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Applications of General Relativity

Sheet 5

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Exercise 1 [More on the linearized EFE] (4 points)

Consider the trace reversed form of the metric perturbations,

$$\gamma_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h^{\sigma}{}_{\sigma}. \tag{1}$$

Starting from the linearized Riemann tensor, show that in the Lorentz gauge, (in which the divergence of $\gamma_{\mu\nu}$ vanishes), the linearized field equations become

$$\gamma_{\mu\nu}{}_{,\alpha}^{;\alpha} = -16\pi T_{\mu\nu}. \tag{2}$$

Exercise 2 [Asymptotic Justice] (8 points)

A colleague of mass μ steals your research and publishes the results before you can. Seeking retribution, and being the heartless, merciless astrophysicist that you are, you decide that dropping him radially into a solar-mass black hole from constant spatial coordinates (r_*, θ_*, ϕ_*) is suitable punishment.

Once you release him however, you begin to doubt your plan, and begin scribbling furiously on a piece of paper.

a) Treating the in-falling physicist as a point particle, when calculating his coordinate time as a function of r, shocked, you realize that the coordinate speed

$$\frac{dr}{dt} = \frac{\dot{r}}{\dot{t}} = -\Delta(r)\sqrt{1 - \frac{\Delta(r)}{\Delta(r_*)}}, \qquad \Delta(r) = 1 - \frac{2GM}{r}, \tag{3}$$

becomes zero as the physicist approaches the horizon, and you will never see him falling in.

b) Perhaps not all hope is lost for your desire for revenge to be satisfacted. A physicist is not a point particle! You recall that the breaking strength of the average astrophysicist of length 1.8 m and mass 80 Kg is $2.5 \cdot 10^3$ N. Assuming $\Delta(r_*) \approx 1$ and using the equation of geodesic deviation, you are able to calculate the radius at which your stomach demands you watch no longer.