MMP I
Exercise Sheet 6

HS 21
Prof. Ph. Jetzer

L. Buonocore, M. Loechner, X. Liu, M. Ebersold
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Exercise 1 [Ordinary Differential Equations (7 points [1, 1.5, 1.5, 1, 2])]

Find a general solution $y(x)$ to the following differential equations:

a) $y' = \frac{1}{xy}$ with $y(1) = -1$

b) $y' = \frac{y}{x} + \frac{x}{y}$

c) $y' = \frac{3x-y+4}{x+y}$

d) $y' = y^2 \cos x$

e) $y' = \cos(x+y) + \sin(x-y)$ (find the solution that passes through the origin).
Hint: Use the parametrization $\sin y = \frac{2t}{1+t^2}, \cos y = \frac{1-t^2}{1+t^2}$

Exercise 2 [Making yogurt (2 points)]

The grandmother prepares home made yogurt for her grandchildren. She first heats the milk up to 85° C to kill all the bacteria in the milk, then she removes it from the cooker to let it chill. The milk must be at a temperature of 45° C to add the milk enzymes. After 5 minutes the grandmother measure the temperature of the milk and it is still 65° C. If the temperature in the kitchen is 21° C, how long should she wait until the milk will reach the temperature of 45° C and she can add the milk enzymes to prepare the yogurt?

Hint: According to Newton's law of cooling, the rate at which the temperature of an object changes is proportional to the difference between its temperature and that of its surrounding.

– please turn over –

Exercise 3 [Parachutist (7 points [2, 1.5, 2.5, 1])]

Once a parachutist jumps from an airplane, there are two forces that determine his motion: the pull of the earth's gravity and the opposing force of air resistance. At high speeds (in the regime of turbulent flow, which corresponds to a high Reynold's number), the strength of the air resistance force can be expressed as $k_1 v^2$, where v is the speed with which the jumper descends and k_1 is a proportionality constant. Once the parachute opens, the descent speed decreases greatly. In this case, the air flow is called "laminar", and the Reynold's number is smaller than 1; the strength of the air resistance force is then given by $k_2 v$.

a) Write down and solve the differential equation for $v(t)$ for freefalling without a parachute. Use the initial condition $v(t = 0) = 0$. After a certain amount of time, the parachutist reaches a stationary velocity v_s. Find v_s and express your solution in terms of it.

   Hint: Use Newton's second law to write down the differential equation for $v(t)$ and try to bring it into the form $\frac{dx}{dt} = \alpha(1 - x^2)$. Then use separation of variables to solve the differential equation.

b) What is the stationary velocity v_{S1} for a freefalling parachutist of total mass 70 kg if $k_1 = 1/4$ kg/m? How long does it take to reach this velocity? How long does it take to reach 99% of the stationary velocity?

c) Write down the differential equation for $v(t)$ once the parachute has opened and solve it employing the method of variation of constants, the initial condition is $v(0) = v_{s_1}$. Find the stationary velocity v_{s_2} and express the solution in terms of v_{s_1} and v_{s_2}. Check whether your result for $v(t)$ satisfies your expectations for the limits $t \to 0$ and $t \to \infty$. What is the parachutist's stationary velocity v_{s_2} under the parachute if $k_2 = 110$ kg/s?

d) Write down and solve the differential equation for the parachutist's altitude $y(t)$ under the parachute (once it has opened), if $y(0) = 0$.