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Applications of General Relativity

Sheet 3

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Exercise 1 [The Reissner-Nordström space-time] (7 points)

The so called "Reissner-Nordström space-time" is a static, spherically symmetric solution of the Einstein's field equations in vacuum with the presence of an electrostatic field:

$$ds^2 = \left(1 - \frac{r_S}{r} + \frac{r_Q^2}{r^2}\right) c^2 dt^2 - \frac{dr^2}{1 - \frac{r_S}{r} + \frac{r_Q^2}{r^2}} - r^2\, d\theta^2 - r^2\, \sin\theta^2\, d\phi^2, \tag{1}$$

where $r_S = 2GM/c^2$ and $r_Q = \sqrt{Q^2 G/c^4}$. This solution represents a non-rotating black hole of mass M and charge Q.

a) Explain why the motion of even *neutral* test particles are affected by the black hole charge.

b) Following the same steps used for the Schwarzschild space-time, describe the possible geodesics of *massive* and *massless* particles in the Reissner-Nordström metric.

c) Following what you learnt about the Schwarzschild metric, discuss geodetic precession in this space-time (i.e. derive the equivalent precession angular frequency ω).

Exercise 2 [The linearized EFE] (5 points)

The linearized Einstein's field equations (EFE) for the perturbation $h_{\mu\nu}$ (remember: $g_{\mu\nu} \simeq \eta_{\mu\nu} + h_{\mu\nu}$) read:

$$\Box h_{\mu\nu} = -\frac{16\pi G}{c^4}\left(T_{\mu\nu} - \frac{T}{2}\eta_{\mu\nu}\right). \tag{2}$$

a) Show that for a static, general source described by $T_{\mu\nu} = \mathrm{diag}\{\rho c^2, 0, 0, 0\}$, the metric $g_{\mu\nu}$ reduces to:

$$ds^2 = \left(1 + 2\phi/c^2\right) c^2 dt^2 - \left(1 - 2\phi/c^2\right)\left(dx^2 + dy^2 + dz^2\right), \tag{3}$$

where ϕ is the newtonian gravitation potential associated to ρ.

b) Show that the, at first order in ϕ/c^2, the effective potential for a particle in the spherically-symmetric metric given by Equation (3) is:

$$V_{\text{eff}} = c^2 + 2\phi + \frac{L^2}{r^2} + \frac{4\phi L^2}{c^2 r^2}, \tag{4}$$

where L is the specific angular momentum of the particle.