

PHY 117 HS2026

stats: students: 690

Quiz 1: 280

Quiz 2: 249

Quiz 3: 140

Quiz 4: 79

Today:

Review of centrifuge

"Pseudoforce"

Fluids in motion

Continuity equation

Bernoulli's equation

Toricelli's law

Venturi effect

Viscosity & resistance

Capillary action

Today: hydrodynamics

Week 5, Lecture 2

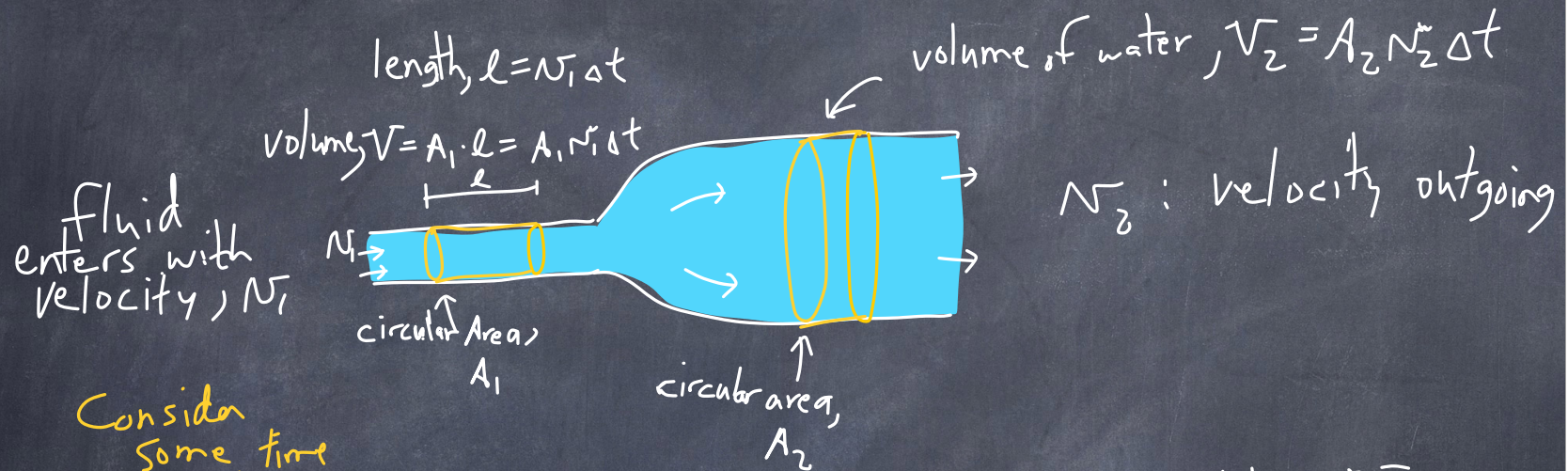
Oct. 14th, 2026

Prof. Ben Kilminster

Prof. Florencia Canelli

Fluids. In. Motion!

consider pipe with flowing fluid



Consider some time Δt

Since fluids are incompressible, $V_1 = V_2$

$$A_1 v_1 \Delta t = A_2 v_2 \Delta t$$

$$A_1 v_1 = A_2 v_2 = \text{constant} \quad (\text{tube area changes})$$

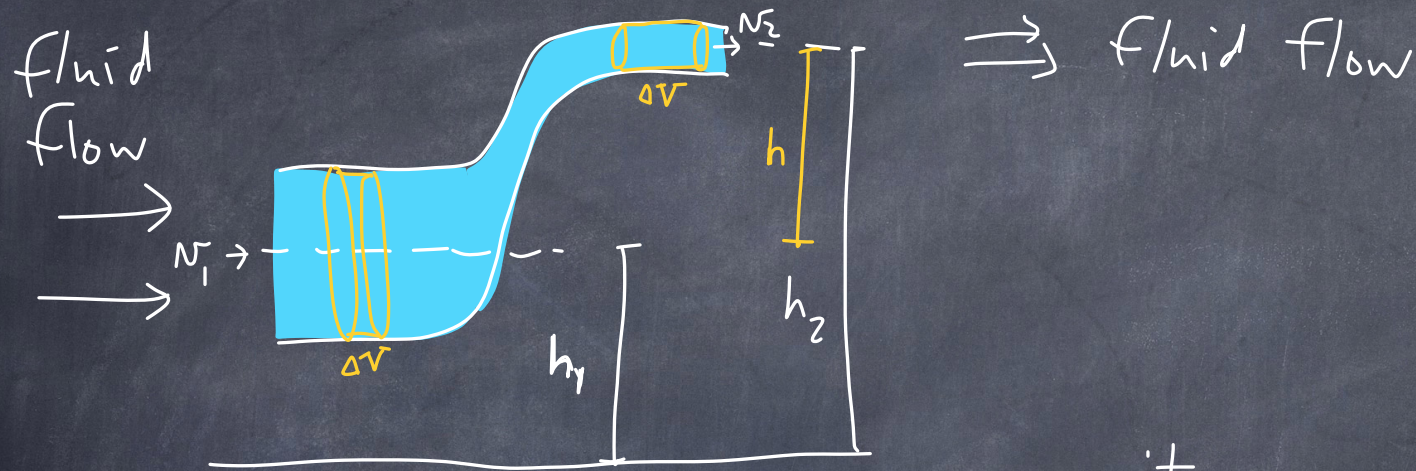
$$I_v \equiv vA : \left[\frac{\text{m}^2 \cdot \text{m}}{\text{s}} \right] \rightarrow \left[\frac{\text{m}^3}{\text{s}} \right] \quad \text{volume flow rate}$$

