Übungen zur **Physik der weichen Materie** , **Serie 3**, FS 2021

Besprechung am 20.04.21

Aufgaben

1. **Diffusion and error propagation**

   In developmental biology, the building of spatial gradients using "morphogens" is of great importance in the determination of spatial information and patterning within developing organisms. In this, the concentration of such a morphogen determines determines whether or not certain other genes are transcribed or not using some threshold level. In order for such a process to be developmentally robust, this concentration gradient needs to be determined with good accuracy. As we will see in later chapters, the concentration dependence of the maternal morphogen Bicoid in an early Drosophila embryo follows an exponential distribution given by $c_{Bcd}(x) = c_0 e^{-x/\lambda_{Bcd}}$, where the different parameters are determined by the diffusion and breakdown properties of the protein. The figure does however also show that there is a certain degree of variation in this concentration gradient.

   a) In the above expression, x is the distance along the anterior-posterior axis of the embryo, $\lambda_{Bcd} = 120\mu$m is the distance after which the concentration has dropped to $1/e$ (roughly equal to 0.37) of the initial concentration c_0. From data like the ones shown in the figure, the uncertainty of λ and c_0 can be determined, yielding $\sigma_\lambda = 20\mu$m and $\sigma_{c_0}/c_0 = 0.05$. How do these uncertainties correspond to an accuracy in positioning in the embryo? Determine the uncertainty of the concentration for a position at $x = 200\mu$m, both with absolute and relative error.

   b) Now let us consider the function of the morphogen. Which position corresponds to a threshold concentration c_T, initiating the transcription of a protein. For this purpose, solve $c_{Bcd}(x_T) = c_T$ for x_T and determine the uncertainty in x_T? For a numerical result, assume that $c_T = 0.2c_0$.

   c) In b), we have assumed the threshold concentration c_T to be exactly given. However, there is also a level of uncertainty involved in this. Do the same as in b), but taking into account the uncertainty in c_T, i.e. σ_{c_T}.

   d) If we are normalizing concentrations to the initial value c_0, we eliminate the uncertainty of c_0, but increase the uncertainty in c_T. Again determine the uncertainty in x_T if c_T is a given fraction of c_0 and compare this with the result from b).

2. **Elastic properties of DNA**

   Consider the force extension curve of a single DNA molecule shown in the text. A DNA molecule, 16 μm long has been stretched in this case.

   (a) Determine the Young's modulus of DNA. For this purpose, assume that DNA is a cylinder with a diameter of 2.0(1) nm and concentrate on the part of the curve, where the molecule is stretched out.

   (b) At the beginning of the curve, the slope is due to a disentanglement of the curled up molecule. For such an entropic spring, we had determined a spring constant of $\frac{3 \cdot 6 \cdot k_B T}{L \xi_P}$ in the text, where ξ_P is the persistence length, T the temperature (in K) and k_B the Boltzmann constant. Assuming the experiment was carried out at room temperature (i.e. 20°C), determine the persistence length of DNA from the slope of the curve.

   (c) For a cylindrical molecule, we have seen that the persistence length is given by $\xi_P = \pi E R^4/(16 k_B T)$, where R is the radius and E the elastic modulus of the molecule. Given the persistence length of DNA (you should have found about 50 nm in c)) determine the elastic modulus.

3. **Packing of long chained molecules**

(a) How long can a piece of DNA be to still fit inside a (spherical) cell with a radius of $R_c = 5(1)\mu$m? Assume that DNA is curled up by thermal fluctuations, leading to a random arrangement corresponding to a Gassian distribution with a radius of gyration $R_g^2 = \xi_P \cdot L/6$. THe persistence length of DNA is roughly $\xi_P = 50(5)$ nm. In order for the entire molecule to fit inside the cell, the radius of the cell has to be about twice the radius of gyration. How many base-pairs does this correspond to?

(b) What energy do you have to spend to bend DNA to a radius of curvature of $r = 5.0(5)$ nm, i.e. the size of a histone? Use that the bending energy is given by $E_{bend} = \pi E R^4 L/(8r^2)$, as well as the definition of the persistence length: $\xi_P = \pi E R^4/(4k_B T) = 50(5)$nm, where $R = 1.0(3)$ nm is the radius and $E = 2.50(3) \cdot 10^8 Pa$ the elastic modulus of DNA. All of this happens at room temperature (293 K). For a turn with radius r you need a length of $L = 2\pi \cdot r$of the molecule.

4. **Heat conduction**

Liquid lava has a temperature of 3300 K, the specific heat of rock is c = 1500 J/(kg K) and its heat conductivity is $\lambda = 1.7$ W/(K m). In a mine shaft one finds a temperature increase of roughly 3 K/100 m and the solid crust of the earth is about 25 km thick. Determine the age of the earth.
