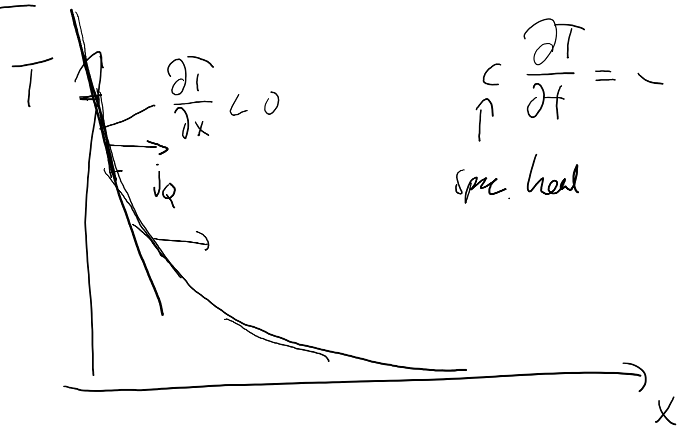
Non-equilibrium systems

gradient in some intensive variable e.g. $\vec{\nabla} T$

$\Rightarrow \vec{j}_Q = -\lambda \vec{\nabla} T$ heat transport equation

$\vec{j}_D = -D \vec{\nabla} c$ Fick's law for diffusion

current of heat



$$c \frac{\partial T}{\partial t} = -\frac{\partial}{\partial x} j_Q = -\vec{\nabla} \cdot \vec{j}_Q$$

spec. heat

$$c \frac{\partial T}{\partial t} = -\vec{\nabla} \cdot \underset{-\vec{\nabla} T \cdot \lambda}{j_Q} = +\lambda \vec{\nabla} \cdot (\vec{\nabla} T) \Leftrightarrow \frac{\partial T}{\partial t} = \underbrace{\frac{\lambda}{c}}_{\alpha} \vec{\nabla}^2 T$$

Diffusion-eq

$$-\vec{\nabla} \cdot \vec{j}_D = \frac{\partial c}{\partial t} \rightarrow \frac{\partial c}{\partial t} = D \vec{\nabla}^2 c$$

(D: diffusivity)

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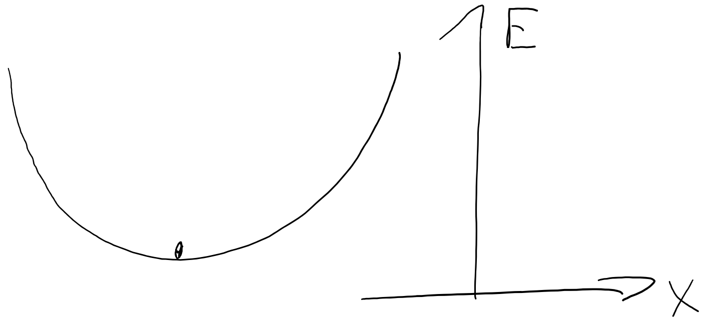
equilibrium

$\downarrow$

Minimum in "Energy" / thermodyn. Potential



$$\rightarrow \left.\frac{\partial \Phi}{\partial x}\right|_{x_0} = 0 \longrightarrow \Phi(x) = \Phi(x_s) + \cancel{\left.\frac{\partial \Phi}{\partial x}\right|_{x_s} (x - x_s)} + \frac{1}{2} \frac{\partial^2 \Phi}{\partial x^2} (x - x_s)^2 + \ldots$$

$$\Rightarrow \frac{\partial \Phi}{\partial x}\Big|_{x_{eq}} = 0 \longrightarrow \Phi(x) = \Phi(x_s) + \cancel{\frac{\partial \Phi}{\partial x}\Big|_{x_s}(x-x_s)} + \frac{1}{2}\frac{\partial^2 \Phi}{\partial x^2}(x-x_s)^2 + \dots$$

$$ds = \frac{1}{T}\,du - \frac{\mu}{T}\,d\rho$$

$$\frac{\partial s}{\partial t} = \frac{1}{T}\frac{\partial u}{\partial t} - \frac{\mu}{T}\frac{\partial \rho}{\partial t} = -\frac{1}{T}\vec{\nabla}\cdot\vec{j}_u + \frac{\mu}{T}\vec{\nabla}\cdot\vec{j}_\rho$$

