.