HS 21
Prof. Ph. Jetzer

MMP I
Exercise Sheet 2

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Exercise 1 [Fourier Analysis and Minimization (4 points)]

f is a 2π periodical function defined as

$$f(x) = \pi - |x|, \quad -\pi \leq x \leq \pi.$$

a) Find the Fourier coefficients for f.

b) Find parameters a and b for which the integral

$$\int_{-\pi}^{\pi} \mathrm{d}x \, (f(x) - a\cos(3x) - b\sin(4x))^2$$

is minimal.

Exercise 2 [Legendre Polynomials (4 points)]

Show that the Legendre polynomials

$$\begin{aligned} P_0(x) &= 1, \\ P_n(x) &= \frac{1}{2^n n!} \left(\frac{\mathrm{d}}{\mathrm{d}x}\right)^n (x^2-1)^n, \quad n \in \mathbb{N} \end{aligned} \tag{1}$$

are orthogonal on the interval $[-1, 1]$.

– please turn over –

Exercise 3 [Recursive Relations for Legendre Polynomials (2 points)]

Using the definition (1) of the Legendre polynomials, find $P_1(x)$, $P_2(x)$ and $P_n(0)$.
Verify the following recursive relations:

$$(xf(x))^{(n)} = xf^{(n)}(x) + nf^{(n-1)}(x) \text{ for any } n \text{ times differentiable function } f \tag{2}$$

$$P'_{n+1}(x) = xP'_n(x) + (n+1)P_n(x) \tag{3}$$

The second relation provides a way to evaluate Legendre polynomials recursively.

Exercise 4 [Symmetry Considerations (2 points)]

Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous 2π periodical function whose graph is symmetric with respect to the line $x = \frac{\pi}{2}$:

$$f(x) \sim \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos(nx) + b_n \sin(nx)\right).$$

What consequences does this symmetry have on coefficients of the Fourier series of f?