$$I_v = vA = \text{constant}$$

known as continuity equation

If A gets bigger, then v gets smaller

What if it changes height?



Fluid gains potential energy, mgh , it must lose kinetic energy (energy conservation)

Let's calculate: In some time Δt , some amount of fluid, Δm_{mass} , gets lifted by a height, h

$$\text{change in potential energy} = \boxed{\Delta U = \Delta mgh = \rho \Delta V gh} \quad \textcircled{1}$$

ΔV : volume of fluid being lifted.

$\Delta K = \text{change in kinetic energy} = \frac{1}{2} \Delta m v_2^2 - \frac{1}{2} \Delta m v_1^2$

$$\Delta K = \frac{1}{2} \rho \Delta V (v_2^2 - v_1^2) \quad (2)$$

The work-energy theorem states that

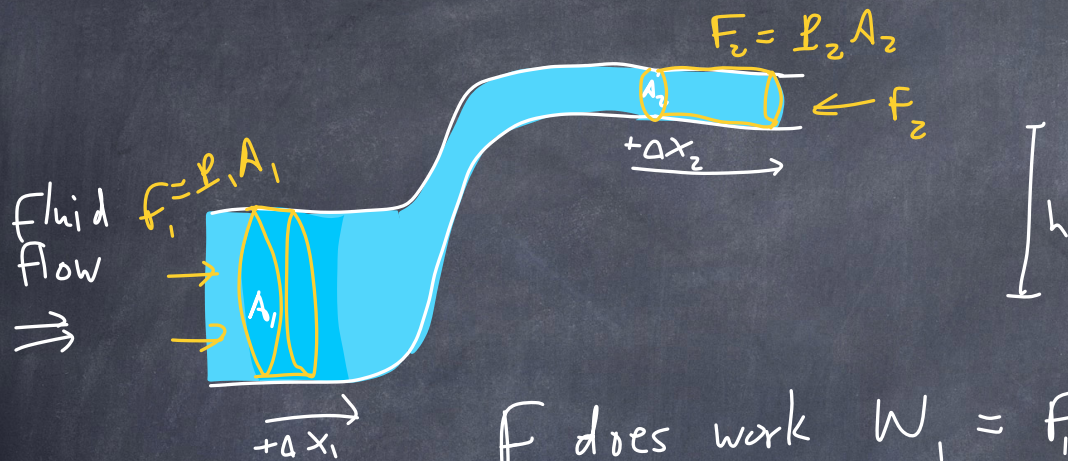
$$W_{\text{TOTAL}} = \Delta U + \Delta K \quad \text{work done by fluid.}$$

$$(1) + (2) \quad W_{\text{TOTAL}} = \rho \Delta V g h + \frac{1}{2} \rho \Delta V (v_2^2 - v_1^2) \quad (3)$$

we know that work comes from a force times a distance. The force, F_1 , comes from the pressure P_1 at bottom.

At the top, there is a force F_2 from the pressure at the top P_2 .

