PHY127 FS2026

Prof. Ben Kilminster

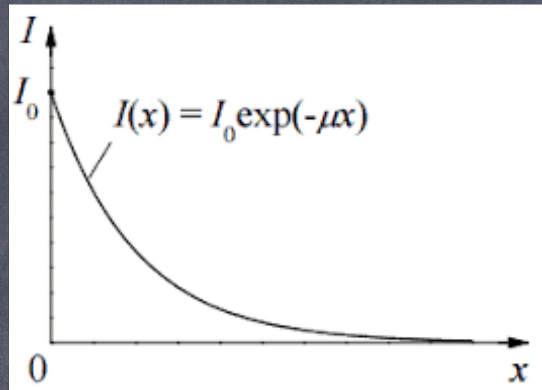
Lecture 9

May 2026

where were we?

- last lecture:
  - quantum numbers of atoms
  - electron transitions → photons
  - X-rays
  - charged-particle detection
  - Compton effect
  - how photons lose energy

Penetration of t-rays



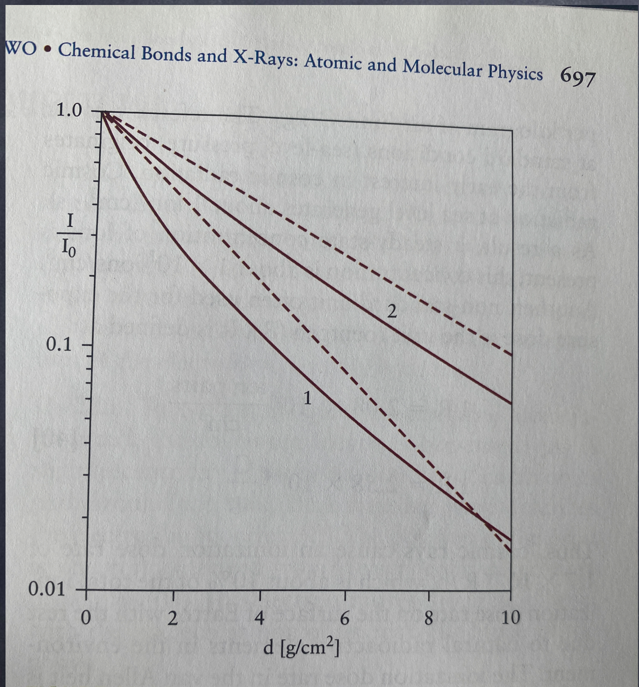


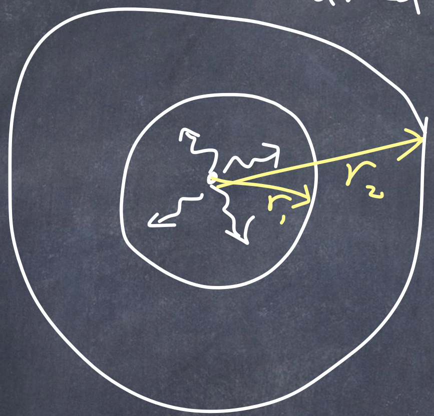
FIGURE 22.25

Intensity attenuation of X-ray beams of 100 keV (solid lines) and 50 keV (dashed lines) in human bones (curves 1) and muscle tissues (curves 2). The depth is given in unit g/cm^2, which results from the product of density and the path length in the tissue.

From [1]: "Physics of the Life Sciences" by Martin Zinke-Allmang

Reminder:

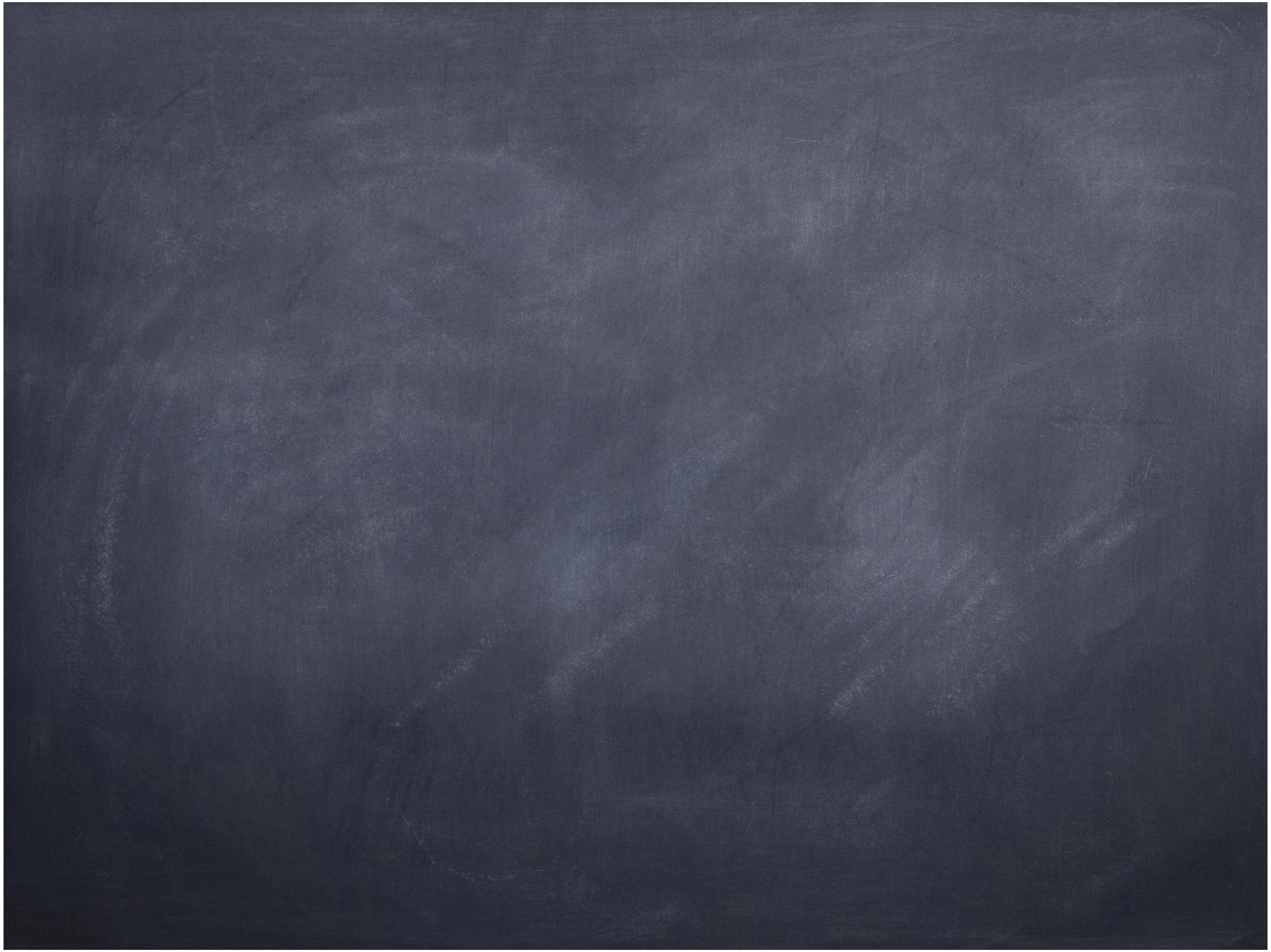
$$\text{Intensity} = \frac{\text{Power}}{\text{area}}$$

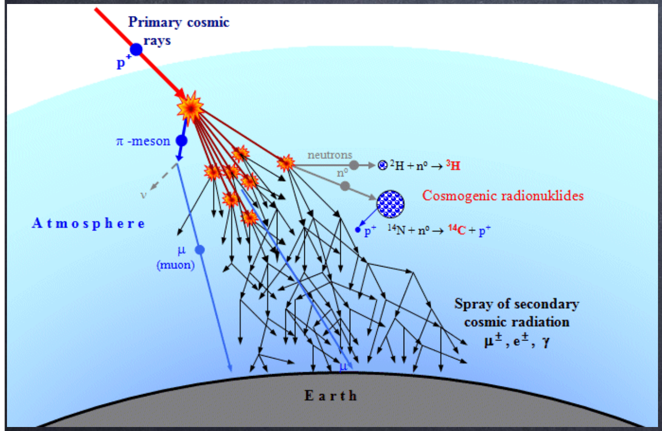


Intensity will decrease like $\frac{1}{r^2}$

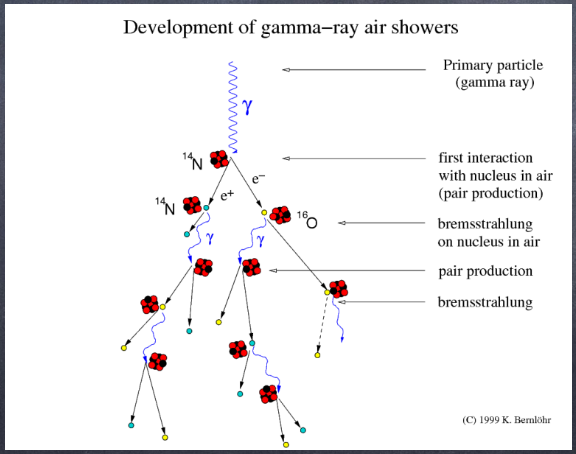
surface Area of a sphere is $A = 4\pi r^2$

