

MECHANICS [G. Isidori, UZH, Autumn Semester 2026]

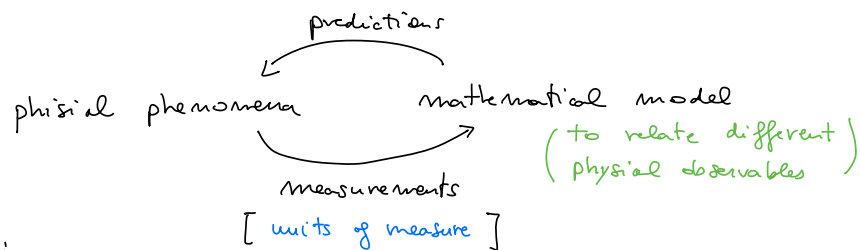
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0. Preliminaries

The aim of this course is to provide you with a first introduction to basic concepts in theoretical physics

Historically, Classical Mechanics is the first area of physics for which a proper mathematical formulation has been obtained (thanks to Galileo, Newton + many others)

As Galileo wrote in 1600, Nature is a book written in a mathematical language. To read this book we must learn this language.



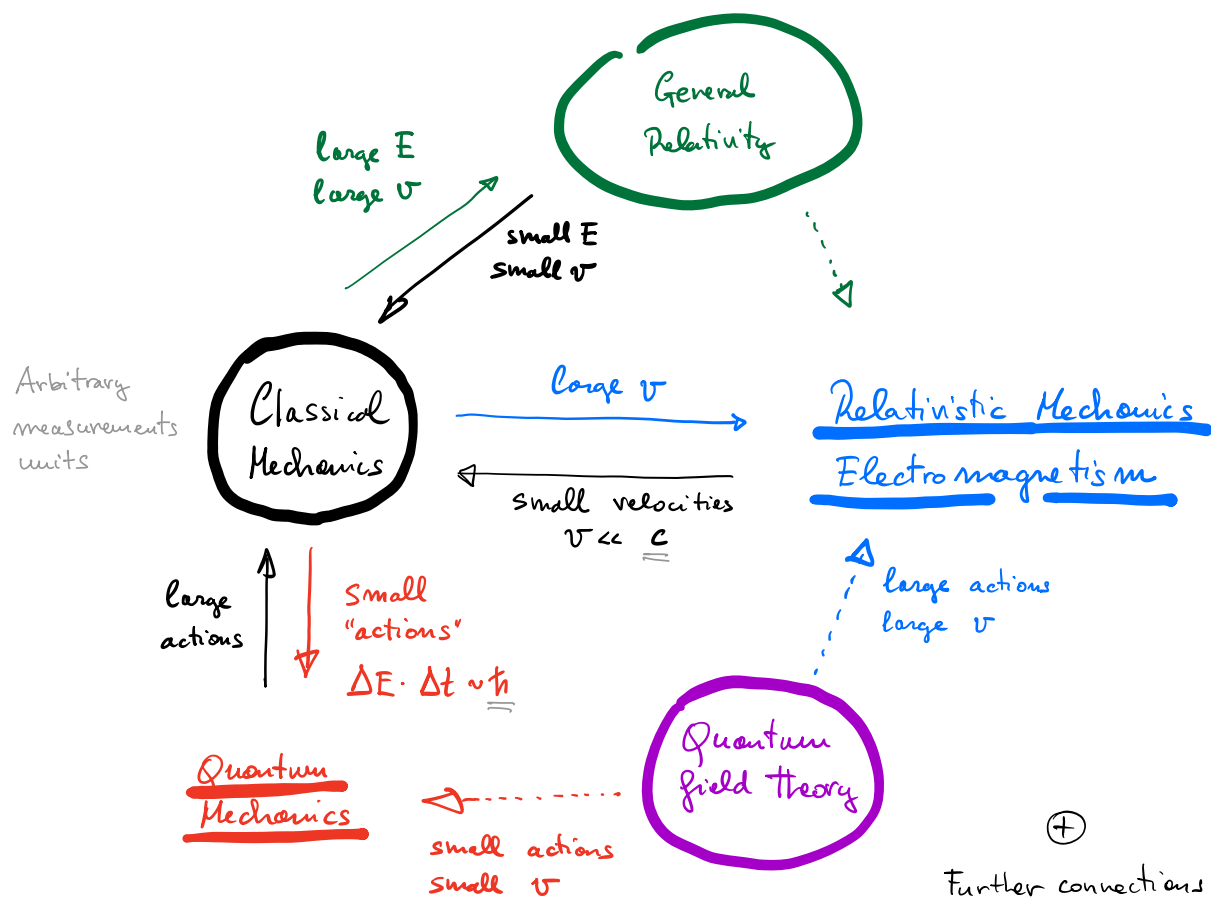
Classical Mechanics:

- Description of the motion of (often idealized) bodies in space & time
- The initial formulation (Newtonian Mechanics) which is appropriate to describe point-like objects, few bodies, in well-defined kinematical limits (e.g. $v \ll c$), has evolved into more sophisticated formulations (Lagrange & Hamilton Mech.)

