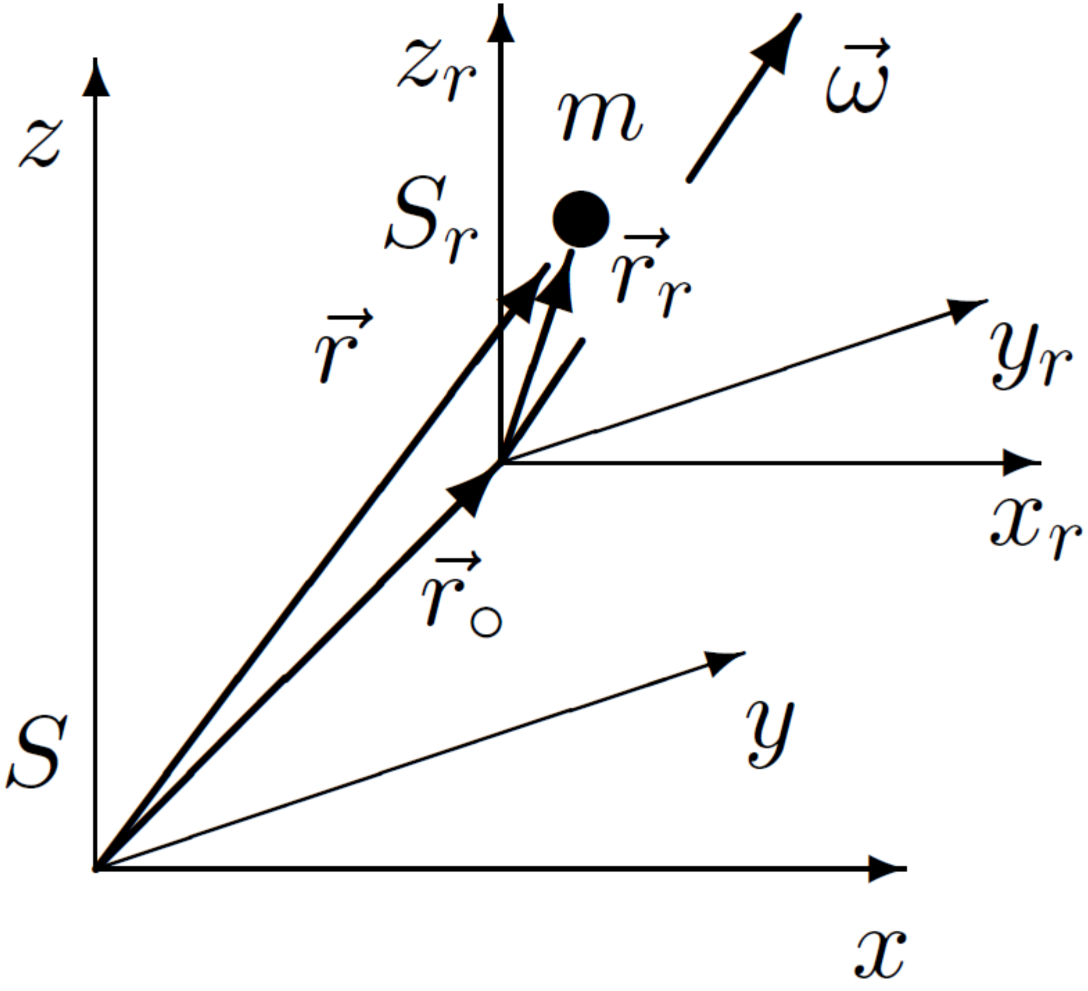
Relativbewegungen



$$\vec{r}(t) = \vec{r}_\circ + \vec{r}_r$$

$$\vec{v} = \frac{d\vec{r}}{dt}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$$

$$\vec{v}_r = \frac{d_r\vec{r}_r}{dt_r} = \frac{d_r\vec{r}_r}{dt}$$

$$\vec{a}_r = \frac{d_r\vec{v}_r}{dt} = \frac{d_r^2\vec{r}_r}{dt^2}$$

$$\vec{v} = \vec{v}_F + \vec{v}_r = \vec{v}_\circ + \vec{\omega} \times \vec{r}_r + \vec{v}_r$$

$$= \underline{\vec{v}_\circ + \vec{\omega} \times \vec{r}_r + \frac{d_r \vec{r}_r}{dt}} = \frac{d}{dt}(\vec{r}_\circ + \vec{r}_r) = \underline{\vec{v}_\circ + \frac{d\vec{r}_r}{dt}}$$