Pressure difference results in a force difference



Fluid is pushed with F_1 by fluid pressure to the right and pushed back by the fluid pressure at F_2

F_1 does work $W_1 = F_1 \Delta x_1 = P_1 A_1 \Delta x_1 = P_1 \underbrace{\Delta V_1}_{\text{Volume of cylinder}} = P_1 \Delta V_1$

F_2 does work $W_2 = -F_2 \Delta x_2 = -P_2 \Delta V_2$ ← (same volumes $\Delta V_1 = \Delta V_2$ in same time)

The total work done is $W_{\text{TOTAL}} = W_1 + W_2 = P_1 \Delta V - P_2 \Delta V$

$$W_{\text{TOTAL}} = (P_1 - P_2) \Delta V \quad (4)$$

we combine (3) + (4) : $(P_1 - P_2) \cancel{\Delta V} = \cancel{\rho \Delta V} g h + \frac{1}{2} \cancel{\rho \Delta V} (v_2^2 - v_1^2)$

$$P_1 - P_2 = \rho g (h_2 - h_1) + \frac{1}{2} \rho (V_2^2 - V_1^2)$$

we move our terms

$$\underbrace{P_1 + \rho g h_1 + \frac{1}{2} \rho V_1^2}_{\text{position 1}} = \underbrace{P_2 + \rho g h_2 + \frac{1}{2} \rho V_2^2}_{\text{position 2}}$$

In other words,

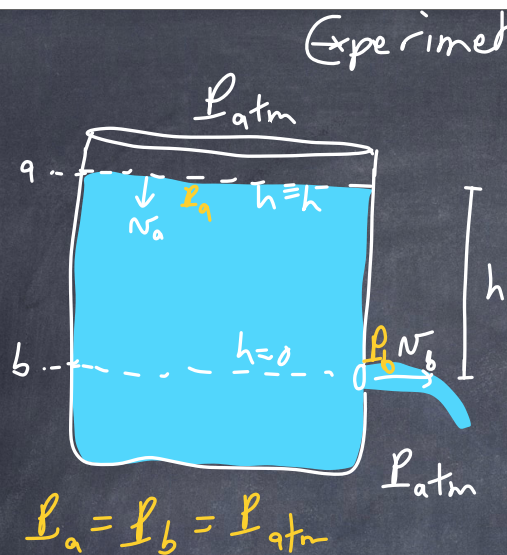
$$P + \rho g h + \frac{1}{2} \rho V^2 = \text{constant}$$

Bernoulli's
equation

- every term has units of pressure.
- neglects friction

→ This combination of quantities stays constant while height, area, velocity, U , K , P changes.

This is a statement of energy conservation.



$$P_a = P_b = P_{atm}$$

Approximation: at the top: $v_a = 0$, valid if the top area is much larger than the hole at point b.

Experiment: Tank with hole in it, open at top.

What is the velocity of water, v_b ?

We apply Bernoulli's equation to solve this at heights a + b.

Level a

$$P_a + \rho gh + \frac{1}{2} \rho v_a^2 =$$

$$\cancel{P_{atm}} + \rho gh + \frac{1}{2} \rho \cancel{v_a^2} =$$

Level b

$$\cancel{P_b} + \rho gh + \frac{1}{2} \rho v_b^2$$

$$\cancel{P_{atm}} + \cancel{\rho g \cdot 0} + \frac{1}{2} \rho v_b^2$$

$$\cancel{\rho gh} = \frac{1}{2} \cancel{\rho} v_b^2$$

$$v = \sqrt{2gh}$$

v_b of water coming out of hole.

Torricelli's Law

Analogy: Imagine an object falling a distance h due to gravity. Start with: $v^2 = v_0^2 + 2ax$

Here:

$$a = g$$

$$v_0 = 0$$

$$x = h$$

$$\text{so } v^2 = 2gh$$

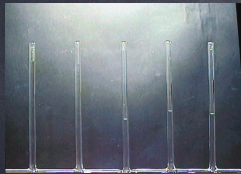
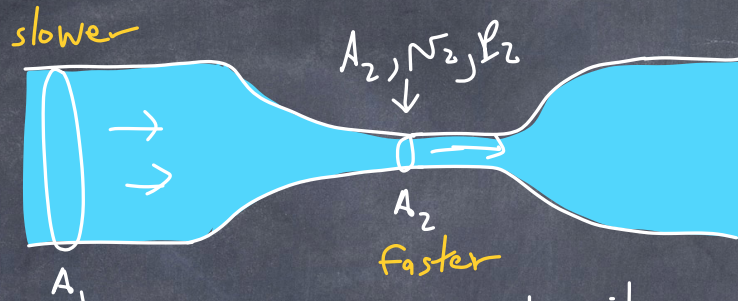
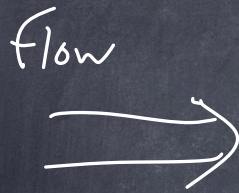
and

$$v = \sqrt{2gh}$$

same result!

Fluid moving in a pipe with changing area

(height doesn't change)



pressure observed by how much water it pushes up.