If you have $r_2 = 2r_1$, the intensity is 4 times less at r_2 than r_1

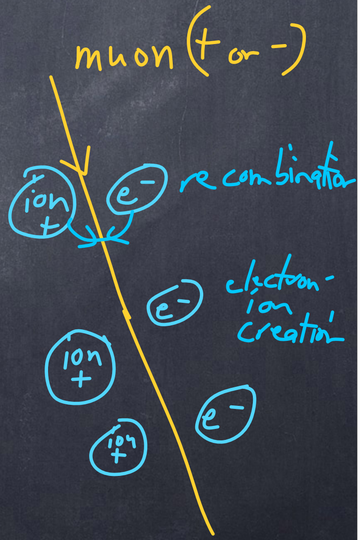




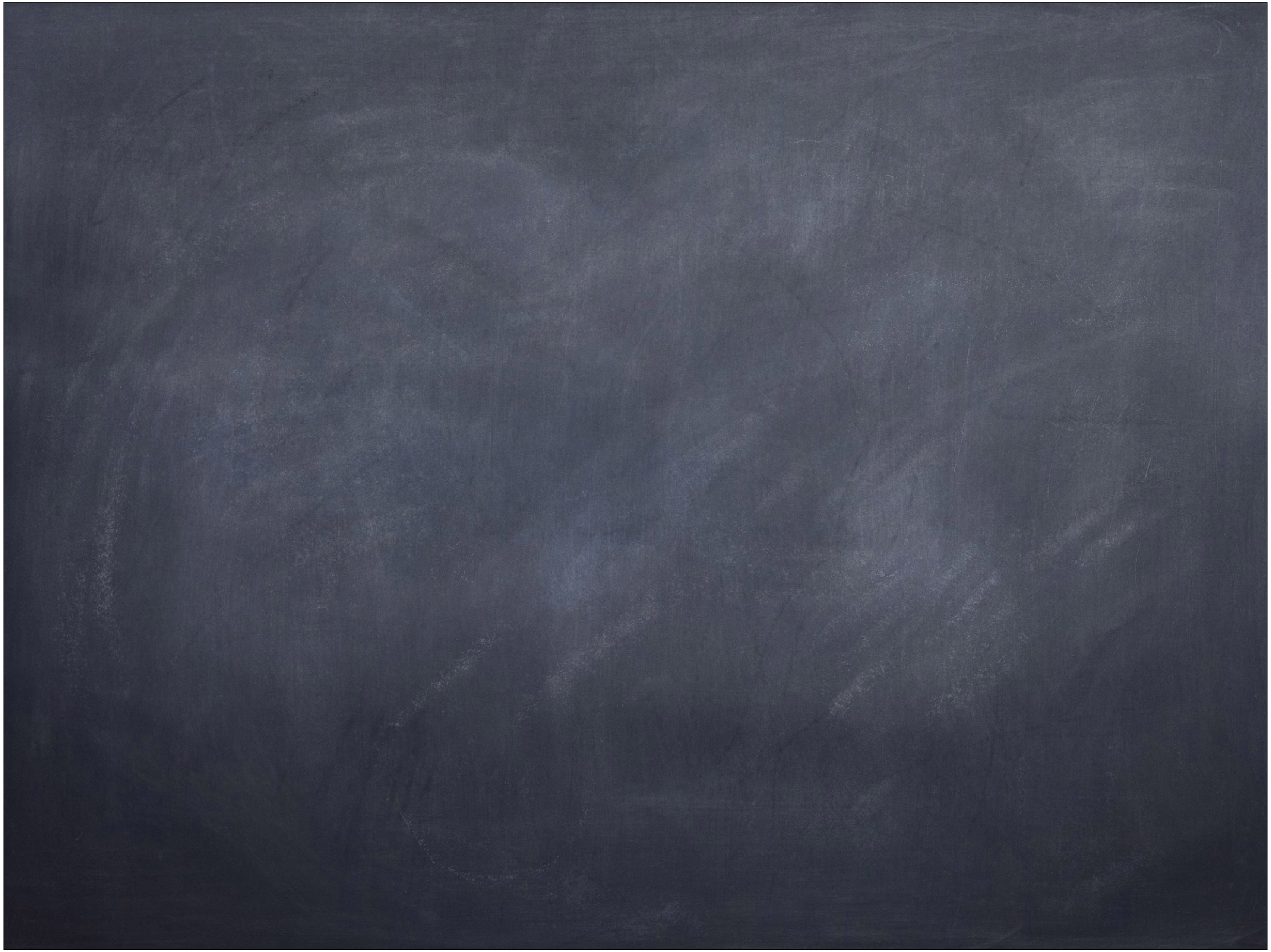
Primary cosmic rays
p+
π -meson
neutrons
n°
²H + n° → ³H
Cosmogenic radionuklides
Atmosphere
ν
μ
(muon)
p+
¹⁴N + n° → ¹⁴C + p+
Spray of secondary cosmic radiation
μ±, e±, γ
Earth

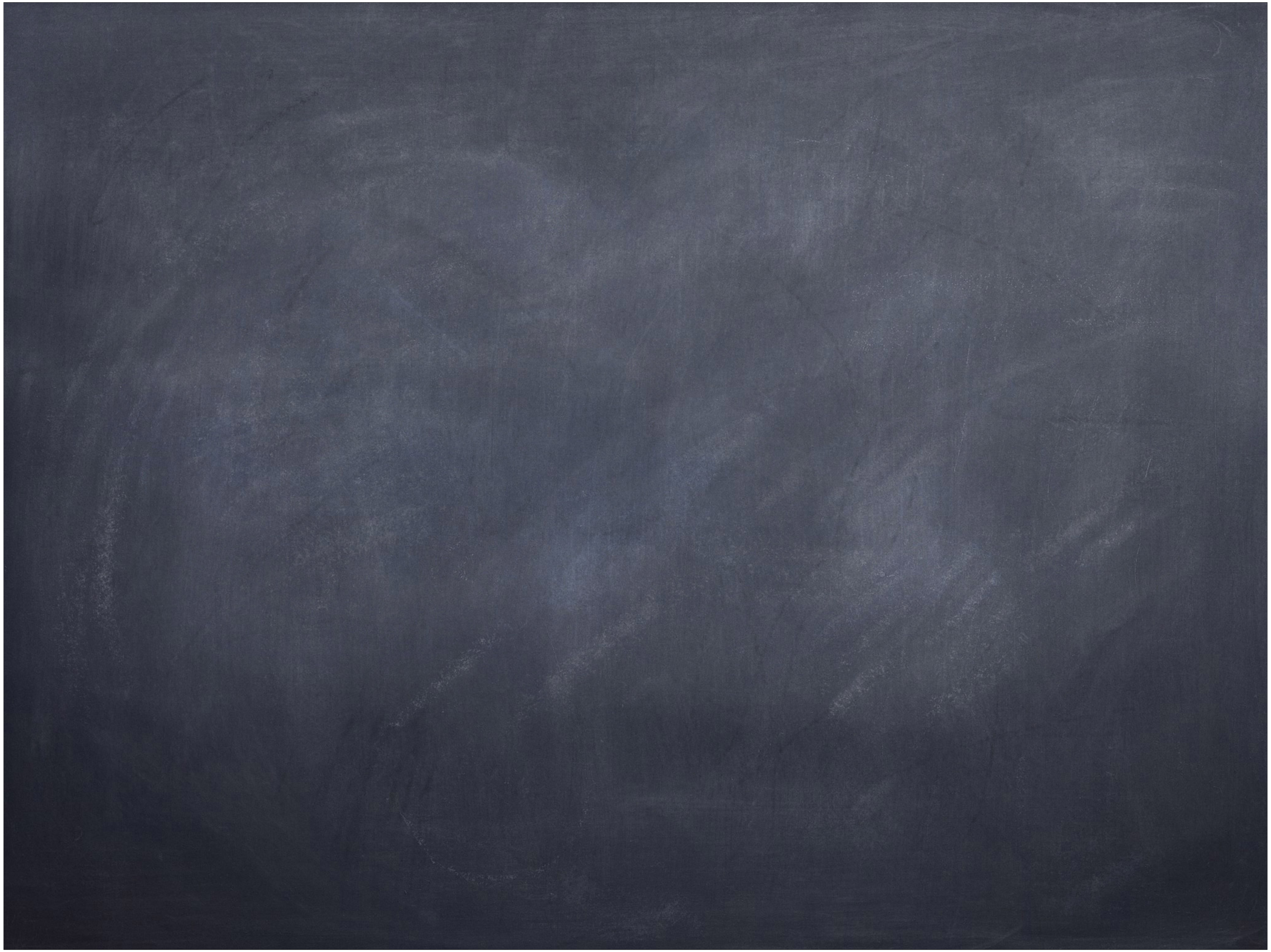


Development of gamma–ray air showers
γ
Primary particle
(gamma ray)
14N
e−
e+
first interaction
with nucleus in air
(pair production)
16O
bremsstrahlung
on nucleus in air
pair production
bremsstrahlung
(C) 1999 K. Bernlöhr