oder allgemein für jeden Vektor

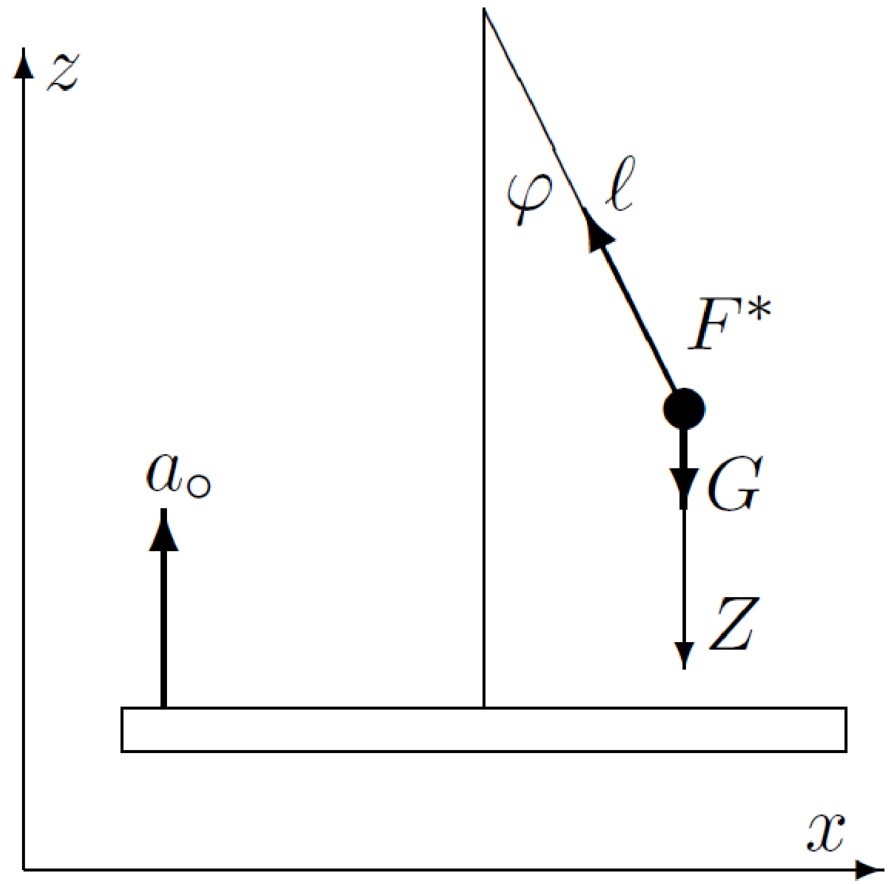
$$\frac{d\vec{A}}{dt} = \frac{d_r \vec{A}}{dt} + \vec{\omega} \times \vec{A}$$

speziell für die Beschleunigung

$$\vec{a} = \underbrace{\frac{d\vec{v}_\circ}{dt} + \frac{d\vec{\omega}}{dt} \times \vec{r}_r + \vec{\omega} \times (\vec{\omega} \times \vec{r}_r)}_{\vec{a}_F} + \underbrace{2 \cdot \vec{\omega} \times \frac{d_r \vec{r}_r}{dt}}_{\vec{a}_C} + \underbrace{\frac{d_r^2 \vec{r}_r}{dt^2}}_{\vec{a}_r}$$