Continuity equation:

$$A_1 v_1 = A_2 v_2 \quad \text{so} \quad v_2 = v_1 \left(\frac{A_1}{A_2} \right)$$

v_2 is higher

Bernoulli's equation

$$P + \frac{1}{2} \rho v^2 + \rho g h = \text{constant}$$

(constant height)

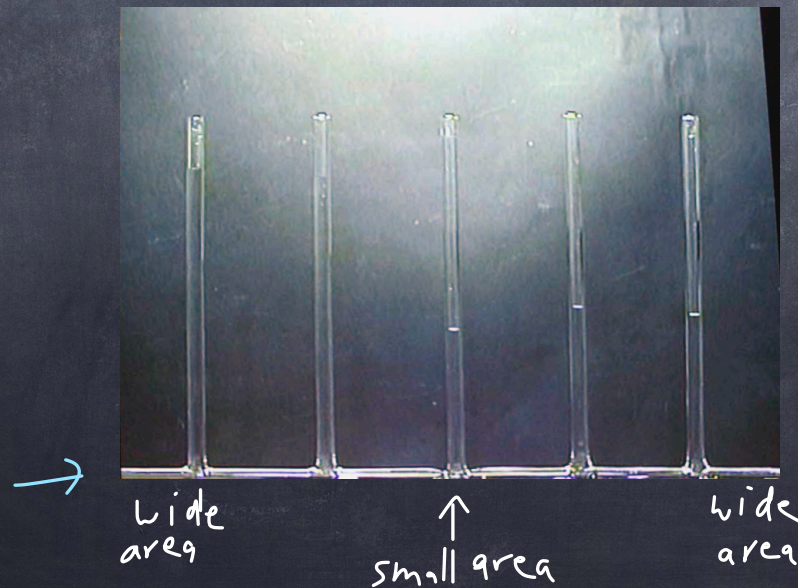
$$P_1 + \frac{1}{2} \rho v_1^2 = P_2 + \frac{1}{2} \rho v_2^2$$

Since $v_2 > v_1$, then it must be that

$$P_2 < P_1$$

Venturi effect: When the speed of a fluid increases, then the pressure gets smaller.
(neglecting viscosity)

But wait. Why does
the pressure not return
to the same value after
the pipe widens?

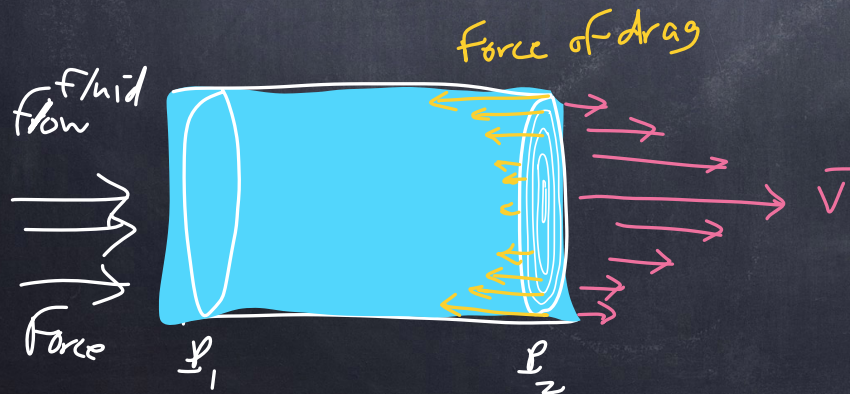


Current of fluid, I_v moves with "viscous" flow.

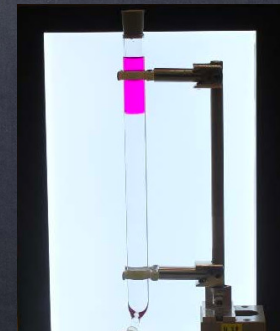
Bernoulli's equation states that the pressure is the same anywhere in a pipe at constant height + area.

$$P_1 \rightarrow \text{---} \rightarrow P_2 \quad P_1 = P_2$$

But, in practice we see a pressure drop. The pressure drop comes from a drag force from the surface of the pipe on the fluid, but also from each layer of the fluid on the next layer.



velocity is faster in the center of the pipe.





Pressure drop from P_1 to P_2

$$P_1 - P_2 = I_v R = (\bar{v} A) R$$

$\bar{v}$ ← average velocity of fluid
 A ← area of pipe
 R ← constant of resistance, depends on fluid + pipe.