muon (+ or −)
ion +
e−
recombination
e−
electron-ion creation
ion +
e−
ion +





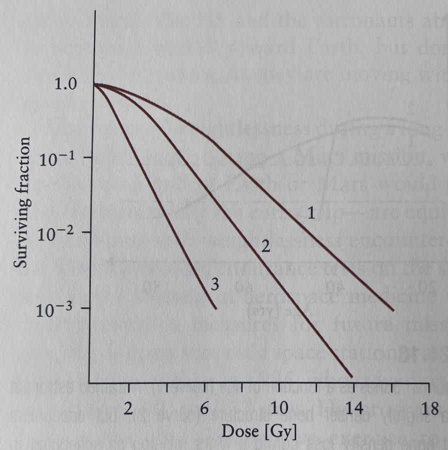


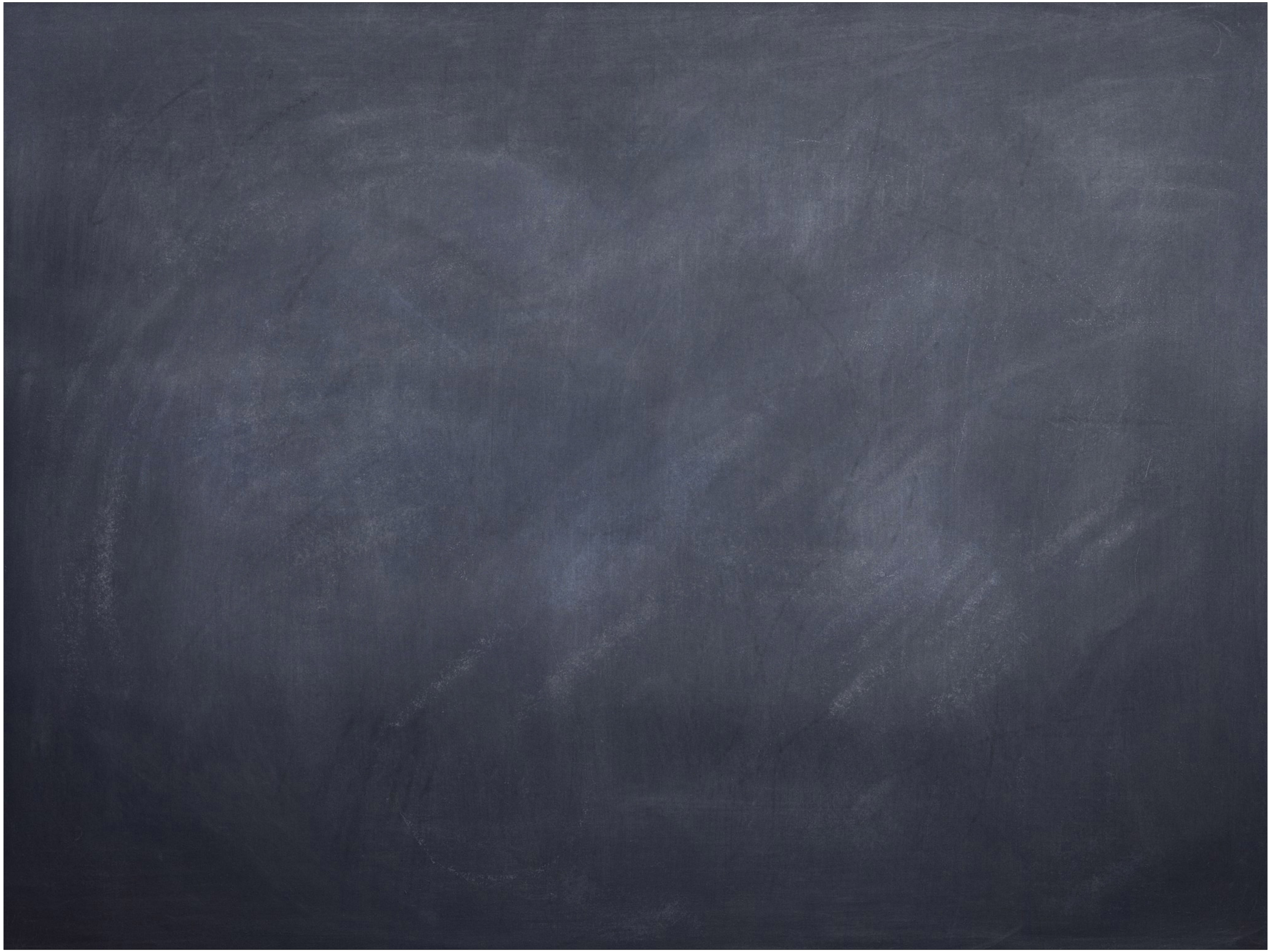
FIGURE 21.15

The surviving fraction of three types of human cells as a function of energy dose in unit Gy. The energy dose is the energy deposited by the radiation per kilogram of tissue. Note the lower steepness at doses below 1 Gy, which is due to self-repair mechanisms in living cells. Various cells respond with different sensitivity to radiation: (1) thyroid cells, (2) mammary cells, and (3) bone marrow.

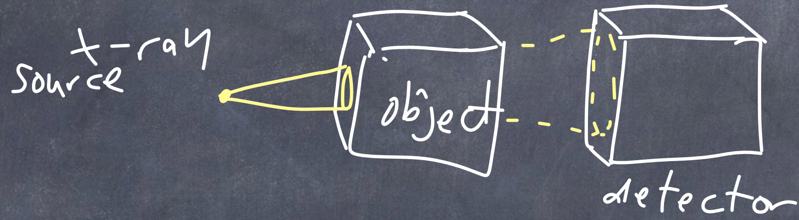
[1]

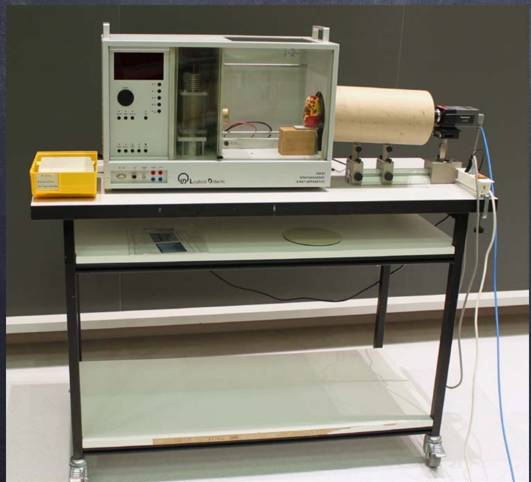
	Female	Male
Testes	0	0.08
Ovaries	0.08	0
Bone surface	0.01	0.01
Bladder	0.04	0.04
Bone marrow, red	0.12	0.12
Brain	0.01	0.01
Breast	0.12	0.12
Colon	0.12	0.12
Liver	0.04	0.04
Lungs	0.12	0.12
Oesophagus	0.04	0.04
Salivary glands	0.01	0.01
Skin	0.01	0.01
Stomach	0.12	0.12
Thyroid	0.04	0.04
Remainder[a]	0.12	0.12

[a]Component organs for remainder in ICRP 103: adrenals, extrathoracic airways, gallbladder, heart, kidneys, lymphatic nodes, muscle, oral mucosa, pancreas, prostate, small intestine, spleen, thymus and uterus/cervix.

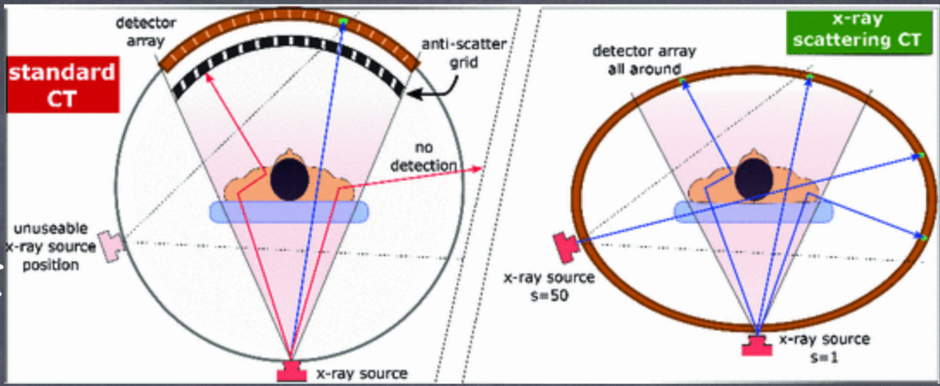


Originally, CT scan would have a single X-ray source, and a detector opposite.