fallendes Pendel

$$m\ell \frac{d_r^2\varphi}{dt^2} = -(mg + ma_\circ)\sin\varphi$$



Kleinwinkelnäherung

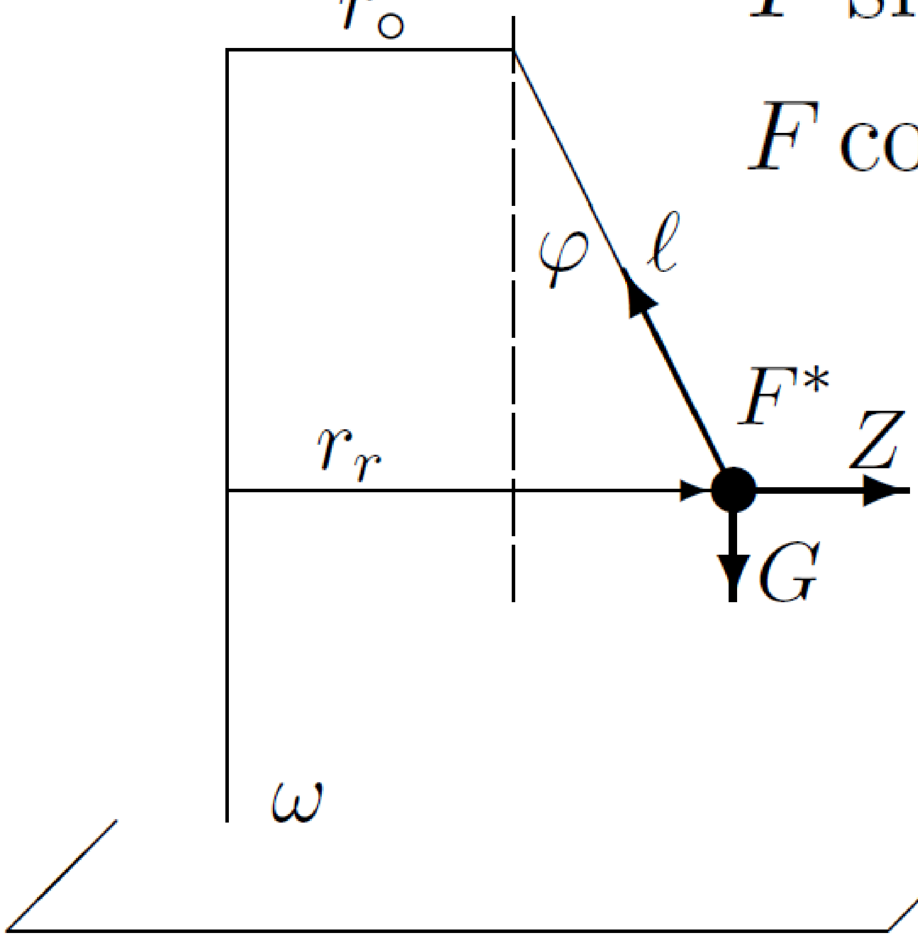
$$\frac{d_r^2\varphi}{dt^2} + \left(\frac{g + a_\circ}{\ell}\right)\varphi = 0$$

$$\varphi(t) = \varphi_\circ \cos(\Omega t - \delta)$$

$$\Omega = \sqrt{\frac{g + a_\circ}{\ell}}$$

im freien Fall keine Schwingung! -> Aequivalenzprinzip

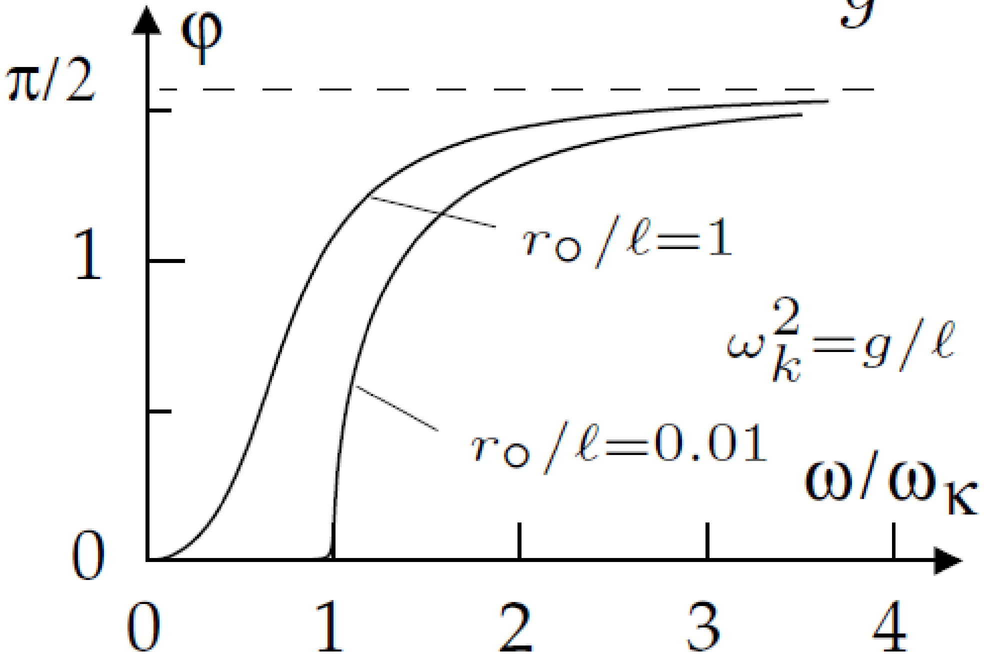
rotierendes Pendel (mit Ausleger; ohne Schwingung)