Known as hydraulic

Ohm's law: $\Delta P = I_v R$

(like $\Delta V = IR$ from voltage = electric current * resistance)
 drop

The resistance for steady flow in a cylindrical pipe is

$$R = \frac{8\eta L}{\pi r^4}$$

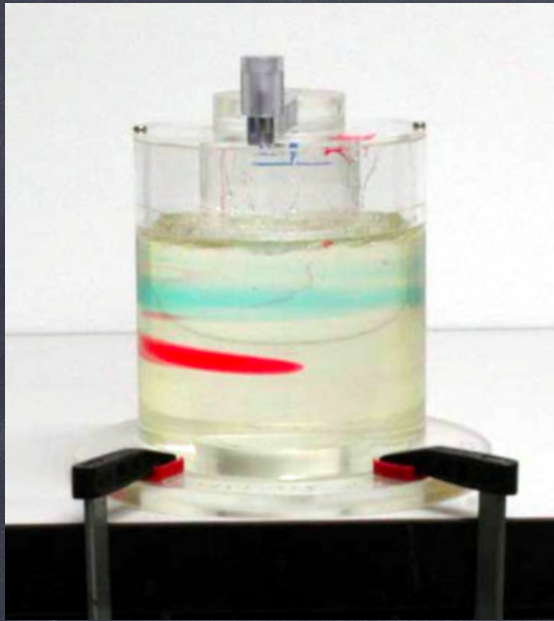
L : length of pipe
 r : radius of pipe
 η : coefficient of viscosity.

η : has units of $\left[\frac{\text{N}\cdot\text{s}}{\text{m}^2}\right] = [\text{Pa}\cdot\text{s}]$

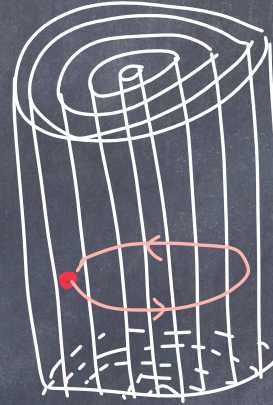
$1 \text{ Pa}\cdot\text{s} = 10 \text{ Poise}$

water: $\eta = 1 \times 10^{-3} \text{ Pa}\cdot\text{s}$

thick syrup: $\eta \sim 10-100 \text{ Pa}\cdot\text{s}$



Thick syrup: $10^3 - 10^4$
larger viscosity
than water



Experiment to understand R by studying $I_v = v \cdot A$.

$$\Delta P = I_v R = I_v \left(\frac{8 \eta L}{\pi r^4} \right)$$

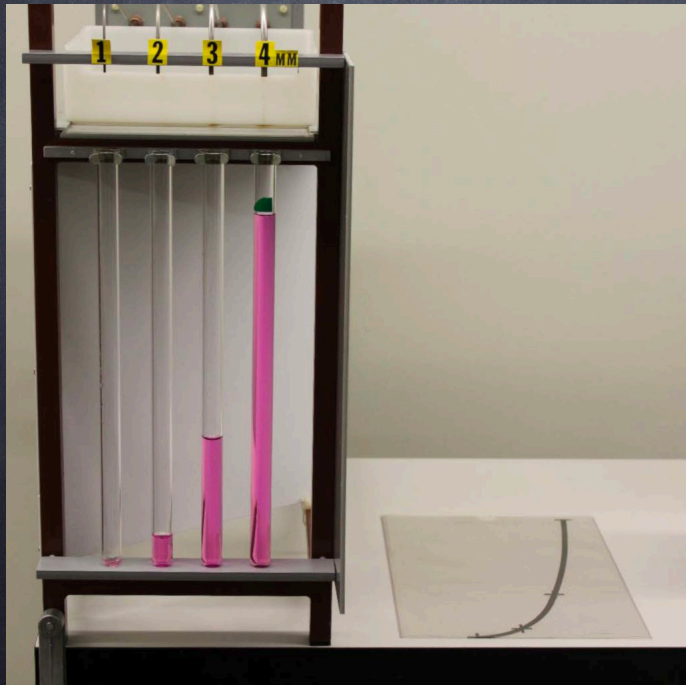
$$\text{So } I_v = \Delta P \left(\frac{\pi r^4}{8 \eta L} \right)$$

$$I_v \sim r^4$$

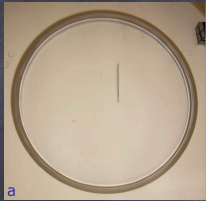
$$I_v \sim \frac{1}{L}$$

$$v \cdot A \sim \frac{1}{L}$$

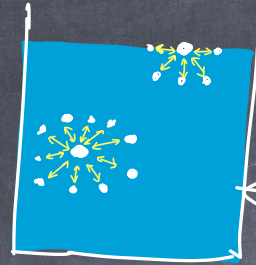
For two pipes of same radius (same area), $v \sim \frac{1}{L}$



surface tension

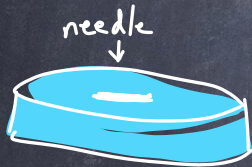


molecules are attracted to each other
a cohesive force



← Here we see an unbalanced force at top
Here, equal force from all directions

Surface tension is the unbalanced force from the cohesive force of molecules below, which makes the surface like a stretchable membrane.