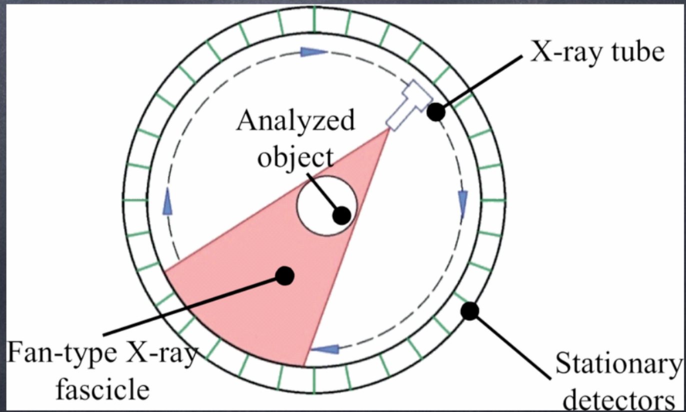


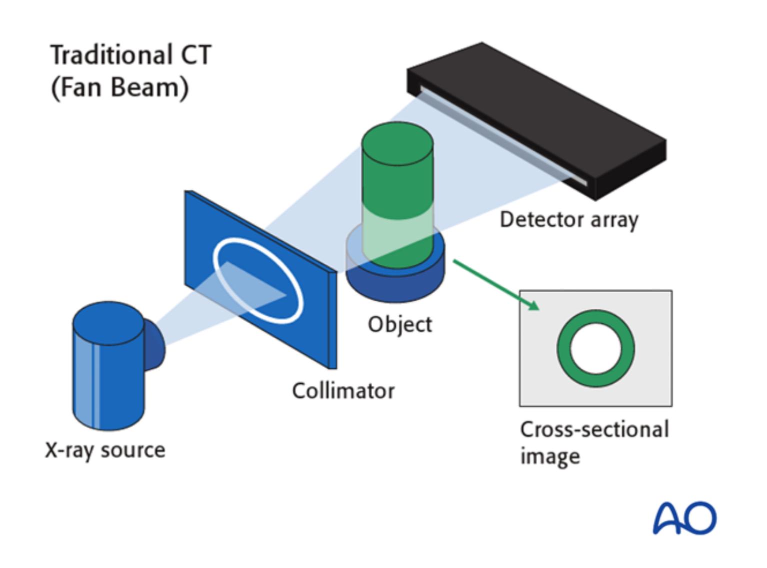


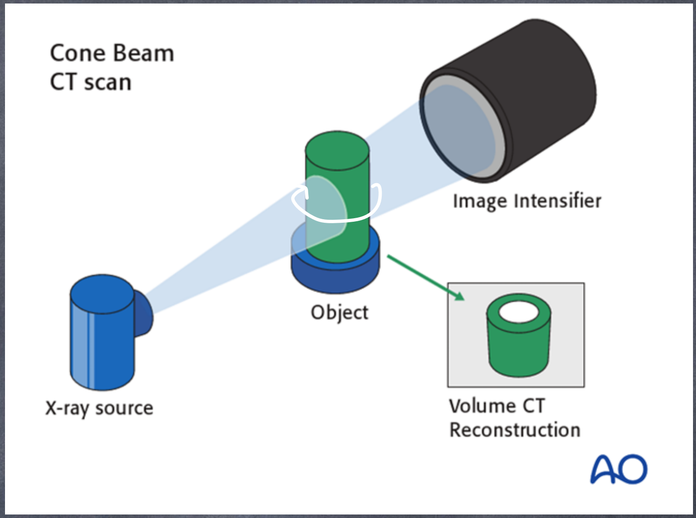
Today, wide fan-like beam, hundreds or thousands of detectors,
⇒ decreases time down to a few seconds



Newest: stationary detectors, and the beam sweeps around the patient.
Typically 50 ms per angle.