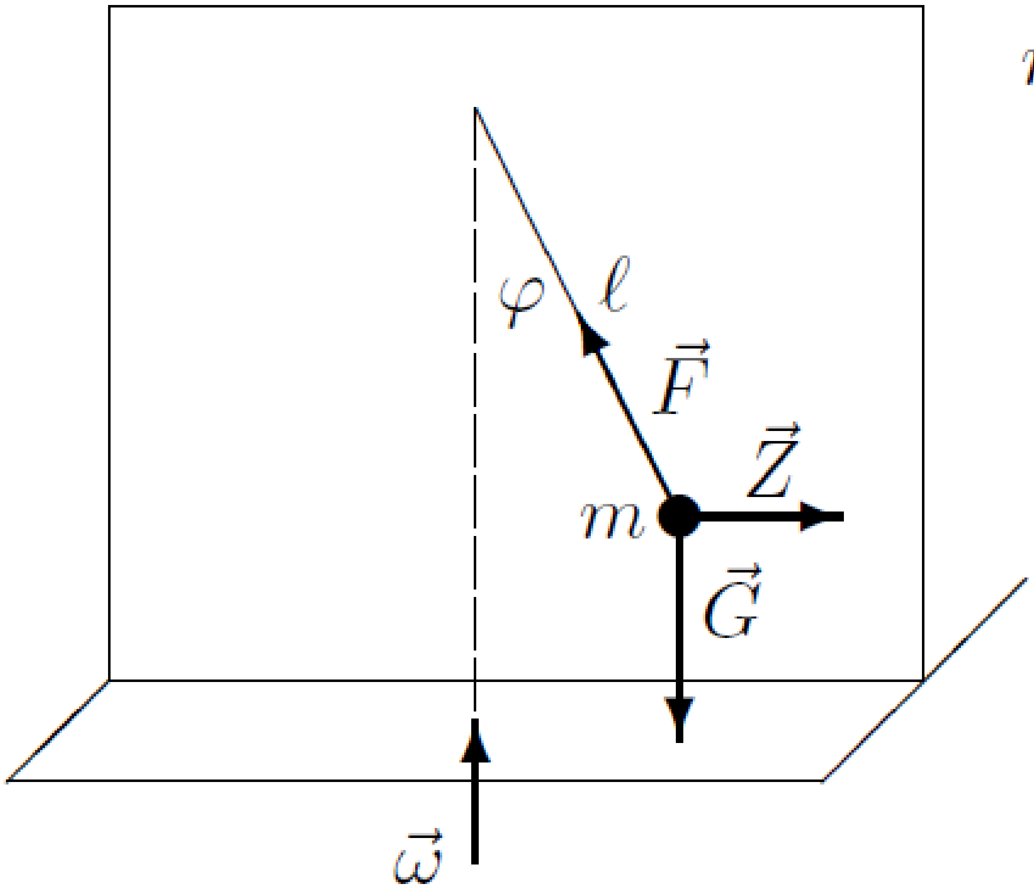
$$F \sin\varphi = m r_r \omega^2 = m(r_\circ + \ell \sin\varphi)\omega^2$$

$$F \cos\varphi = mg$$

$$\tan\varphi = \frac{(r_\circ + \ell \sin\varphi)\omega^2}{g}$$



rotierendes Pendel (ohne Ausleger; mit Schwingung)



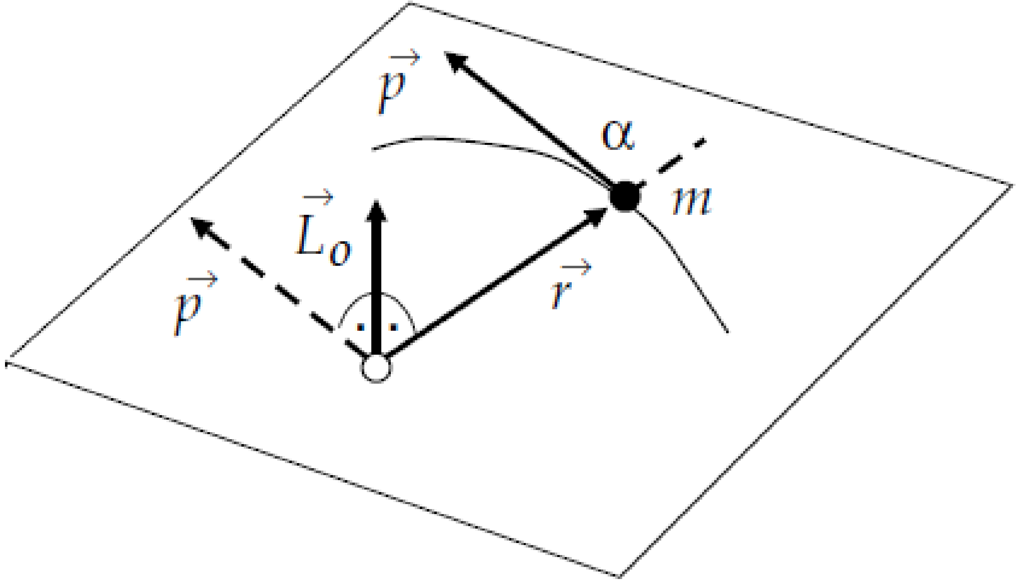
$$m\ell \frac{d_r^2 \varphi}{dt^2} = -mg\,\sin\varphi + Z\,\cos\varphi =$$
$$= -mg\sin\varphi + \ell\omega^2 m\,\sin\varphi\,\cos\varphi$$