$$\begin{aligned} L &= 4 \text{ cm} \\ \gamma &= 0.073 \text{ N} \\ F_{ST} &= 6 \times 10^{-3} \text{ N} \\ m &= 0.5 \text{ g} \rightarrow F_g = 5 \times 10^{-3} \text{ N} \end{aligned}$$

longer L , more F_{ST}



side view



There is a restoring force F_{ST} , proportional to the total length of the object.

$$F_{ST} = 2\gamma L$$

where γ is a constant that depends on the fluid + its temperature + gas in contact.

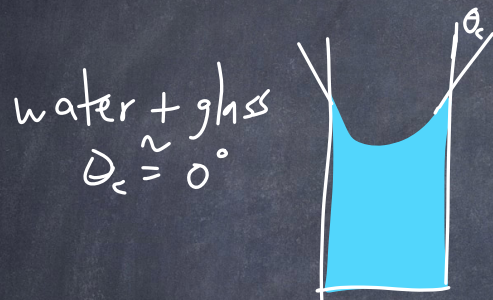
$$\gamma = 0.073 \frac{\text{N}}{\text{m}}$$

For water at room temperature in contact with air.

If we add detergent, γ decreases...

There are cohesive forces (of the liquid) and adhesive forces (between liquid and container).

One consequence of these is capillary action.

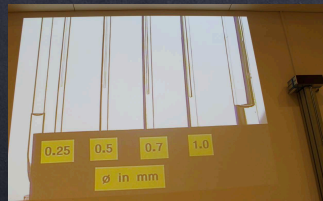


← If adhesive forces > cohesive forces, this shape is seen, known as the (concave) meniscus.
 θ_c : contact angle.

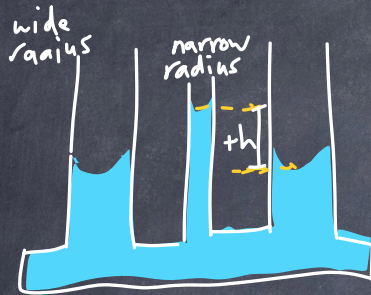


If adhesive forces < cohesive forces.
e.g. mercury + glass,
($\theta_c = 140^\circ$)

then we see a different (convex) shape (meniscus).



But why does this happen?



$$h = \frac{2\gamma \cos \theta_c}{\rho g} \left(\frac{1}{r_{\text{narrow}}} - \frac{1}{r_{\text{wide}}} \right)$$

change in liquid height
between 2 tubes of different
radii.

Note: $h \propto \frac{1}{r}$

$$h \propto \cos \theta_c \begin{cases} + \text{ for water + glass } (\theta_c = 0^\circ) \\ \quad \rightarrow \text{rise} \\ - \text{ for mercury + glass } (\theta_c = 140^\circ) \\ \quad \rightarrow \text{fall} \end{cases}$$

Section: Quiz 03

Energy

Question

Which of the following are types of energy ?

Unanswered	Right	Wrong	
☒	☐	☐	Potential
☒	☐	☐	Kinetic
☒	☐	☐	Momentum
☒	☐	☐	Work due to friction
☒	☐	☐	Force of a spring

Answers	Unanswered	Right	Wrong
Potential	2	134	4
Kinetic	2	136	2
Momentum	4	10	126
Work due to friction	3	63	14
Force of a spring	3		99

