High-precision scattering amplitudes from automated tools

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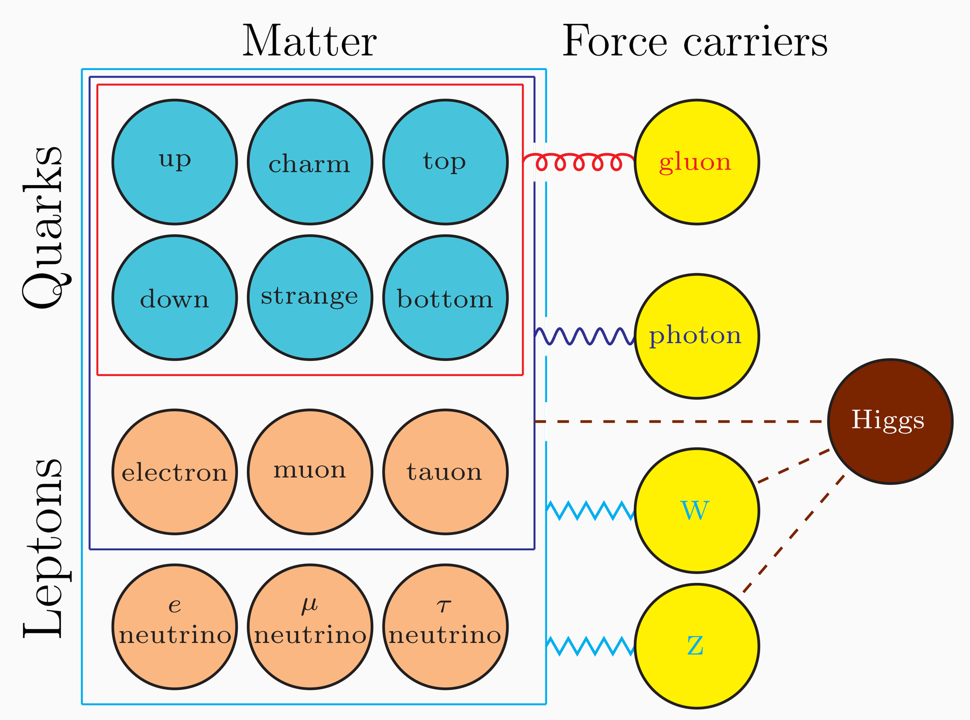
The Standard Model of Particle Physics

The Standard Model describes the known forms of matter and forces with only 17 elementary particles.



But some **big puzzles are beyond the Standard Model**

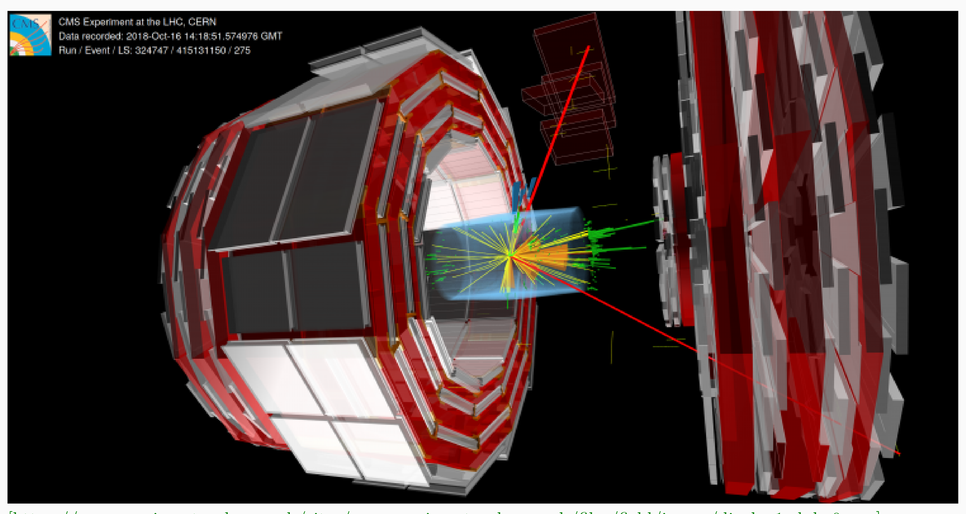
- What is **Dark Matter**?
- Why more **Matter** than **Antimatter**?
- **Quantum nature of gravity?**
- ...

Testing the Standard Model at the LHC in search of new particles

High-energy collisions of protons (14 TeV) produce **huge amount of particles**

→ measured in highly sophisticated detectors (ATLAS, CMS, etc.)