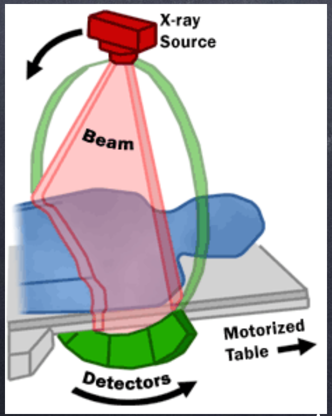






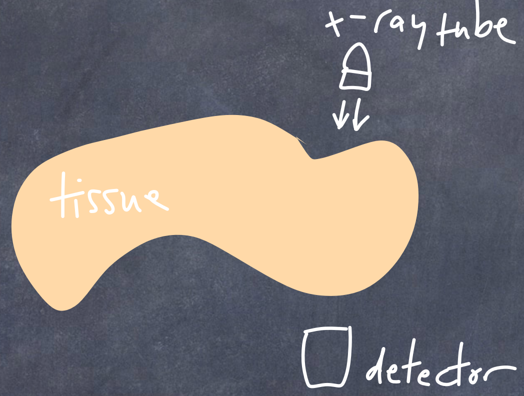
rotating source

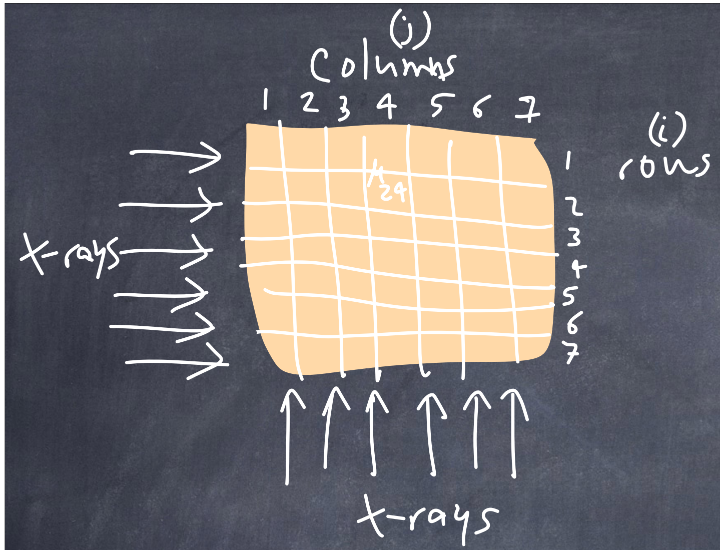
fan beam



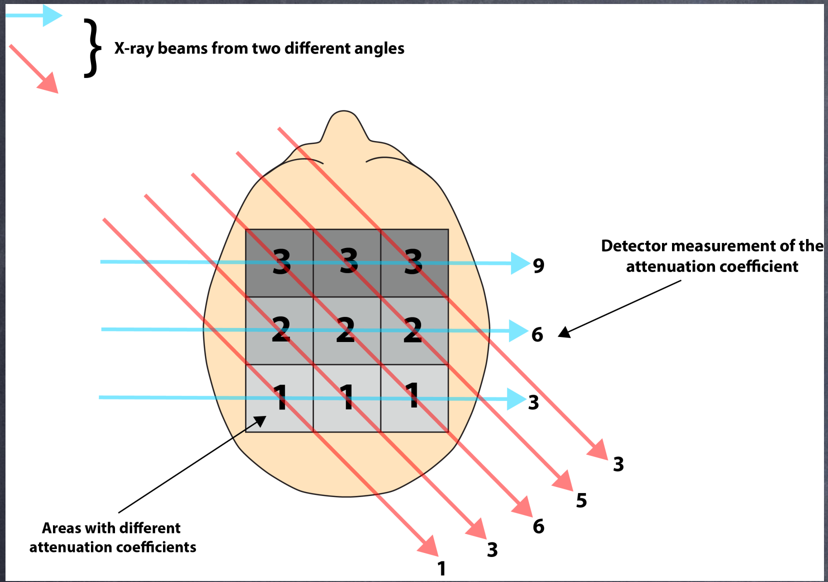
detectors opposite source

How to do and reconstruct a CT scan

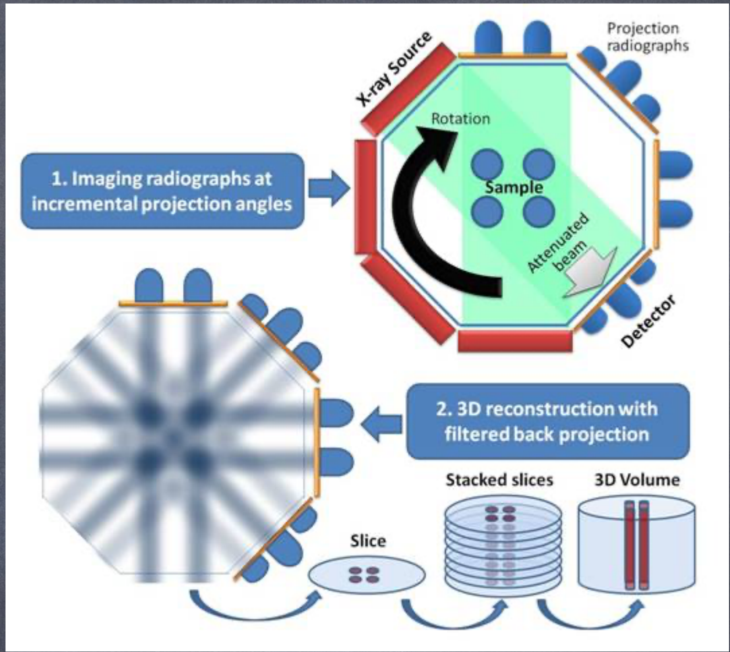



(j)
Columns
1 2 3 4 5 6 7
(i)
rows
1 2 3 4 5 6 7
x-rays
x-rays

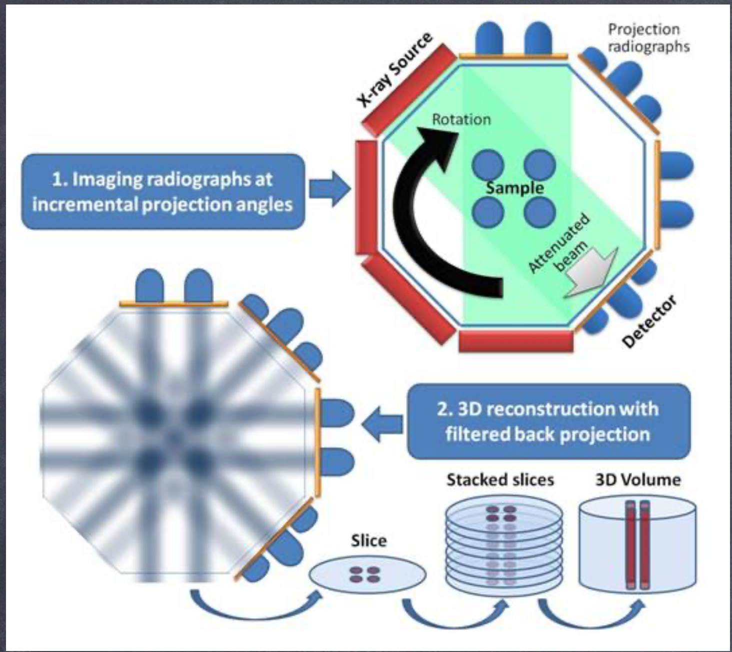
2D → 3D

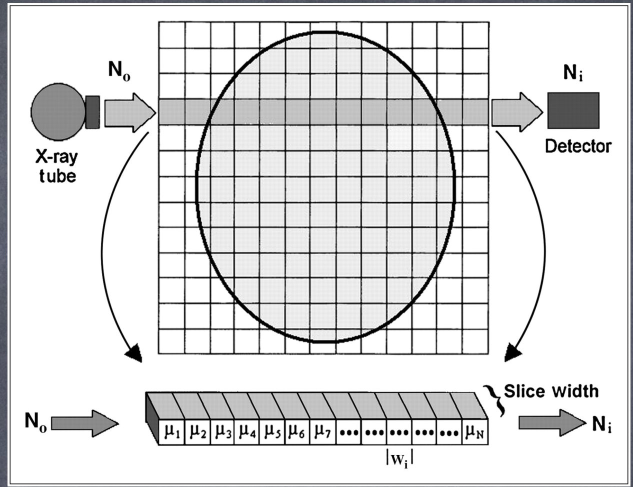


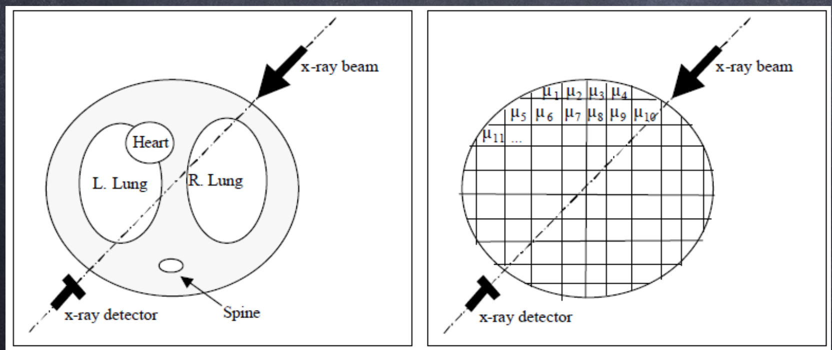
X-ray beams from two different angles
3 3 3
2 2 2
1 1 1
9
6
3
3
5
6
3
1
Detector measurement of the attenuation coefficient
Areas with different attenuation coefficients

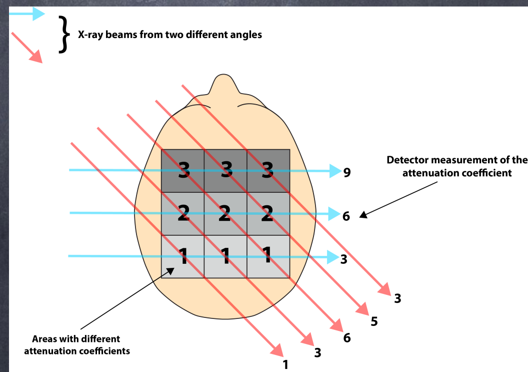


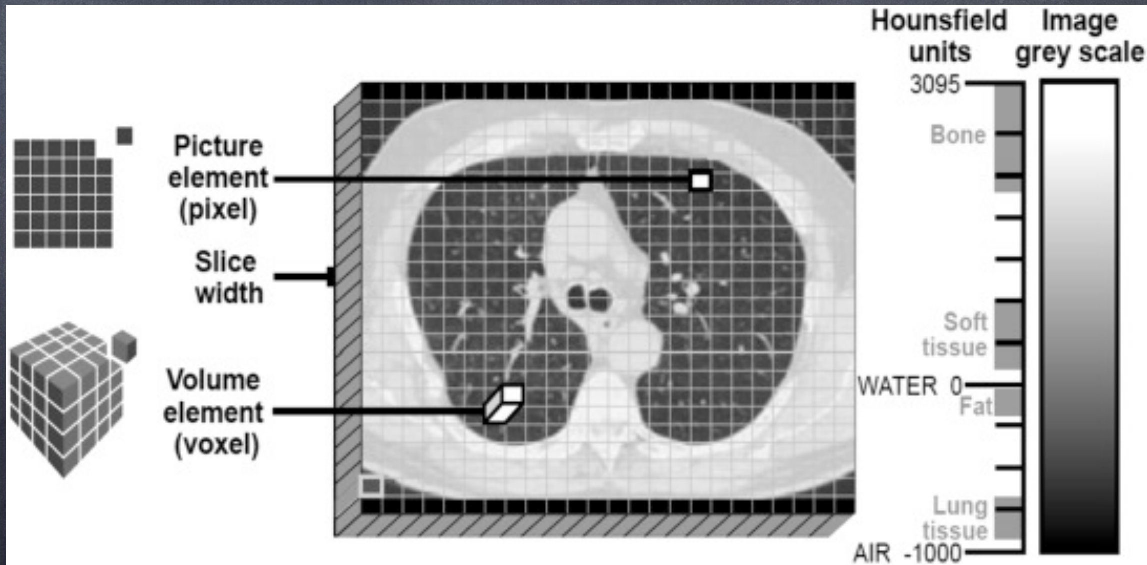
X-ray Source
Projection radiographs
Rotation
Sample
Attenuated beam
Detector
1. Imaging radiographs at incremental projection angles
2. 3D reconstruction with filtered back projection
Stacked slices
3D Volume
Slice


X-ray Source
Projection radiographs
Rotation
Sample
Attenuated beam
Detector
1. Imaging radiographs at incremental projection angles
2. 3D reconstruction with filtered back projection
Stacked slices
3D Volume
Slice


N0
Ni
X-ray tube
Detector
Slice width
μ1 μ2 μ3 μ4 μ5 μ6 μ7 ••• ••• ••• ••• ••• μN
|wi|

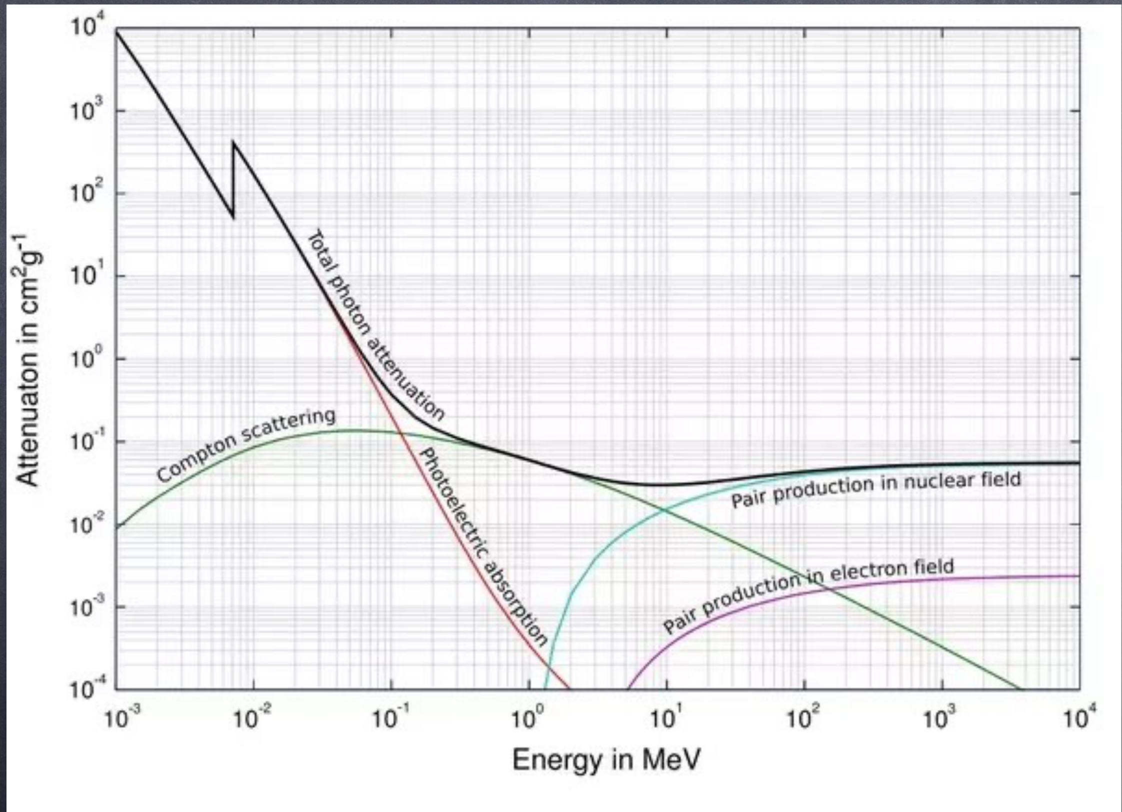

x-ray beam
Heart
L. Lung
R. Lung
x-ray detector
Spine
x-ray beam
μ1 μ2 μ3 μ4
μ5 μ6 μ7 μ8 μ9 μ10
μ11 ...
x-ray detector