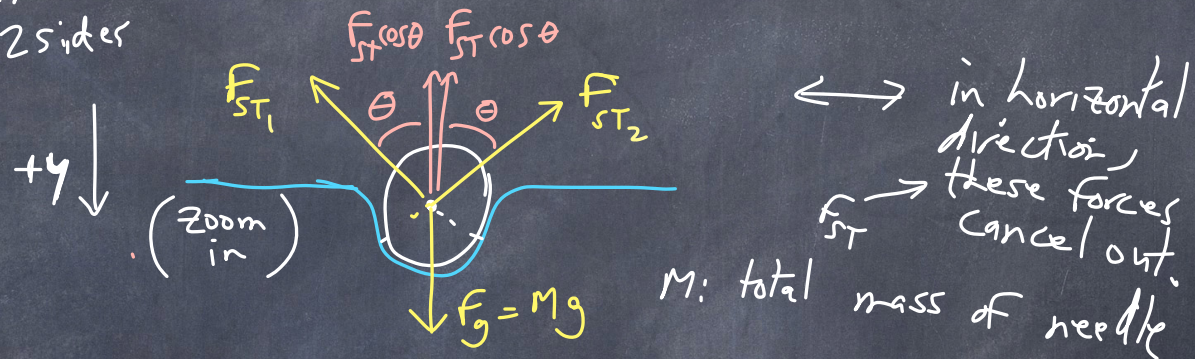
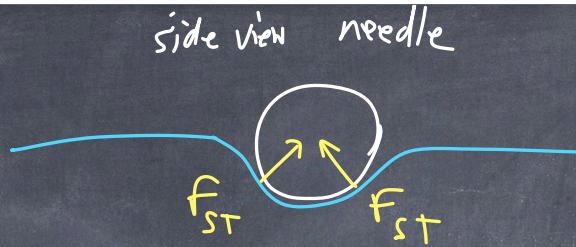
- work has units of energy
- but "work is energy transfer, not stored energy"
- Not a good question as worded ...

After this, there are a few derivations
for your information.

There is a surface force on both sides of the needle, so

$$F_{ST} = \gamma L = \gamma 2l$$

l , length of the needle
 2 : comes from 2 sides



In the y -direction, the total surface tension is

$$F_{ST_y} = F_{ST1} \cos \theta + F_{ST2} \cos \theta = 2 F_{ST} \cos \theta = 2 \gamma l \cos \theta$$

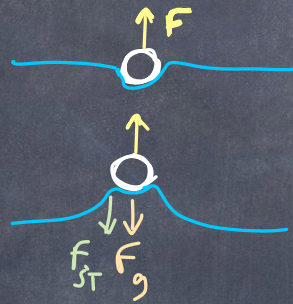
The needle floats as long as $F_{ST_y} > F_g$



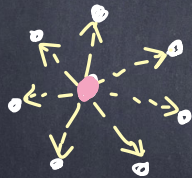
As M gets larger, θ decreases.

When $\theta = 0^\circ$, $\cos \theta = 1 \Rightarrow F_{ST} = 2\gamma l$
 The maximum mass allowed is when
 $Mg = 2\gamma l \cos \theta^\circ \quad m_{max} = 2\gamma l / g$

The force to lift the needle off the surface
is $F = mg + \gamma 2L$

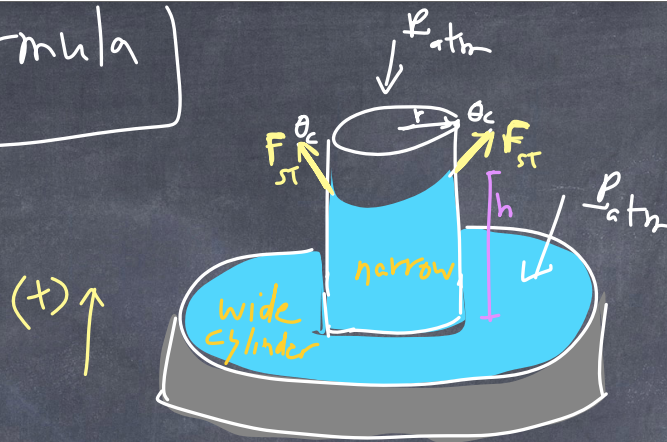


In this case, the surface
tension resists us pulling
the needle up, because
we are stretching the
fluid membrane upward.



cohesive force on one molecule
is coming from the surrounding molecules.

Derivation of formula for h.



Adhesive force pulls fluid upward

F_{ST} : adhesive force pulling upward

vertical direction $\left[\sum F = F_{ST} \cos \theta_c - mg = 0 \right.$

$$\gamma L \cos \theta_c = mg$$

$$\gamma 2\pi r \cos \theta_c = \rho V g$$

$$\gamma 2\pi r \cos \theta_c = \rho (\pi r^2 h) g$$

$$h = \frac{2\gamma \cos \theta_c}{\rho r g}$$

when $\theta_c < 90^\circ$, $\cos \theta_c > 0 \Rightarrow h$ is (+)
 when $\theta_c > 90^\circ$, $\cos \theta_c < 0 \Rightarrow h$ is (-)