Kleinwinkelnäherung

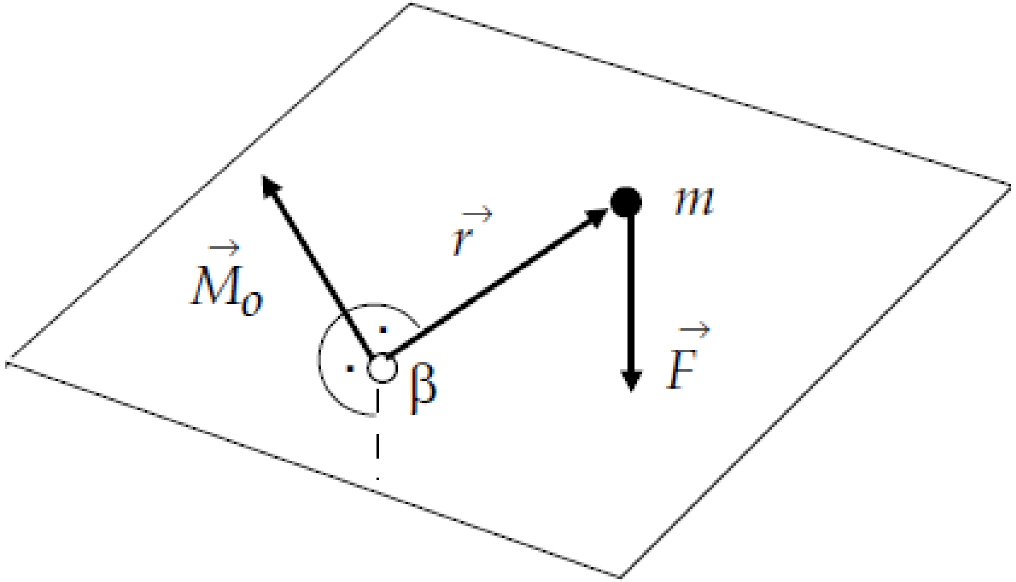
$$\frac{d_r^2 \varphi}{dt^2} + \left(\frac{g}{\ell} - \omega^2\right)\varphi = 0$$

$$\varphi(t) = \varphi_\circ \cos(\Omega t - \delta) \qquad \Omega = \sqrt{\frac{g}{\ell} - \omega^2}$$

