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Applications of General Relativity

Sheet 4

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Exercise 1 [Out of plane precession of S2 orbit]
The center of our galaxy harbours a rotating black hole. While the exact Kerr solution for rotating black holes will be discussed at the end of the semester we will consider an interesting effect using the weak field results derived in the lecture.

Many of the stars within 1 pc from the central black hole have orbits that lie in a plane. This suggests that these stars formed from gas that is arranged in a disk-like structure. However, the orbit of the star S2 with semi-major axis $a_r = 4.7 \times 10^{-3}$ pc and eccentricity $e = 0.88$ is inclined by 75° with respect to this plane. One possible origin for this inclination could be the precession of the orbital plane due to the black hole's spin. Using the age estimate of $t_{\mathrm{S2}} \approx 3 \times 10^6$ yr, calculate a lower bound on the black hole spin using the black hole mass $M_{\mathrm{BH}} = 4.2 \times 10^6\ M_\odot$.

Hint: As shown in the lecture, the "gravitomagnetic" equation of motion can be written as

$$\frac{d\boldsymbol{v}}{dt} = -\nabla\phi + 2\boldsymbol{\Omega} \wedge \boldsymbol{v}, \tag{1}$$

where

$$\boldsymbol{\Omega} = \frac{G}{c^2}\left[\frac{3\boldsymbol{r}(\boldsymbol{r}\cdot\boldsymbol{S})}{r^5} - \frac{\boldsymbol{S}}{r^3}\right]. \tag{2}$$

Here, $\boldsymbol{S}$ is the spin of the black hole. Since you are searching a lower bound for the spin, it is sufficient to estimate the precession rate for the configuration where the precession is maximal, i.e., when $\boldsymbol{S}$ lies on the orbital plane. Furthermore, you are allowed to simplify the system assuming circular orbits. Note that the black hole spin is conveniently parametrized in terms of the spin parameter

$$a = \frac{Sc}{GM^2}, \tag{3}$$

where $a < 1$. A black hole with $a = 1$ is said to be maximally spinning.