X-ray beams from two different angles
Detector measurement of the attenuation coefficient
Areas with different attenuation coefficients
3 3 3
2 2 2
1 1 1
9
6
3
3
5
6
3
1



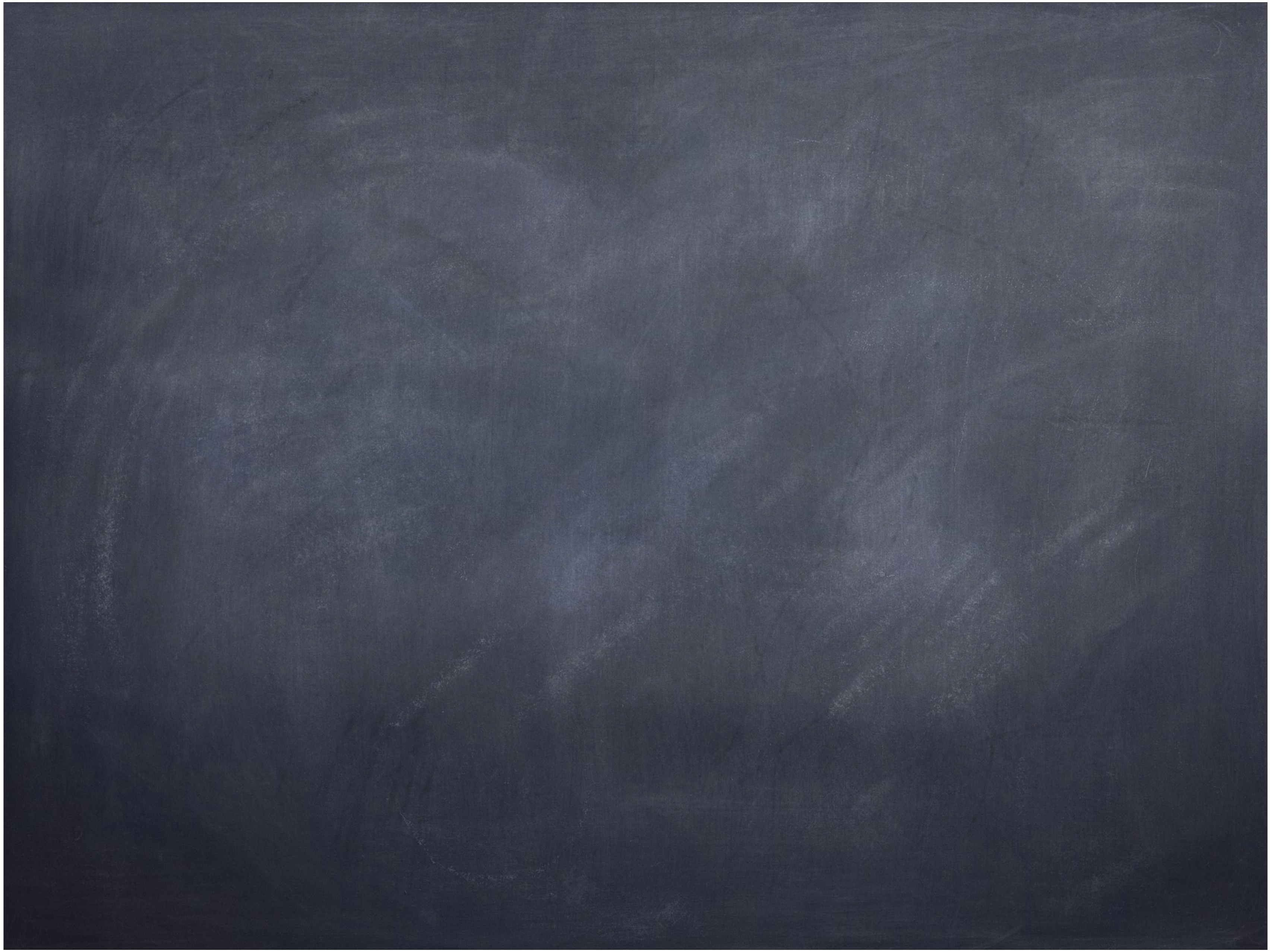
Hounsfield units
Image grey scale
3095
Bone
Picture element (pixel)
Slice width
Soft tissue
WATER 0
Fat
Volume element (voxel)
Lung tissue
AIR -1000

Reminder of attenuation of x-rays + gamma rays



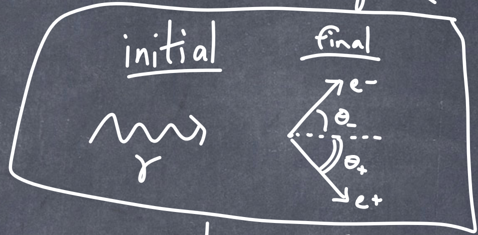
x-rays ↑ gamma rays

pair production starts here



start S3: supplementary proof that $\gamma \to e^+ + e^-$ can't happen in free space

we assume no nucleus!



initial: photon has energy : $h\nu$

final : no photon anymore,

electron has energy E_- and momentum $\bar{p}_-$

positron has energy E_+ and momentum $\bar{p}_+$

$m = m_e$: mass of electron

From conservation laws:

	initial	final	
Energy :	$h\nu =$	$E_+ + E_-$	①
momentum x :	$\frac{h\nu}{c}$	$= p_- \cos\theta_- + p_+ \cos\theta_+$	②
momentum y :	0	$= p_- \sin\theta_- + p_- \sin\theta$	③

Rewrite ② we get : $h\nu = cp_- \cos\theta_- + cp \cos\theta_-$ ④

Insert formula for relativistic energy ($E^2 = (cp)^2 + (mc^2)^2$) into ①:

$$h\nu = \sqrt{(cp_+)^2 + (mc^2)} + \sqrt{((cp_-)^2 + (mc^2)^2} \quad ⑤$$

The maximum value of $h\nu$ in ④ is when
$\cos\theta_- = \cos\theta_+ = 1$.

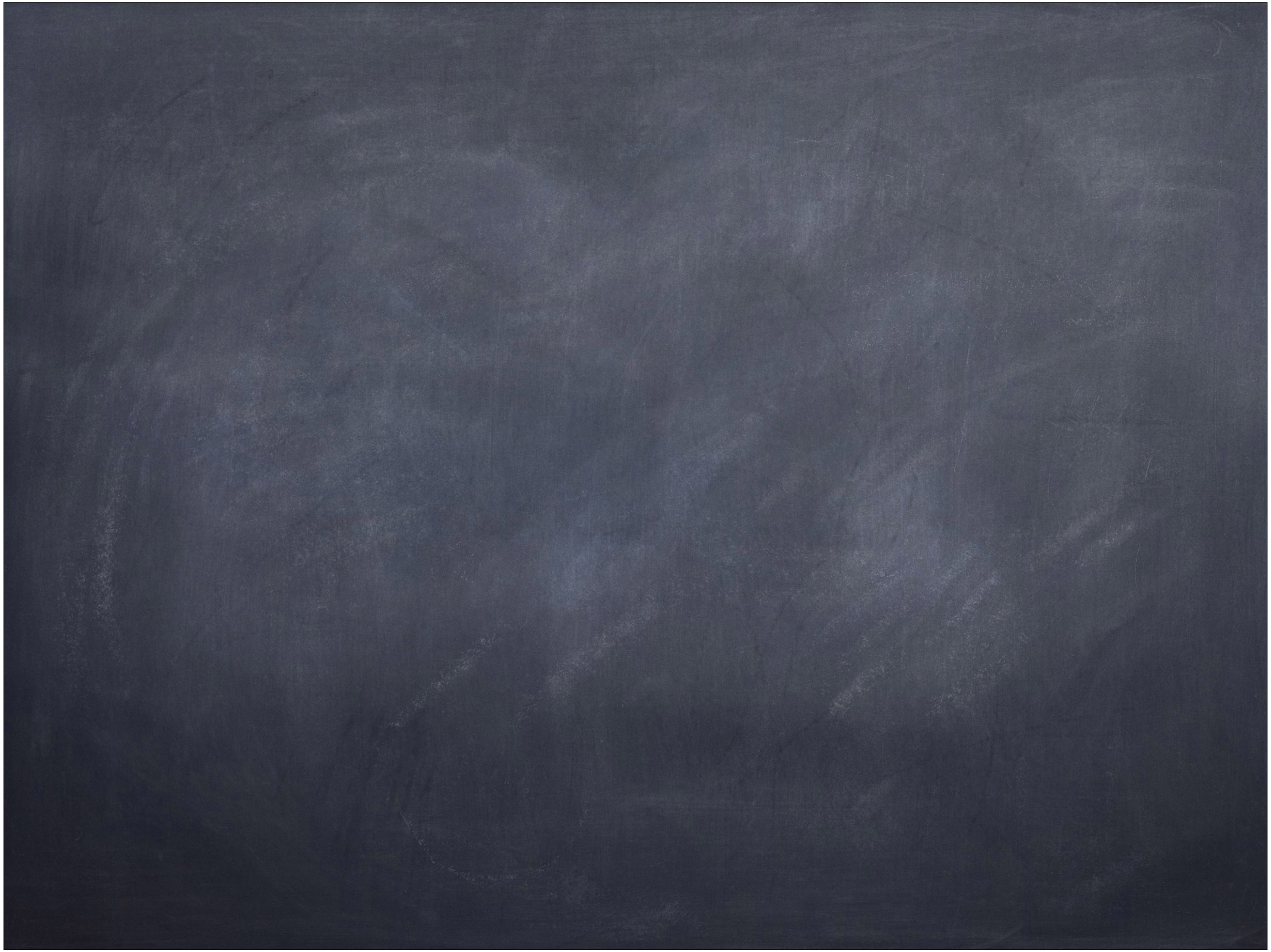
Then ④ becomes $\boxed{h\nu = cp_- + cp_+}$ ⑥