Drehimpuls und Drehmoment

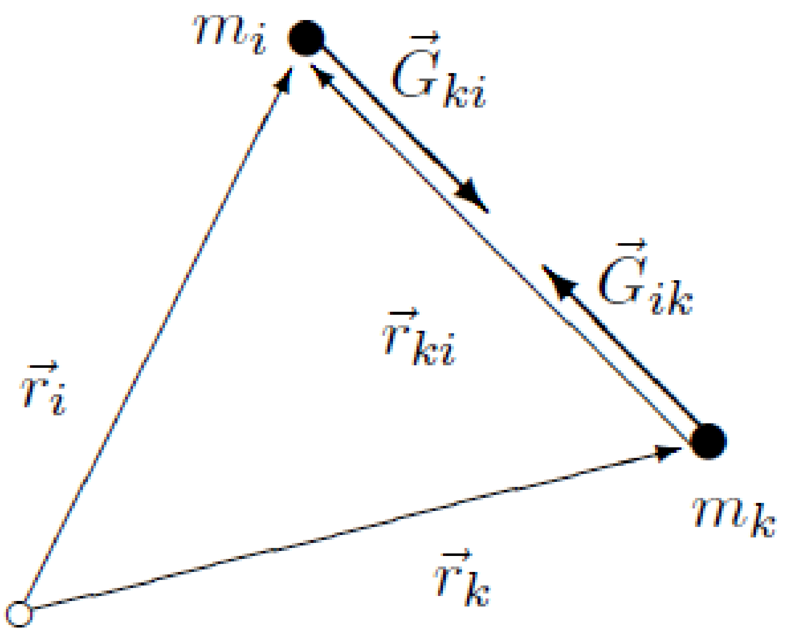


$$\vec{L}_\circ \doteq \vec{r} \times \vec{p}$$



$$\vec{M}_\circ \doteq \vec{r} \times \vec{F}$$

Der Drallsatz

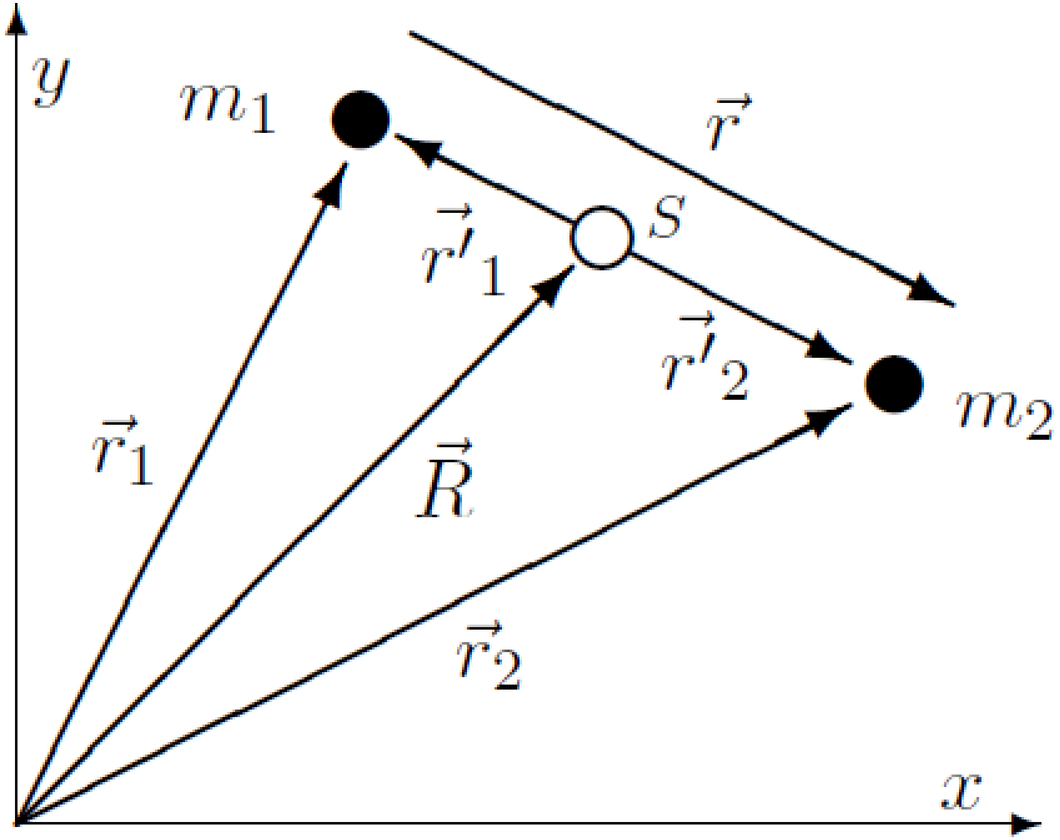


$$\sum_{i=1}^{N} \vec{r}_i \times \vec{F}_i = \sum_i \vec{M}_{\circ i}$$

$$\sum_{i=1}^{N} r_i \times \frac{d\vec{p}_i}{dt} = \sum_i \frac{d}{dt}(\vec{r}_i \times \vec{p}_i) = \frac{d}{dt} \sum_i \vec{r}_i \times \vec{p}_i = \frac{d}{dt} \sum_i L_{\circ i}$$