These have a twofold advantage:

- * Easier to handle more complicated problems (many bodies, constraints, ...)
- * Transparent connection to more "complete" theories such as Quantum Mechanics or General relativity

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Tentative plan of the lectures :

- Newtonian Mechanics
- Lagrangian Mechanics
 - Conservation laws
 - The two-body problem
 - Small oscillations
 - Rigid Bodies
- Hamiltonian Formulation
- Relativistic Mechanics

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1. Newtonian Mechanics

1.1 Galileo's principle of relativity & the Galilean group

Basic assumption about space & time in Classical Mechanics:

Space = 3D Euclidean

Time = 1D

Galileo's principle of relativity:

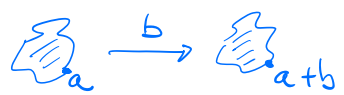
- There exist coordinate systems, called inertial, possessing the following two properties:
- I. All the laws of Nature, at any given time, are the same in all inertial systems
 - II. All coordinate systems in uniform rectilinear motion with respect to an inertial system are inertial

This principle defines what is technically known as the "affine" space A^4 :

→ no preferred origin, neither in time, nor in space

→ the group $\mathbb{R}^4$ acts on A^4 as the group of parallel displacements

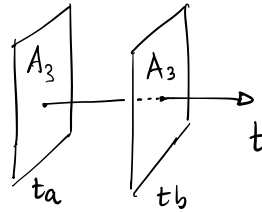
$$\left\{ \begin{array}{l} \text{given an event } a \equiv (\vec{x}, t) \quad a \in A^4 \\ \text{and given } b \in \mathbb{R}^4 \text{ then } a + b \in A^4 \end{array} \right.$$



In A^4 we can define two type of "distances" (or intervals)

- $\Delta t = t_a - t_b$ time separation between two events
(independent of their location in space)
- $|\Delta \vec{x}| = \sqrt{|\vec{x}_a - \vec{x}_b|^2}$ space separation between simultaneous events

The structure of A^4 is of the type:



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N.B: We cannot unambiguously

define a space distance between non-simultaneous events
(it is a quantity that change if we change reference system)

the parallel displacement is a sub-group of the more general

Galilean group = group of coord. transformations that preserve
the structure of A^4 = " " " " under which physical
laws must be invariant:

$$\left\{ \begin{array}{ll} (3) \text{ Space translations} & \vec{x}' = \vec{x} + \vec{x}_0 \\ (3) \text{ Space Rotations} & \vec{x}' = O \cdot \vec{x} \\ (3) \text{ Boosts} & \vec{x}' = \vec{x} + \vec{v}_0 \cdot t \\ (1) \text{ Time translations} & t' = t + t_0 \end{array} \right. \quad \begin{array}{l} 3 \text{ possible directions for } \vec{x}_0 \neq \vec{v}_0 \\ O = 3 \times 3 \text{ orthogonal matrix (3 angles)} \end{array}$$

As we will see in the next lectures, the requirement of invariance of physical laws under these transformations is the reason behind a series of conservation laws

In the next lecture we will discuss Newton's laws to describe the motion of point-like particles (= idealized object fully characterized by their position in space). This motion is described by trajectories, $\vec{x}(t)$, which are paths in $\mathbb{R}^3$

$$\vec{x}: \mathbb{R} \longrightarrow \mathbb{R}^3 \\ (t)$$

(5)

1.2 Newton's first & second principle

Let's define

① velocity $\vec{v} = \frac{d\vec{x}}{dt} \equiv \dot{\vec{x}}$ ② momentum $\vec{p} = m \cdot \vec{v}$

where m is the mass of the particle.

The 1st principle of Newton's Mechanics (or Newton's 1st Law) states that in inertial systems $\vec{p} = \text{const.}$ ("particles at rest stay at rest, particles in motion remain in motion"), unless there is a force acting on the particle.

Newton's 2nd Law is the statement that in inertial systems

$$\dot{\vec{p}} = \vec{F}(\vec{x}, \dot{\vec{x}}) \quad (1.1)$$

that in the limit where $m \neq 0$, implies the familiar

$$\vec{F} = m\vec{a} \quad (\vec{a} \equiv \ddot{\vec{x}})$$

Note that this equation defines the force, for which already Newton realized he could only give a "circular" definition: the force is what causes $\vec{p}$ to change with time.

What is somehow more important is that (1.1) implies

$$\ddot{\vec{x}} = \vec{f}(\vec{x}, \dot{\vec{x}}) \quad (1.2)$$

which can be read as the declaration that the evolution of the trajectories is completely determined by initial position & velocity.

By theorems on differential eq's, we know indeed that the solution $\vec{x}(t)$ to (1.2) is completely determined by $\vec{x}(t_0)$ & $\dot{\vec{x}}(t_0)$.

Remark: The above declaration would not prevent $\vec{f}$ to be an explicit function of t , but this would contradict Galileo's principle of relativity, which implies that if $\vec{x}(t)$ is a solution, then also $\vec{x}(t+s)$ for any $s \in \mathbb{R}$ is a solution [Discuss closed vs. open systems]

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(1.3)

$$\vec{r} = \vec{x} - \vec{x}_0$$

(distance with respect
to the fixed point)

and correspondingly

① torque (moment of force)

$(1, 4)$

From Newton's second law (since $\dot{\vec{r}} \times \vec{p} = 0$) it follows

(1.5)

From (1.1) & (1.5) we deduce the following two conservation laws (which appear trivial at this stage, but later on we will show can be derived directly from invariance under Galilean transf.):

$$\left\{ \begin{array}{ll} \text{I.} & \text{If } \vec{F} = 0 \text{ then } \vec{p} = \text{constant throughout the motion} \\ \text{II.} & \text{If } \vec{v} = 0 \text{ then } \vec{L} = \text{" " " "} \end{array} \right.$$

Note that $\vec{r} = \rho$ does not require $\vec{F} = \rho$, but only $\vec{r} \times \vec{F} = 0$, which can be obtained with $\vec{F} \neq 0$ parallel to $\vec{r}$. A force of this type is called central force [mention examples].