$$\vec{j}_s = \frac{1}{T}\vec{j}_u - \frac{\mu}{T}\vec{j}_\rho \qquad \vec{\nabla}\cdot\vec{j}_s = \frac{1}{T}\vec{\nabla}\cdot\vec{j}_u + \vec{j}_u\cdot\vec{\nabla}\left(\frac{1}{T}\right) - \frac{\mu}{T}\vec{\nabla}\cdot\vec{j}_\rho - \vec{j}_\rho\cdot\vec{\nabla}\left(\frac{\mu}{T}\right)$$

$$\underbrace{\frac{\partial s}{\partial t} + \vec{\nabla}\cdot\vec{j}_s}_{=\frac{\partial s_c}{\partial t}} = \cancel{-\frac{1}{T}\vec{\nabla}\cdot\vec{j}_u} + \cancel{\frac{\mu}{T}\vec{\nabla}\cdot\vec{j}_\rho} + \cancel{\frac{1}{T}\vec{\nabla}\cdot\vec{j}_u} + \vec{j}_u\cdot\vec{\nabla}\left(\frac{1}{T}\right) - \cancel{\frac{\mu}{T}\vec{\nabla}\cdot\vec{j}_\rho} - \vec{j}_\rho\cdot\vec{\nabla}\left(\frac{\mu}{T}\right)$$

$$\Rightarrow \boxed{\frac{\partial s_c}{\partial t} = \vec{j}_u\cdot\underbrace{\vec{\nabla}\frac{1}{T}}_{=\vec{F}_u} + \vec{j}_\rho\cdot\underbrace{\vec{\nabla}\left(\frac{-\mu}{T}\right)}_{\vec{F}_\rho}} = \underbrace{\vec{j}_u}_{L_{uu}\vec{F}_u}\cdot\vec{F}_u + \underbrace{\vec{j}_\rho}_{L_{\rho\rho}\vec{F}_\rho}\cdot\vec{F}_\rho$$

$$= \underline{L_{uu}|\vec{F}_u|^2 + L_{\rho\rho}|\vec{F}_\rho|^2} > 0$$ 2. law of Thermodynamics

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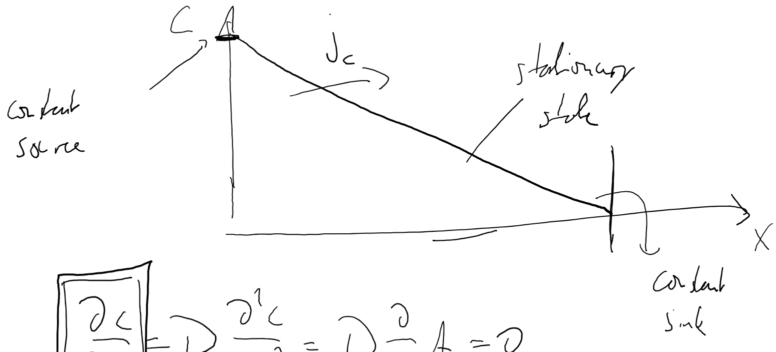

$$\frac{\partial c}{\partial x} = A = \text{constant}$$

$$\boxed{j_c = -D\frac{\partial c}{\partial x} = -DA}$$

$$\boxed{\frac{\partial c}{\partial t}} = D\frac{\partial^2 c}{\partial x^2} = D\frac{\partial}{\partial x}A = 0$$

$$\frac{\partial c}{\partial t} = D\frac{\partial^2 c}{\partial x^2} - \underline{kc} \longrightarrow \text{stationary state } \frac{\partial c}{\partial t} = 0$$

$$\frac{\quad}{\partial t} = D \frac{\quad}{\partial x^2} - \underline{k c} \quad \rightarrow \text{stationary state} \quad \frac{\quad}{\partial t} = 0$$

$$\underline{c(x) \sim e^{-x/\sqrt{D/k}}}$$