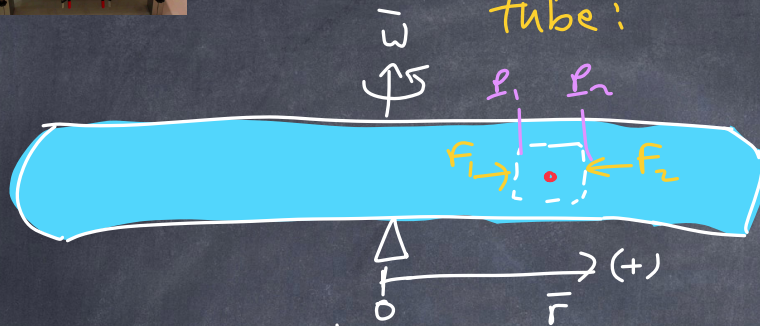
what is

L ? It's the length of contact between fluid & container
 $L = 2\pi r$

The height that the adhesive force raises a fluid in a cylinder compared to an infinitely wide cylinder ($r \rightarrow \infty$)



consider empty spinning tube:

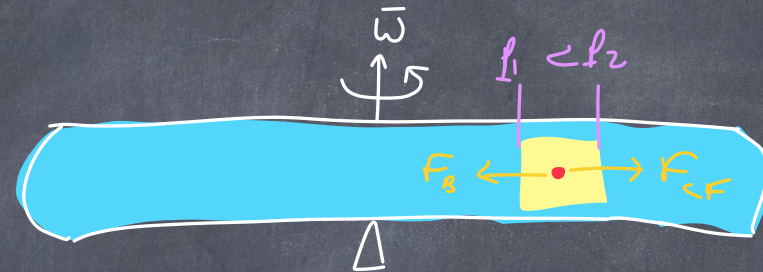


The buoyant force comes from the centripetal force of the mass of fluid that is displaced

$$F_B = F_1 - F_2$$

$$F_B = m_e \frac{v^2}{r} = m_e \omega^2 r (-\hat{r})$$

F_B is bigger at large r



An object in the centrifuge feels a buoyant force and an apparent force the "centrifugal pseudo force" outward

$$F_{cf} = m_o \omega^2 r (\hat{r})$$

$$\Sigma F = F_c - F_B = m_o \omega^2 r - m_e \omega^2 r = V \rho_o \omega^2 r - V \rho_e \omega^2 r = V \omega^2 r (\rho_o - \rho_e) \hat{r}$$

If $\rho_o > \rho_e$, the force is outward in $(+\hat{r})$ direction

If $\rho_o < \rho_e$, force is inward $(-\hat{r})$

why did I call it a pseudoforce?

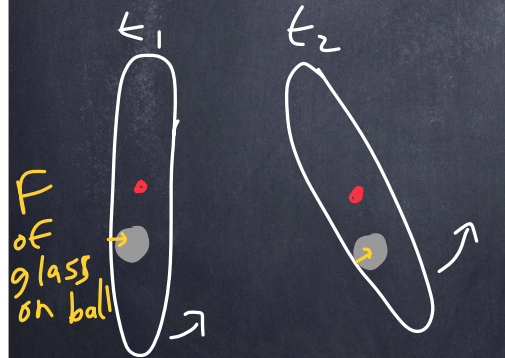
Because we are viewing the object in its reference frame, which is accelerating, we see this apparent pseudoforce.

In the rest frame of the tube

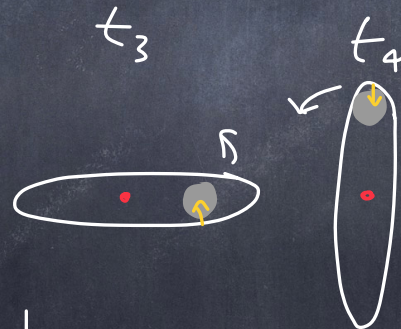


As seen by camera above tube rotating with tube.

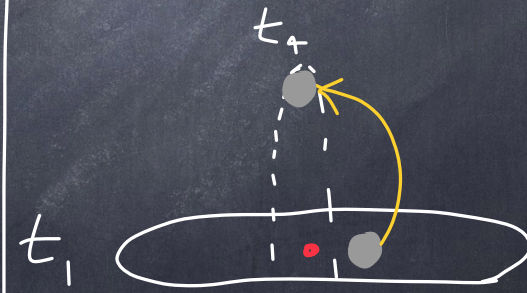
But From above see that, in a fixed frame that is not rotating, we see that the movement is caused by centripetal force. Movement is:



Metal ball slides outward



After t_4 , ball now moves in a circle with wall providing $F_{\text{centripetal}}$



due to force of glass on ball