$$\frac{d\vec{L}_\circ}{dt} = \vec{M}_\circ$$

Zentralkräfte/Zweikörperproblem



$$\vec{R} = (m_1\vec{r}_1 + m_2\vec{r}_2)/(m_1 + m_2)$$

$$\vec{r} = \vec{r}_2 - \vec{r}_1$$

$$\vec{F}_{21} = +f(r)\vec{r} = m_1\ddot{\vec{r}}_1$$

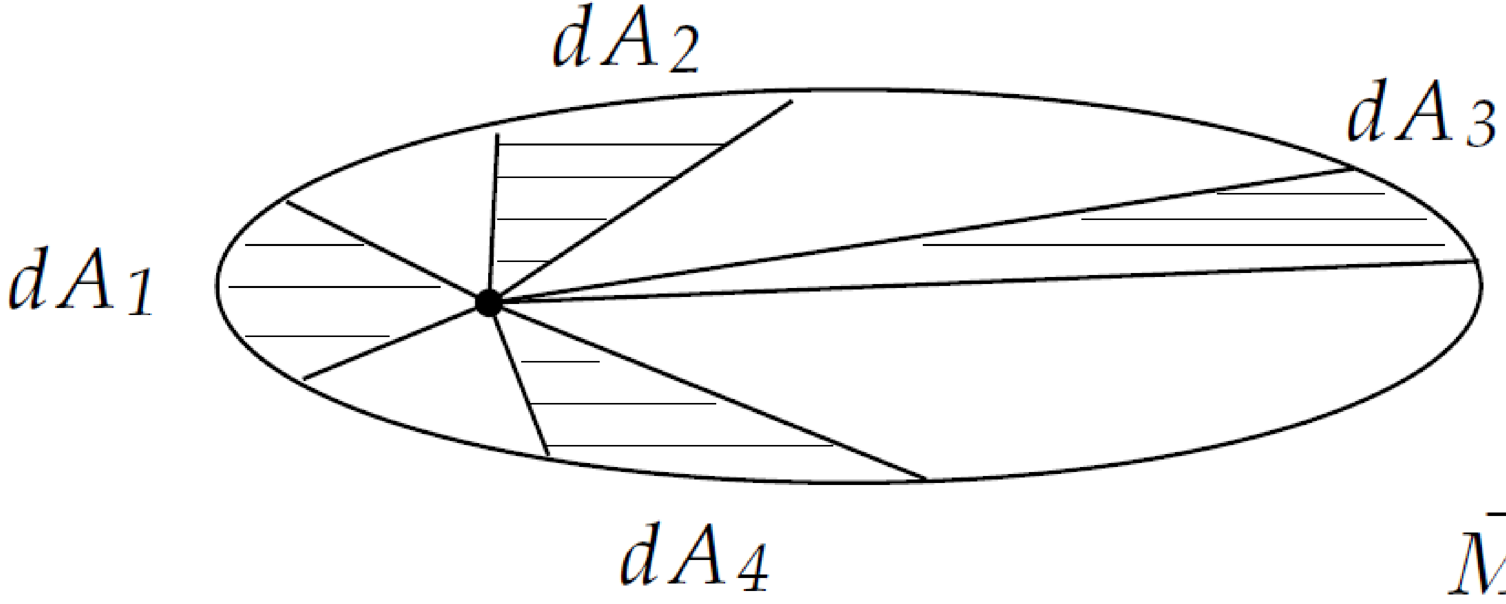
$$M\ddot{\vec{R}} = \dot{\vec{p}} = 0$$

$$\vec{F}_{12} = -f(r)\vec{r} = m_2\ddot{\vec{r}}_2$$

$$m_1 m_2 \underbrace{(\ddot{\vec{r}}_2 - \ddot{\vec{r}}_1)}_{\ddot{\vec{r}}} = m_1\vec{F}_{12} - m_2\vec{F}_{21} = \underbrace{(m_1 + m_2)}_{M}\vec{F}_{12}$$

$$\vec{F}(\vec{r}) = \mu\ddot{\vec{r}}$$

Der Kepler'sche Flächensatz



$$\mu \frac{d^2\vec{r}}{dt^2} = \vec{F}(\vec{r}) = f(r) \cdot \vec{r}.$$

$$\vec{M}_\circ = \frac{d\vec{L}_\circ}{dt} = \vec{r} \times \vec{F} = 0$$

$$|\vec{L}_\circ| = \mu |\vec{r} \times \frac{d\vec{r}}{dt}| = 2\,\mu\,\frac{dA}{dt} = \text{konst}$$