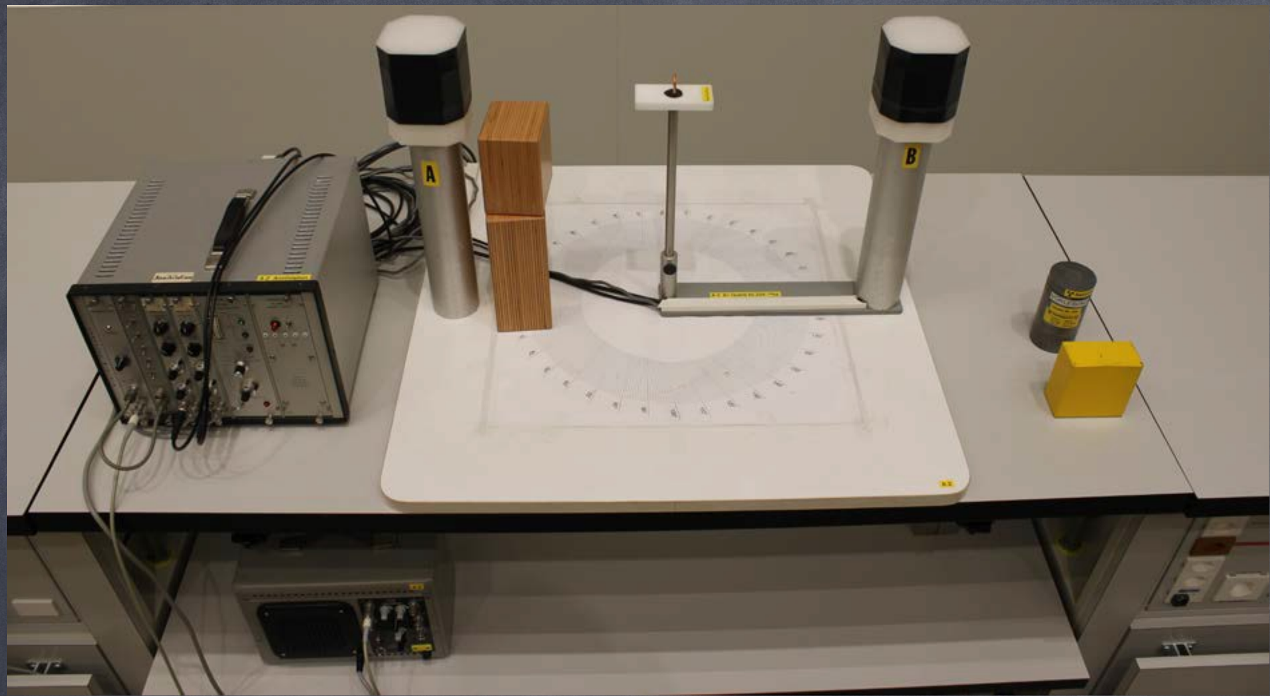
But if we look at eq. ⑤, we see that
$(h\nu)^2$ must be greater than $(cp_-)^2 + (cp_+)^2$
because of the electron + positron masses.

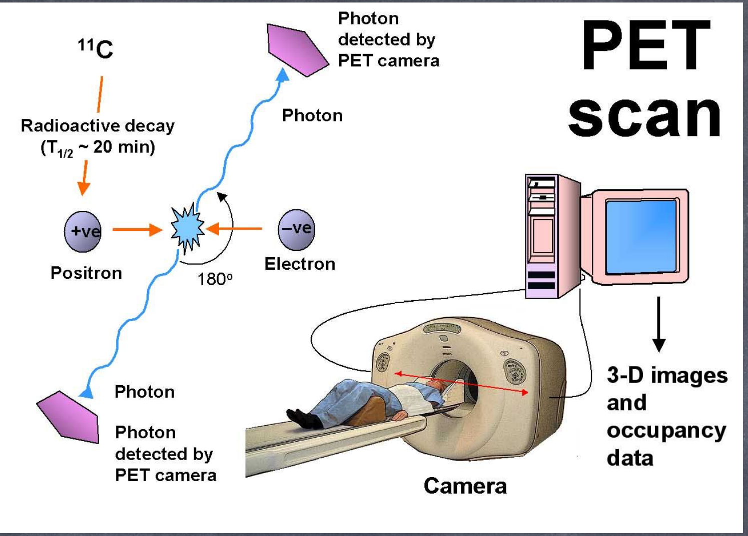
Therefore, since we have 2 equations, ⑤ and ⑥, which can't be both true at the same time, this reaction is not valid, because energy & momentum can't be conserved simultaneously.

end

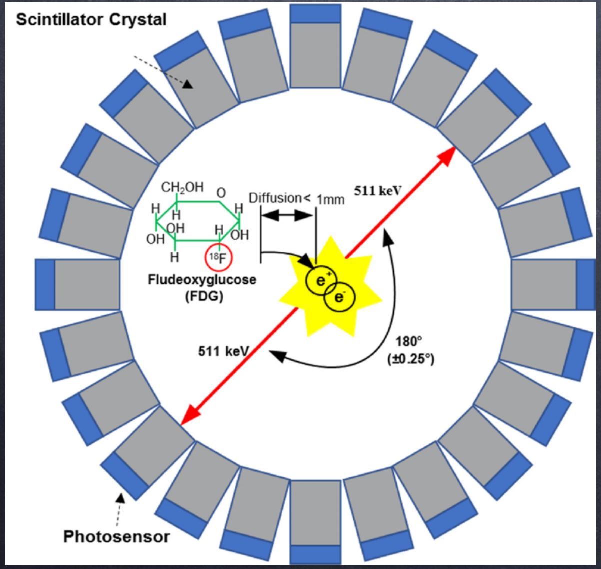
(53) Finished



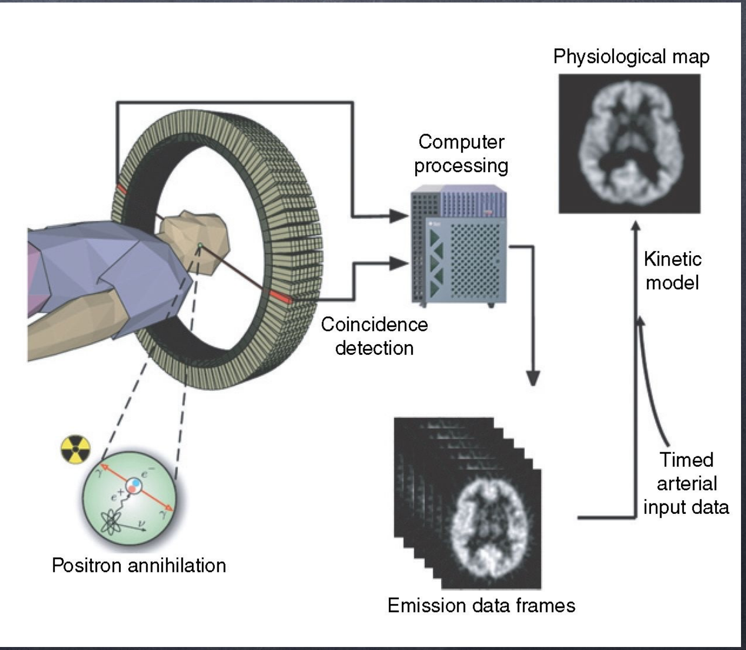

A
B



PET scan
11C
Radioactive decay (T1/2 ~ 20 min)
+ve
Positron
-ve
Electron
180°
Photon
Photon detected by PET camera
Photon
Photon detected by PET camera
Camera
3-D images and occupancy data



Scintillator Crystal
CH2OH
Diffusion< 1mm
511 keV
Fludeoxyglucose (FDG)
511 keV
180° (±0.25°)
Photosensor



Physiological map
Computer processing
Kinetic model
Coincidence detection
Timed arterial input data
Positron annihilation
Emission data frames

Supplement on Feynmann diagrams follows

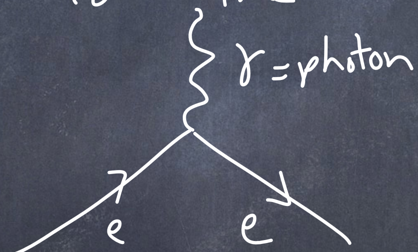
Light is an electromagnetic wave.

