MMP I
Tutorial 5

HS 2019
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https://www.physik.uzh.ch/en/teaching/PHY312/HS2019.html

Issued: 15.10.2017
Due: 22.10.2017 10:15

Exercise 1: Ordinary differential equations (4 Pts.)

a) Solve the differential equation

$$y' = \frac{\cos x \cos y + \sin^2 x}{\sin x \sin y + \cos^2 y} \tag{1.1}$$

An implicit solution of the form $f(x, y(x)) = 0$ is sufficient.

b) A particle of mass m and charge q moving in a magnetic field $\vec{B}$ experiences a force

$$\vec{F} = q \cdot \vec{v} \times \vec{B}, \text{ with } \vec{v} = \dot{\vec{r}} \tag{1.2}$$

Solve the equation of the motion

$$m\ddot{\vec{r}} = q \cdot \dot{\vec{r}} \times \vec{B} \tag{1.3}$$

for a constant $\vec{B}$, by choosing a convenient coordinate system which is at rest with respect to the magnetic field.

Exercise 2: Differential equations (6 Pts.)

Find the general solution (in the form $y = y(x)$ or $f(x, y) = 0$) for the following differential equations

a) $y' = \arctan(x)$ (1 Pt.);

b) $y' = \frac{2}{x}y - \frac{\log x}{x}y^2$ (2 Pts.);

c) $xy' - 2y = x^3 e^x$ (1 Pt.).

d) $x\mathrm{d}x + y(1 + x^2 + y^2)\mathrm{d}y = 0$, (Hint: find the integrating factor) (2 Pts);

– please turn over –

Exercise 3: Systems of differential equations (4 Pts.)
Find the fundamental system of solutions for the following systems of linear differential equations:

a)

$$\begin{aligned} x' + x - y &= 0 \\ y' - 4x + y &= 0 \end{aligned} \tag{3.1}$$

b)

$$\begin{aligned} x' &= y \\ y' &= -2x + 2y \end{aligned} \tag{3.2}$$

c)

$$\begin{aligned} x' &= x - y + z \\ y' &= 2y \\ z' &= -2x + y - z \end{aligned} \tag{3.3}$$

d)

$$\begin{aligned} x' &= x + y + 2z \\ y' &= x + 2y + z \\ z' &= 2x + y + z \end{aligned} \tag{3.4}$$