Bahnkurven für Zentralprobleme

$$\frac{\mu}{2}\left[\left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\varphi}{dt}\right)^2\right] + V = E_\circ \qquad L_\circ = \mu \mid \vec{r} \times \vec{v} \mid = \mu r^2 \frac{d\varphi}{dt}$$

$$\frac{dr}{dt} = \sqrt{\frac{2}{\mu}(E_\circ - V) - \left(\frac{L_\circ}{\mu r}\right)^2}$$

$$\frac{d\varphi}{dr} = \frac{L_\circ}{\mu r^2 \sqrt{\frac{2}{\mu}(E_\circ - V) - \left(\frac{L_\circ}{\mu r}\right)^2}}$$

Planetenbewegungen

$$V(r) = -\Gamma \frac{mM}{r}$$

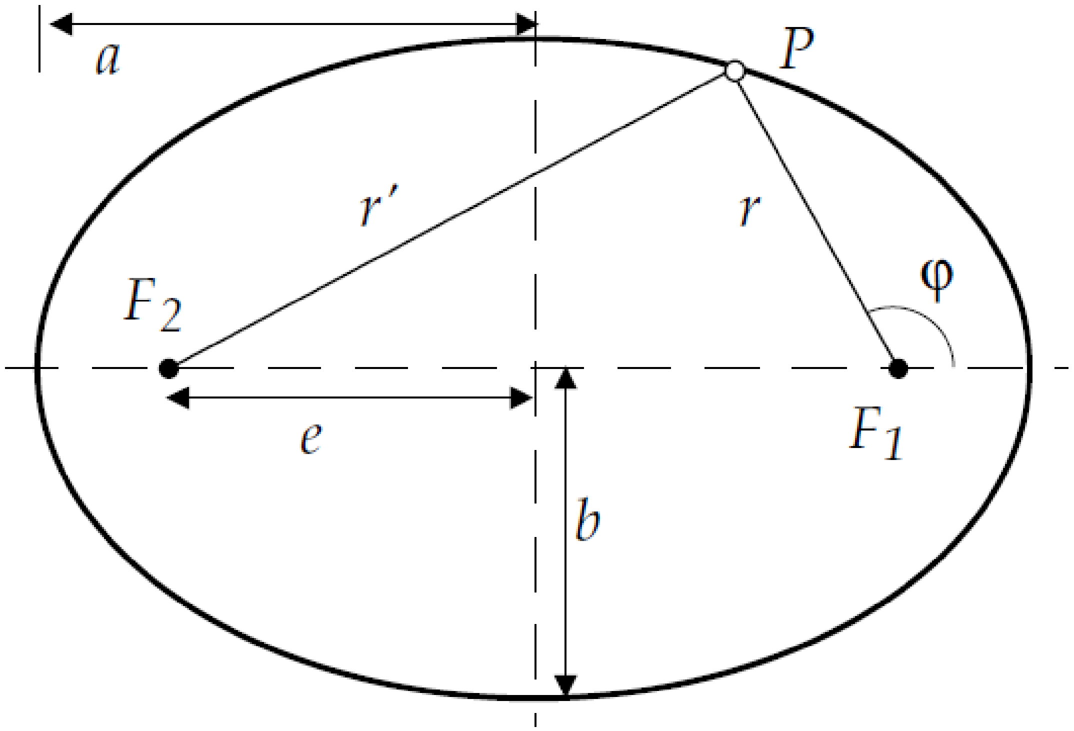
$$\frac{d\varphi}{dr} = \frac{L_\circ}{\mu r^2 \sqrt{\frac{2}{\mu}\left(E_\circ + \frac{\Gamma mM}{r}\right) - \left(\frac{L_\circ}{\mu r}\right)^2}}$$

$$u = \frac{\frac{L_\circ}{\mu} x - \frac{\Gamma mM}{L_\circ}}{\sqrt{\frac{2E_\circ}{\mu} + \left(\frac{\Gamma mM}{L_\circ}\right)^2}} \qquad 1/r = x$$

$$d\varphi = \frac{-du}{\sqrt{1-u^2}} \qquad \cos\varphi = u = \frac{\frac{L_\circ}{\mu} x - \frac{\Gamma mM}{L_\circ}}{\sqrt{\frac{2E_\circ}{\mu} + \left(\frac{\Gamma mM}{L_\circ}\right)^2}}$$

Planetenbahnen sind Elipsen

$$r = \frac{L_\circ^2}{\Gamma m M \mu} \cdot \frac{1}{1 + \cos\varphi \sqrt{1 + \frac{2E_\circ L_\circ^2}{\Gamma^2 \mu m^2 M^2}}}$$



$$r = \frac{b^2/a}{1 + e/a \cos\varphi}$$