Light is quantized. The unit of light is a photon.

Quantum electrodynamics (QED)

All electromagnetic phenomena are ultimately reducible to the following elementary process.

(with an electron)



⟶ time

Time flowing horizontally to the right

This diagram reads "an electron enters, emits or absorbs a photon, and exits."

This diagram can be flipped or rotated, and the process still happens.

A particle moving backwards in time is interpreted as an antiparticle moving forwards in time.

electron = e^-

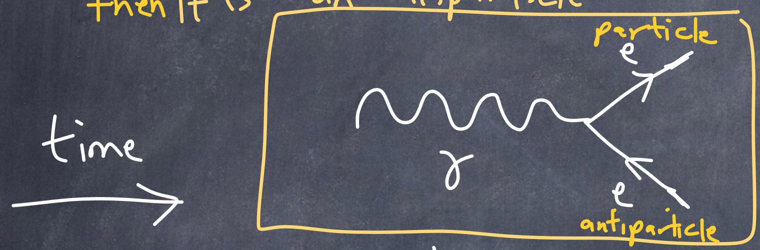
positron = e^+ antiparticle of the electron

A photon does not need an arrow since it is its own antiparticle.

So this diagram reads " a positron enters, emits or absorbs a photon, and exits. "

The positron was predicted in 1928
by Dirac because his formulas had 2 solutions: +, -
Discovered in 1932 by Anderson

If arrow is moving opposite to time, then it is an antiparticle.



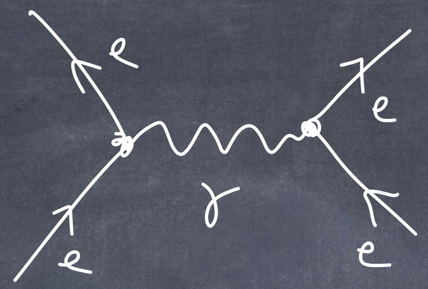
Can happen but must obey energy & momentum conservation.

"A photon enters, decays into an electron and a positron, and they exit."

These diagrams are called Feynmann diagrams.

Some people label these diagrams with e^- & e^+, but I find this dangerous.

Since a positron moving backwards in time would be an electron



Here, an electron and positron annihilate into a photon, and then the photon decays into a new electron and positron.

Note: the electric charge is conserved.

we can write this diagram as:

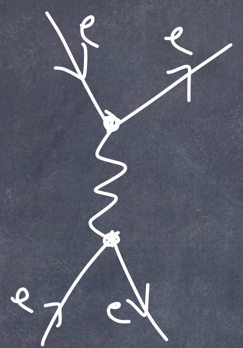
$$e^- + e^+ \rightarrow \gamma \rightarrow e^- + e^+$$

electric charge $-1 + 1 = 0 = 0 = -1 + +1$

Energy + momentum are conserved in this process.

This diagram can be rotated.

time →

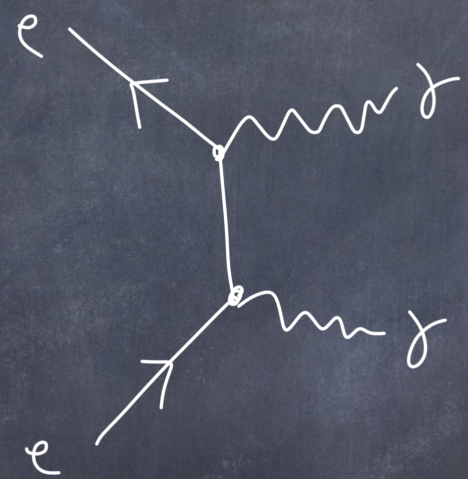


Here, two electrons enter, exchange a photon and continue as electrons.

Here, the electrons repel can be seen to repel each other.

In quantum physics, forces are mediated by particles.
The photon mediates the electromagnetic force.

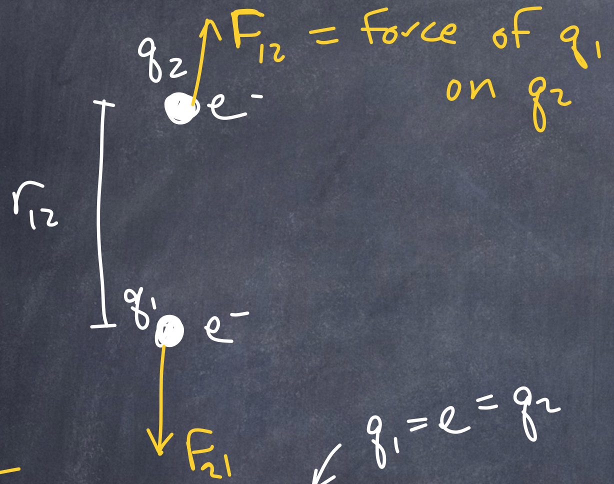
How do we draw the Feynman diagram for electron-positron annhilation (as in PET scan)?



read: an electron and positron annhilate into 2 photons.

But does this mean classical physics is wrong?

classical physics view: (PHY 117)

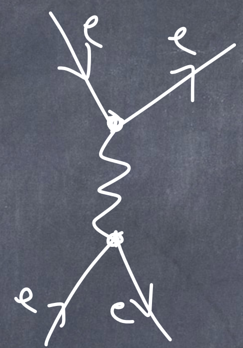


$q_1 = e = q_2$

$$-\bar{F}_{21} = \bar{F}_{12} = k \frac{q_1 q_2}{r_{12}^2} \hat{r}_{12}$$

Electrons are repelled by a force, which we can calculate.

quantum physics view: time →



Here, two electrons exchange a photon and continue as electrons.

Here, the electrons repel each other.

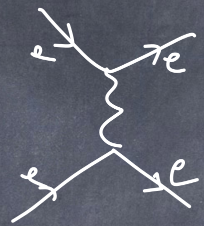
In quantum physics, forces are mediated by particles.

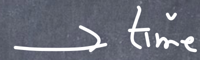
The photon mediates the electromagnetic force.

In practice, classical physics is easier to calculate for most everyday situations

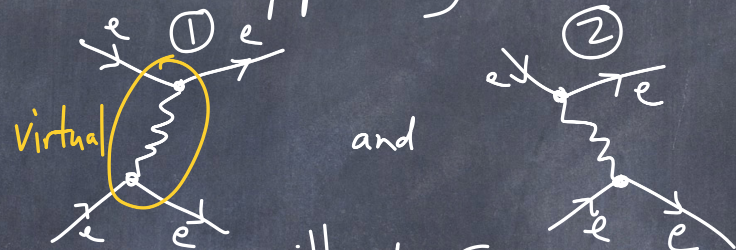
What happens if a particle moves perpendicular to time?

A: Here a photon moves vertically.



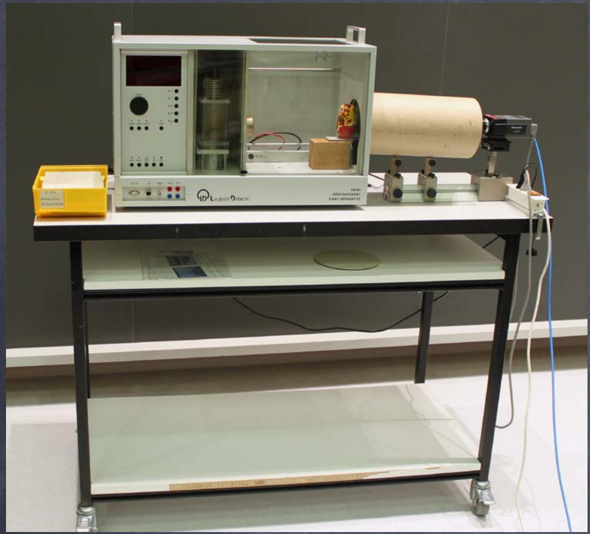


Really, what is happening is both of these:



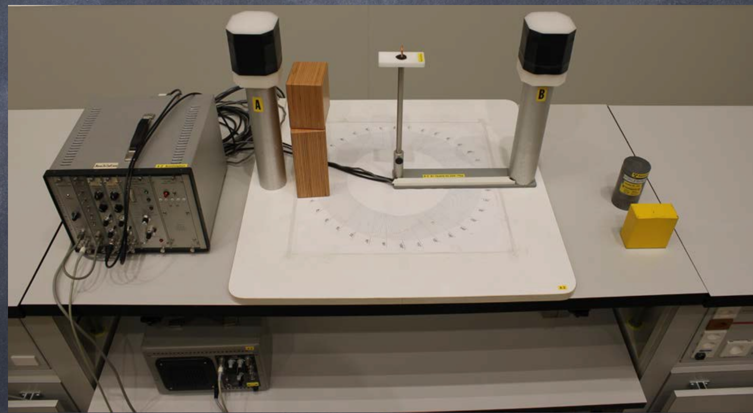
In ①, the photon is emitted from the below electron.
In ②, the photon is emitted from the above electron

The photon is not observable, we call it virtual.
we can't tell if ① or ② happens, so we use quantum mechanics to consider both.
But we draw it like this



A24

ES90



A2