Experimental data compared to **theoretical simulations**



[https://cmsexperiment.web.cern.ch/sites/cmsexperiment.web.cern.ch/files/field/image/display1_hl_0.png]

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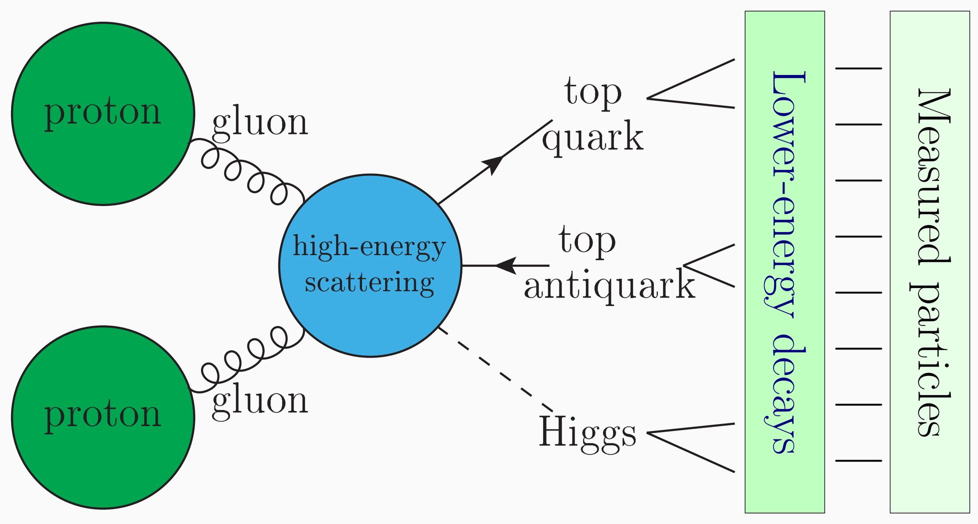


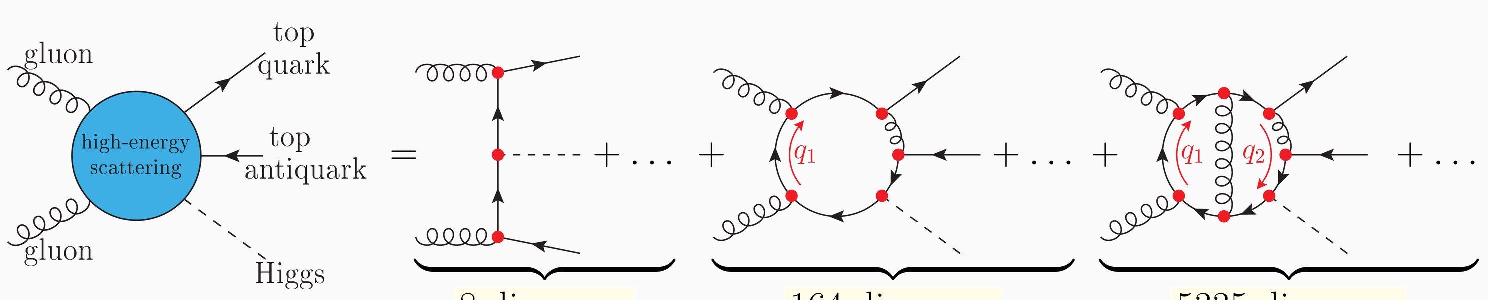
The search for small deviations from the Standard Model makes experimental data and theoretical simulations for a very wide range of processes at the highest precision crucial.

Scattering amplitudes in perturbation theory

Elementary building blocks: Feynman rules for propagation and interaction

$$\text{(gluon)} = \frac{-ig^{\mu\nu}}{p^2}, \quad \text{(fermion)} = \frac{-i\not{p}}{p^2-m^2}, \quad \text{(vertex)} \propto g_s, \quad \text{(4-gluon vertex)} \propto g_s, \quad \text{etc.}$$

Compute amplitude of a scattering process from sum of Feynman diagrams, e.g.



8 diagrams	164 diagrams	5335 diagrams
↓	↓	↓
Leading order (LO)	**Next-to-leading order (NLO)**	**NNLO**
↓	↓	↓
~ 100% **uncertainty**	~ 10% **uncertainty**	~ 1% **uncertainty**

- Expansion in coupling constants, e.g. g_s. **Higher precision** $\widehat{=}$ **more loops**
- Each loop: Integration over D-dimensional energy-momentum vector q_i. Integrals are computed in $D = 4-2\varepsilon$ dimensions, where ε regularizes divergences.

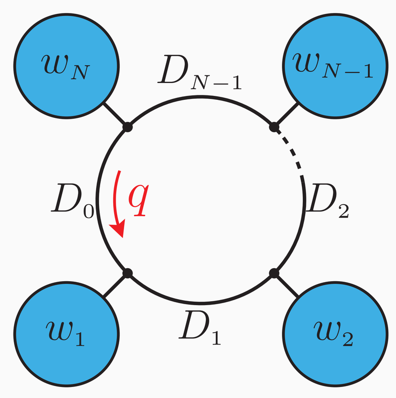
Complexity grows strongly with number of loops and external particles limiting analytical calculations

⇒ **Automated numerical tools** enable studies of many processes @ NLO in a short time, e.g. OpenLoops 2 [Buccioni, Lang, Lindert, Maierhöfer, Pozzorini, Zhang, M.Z. 2019].

- Numerical calculations are performed in integer dimensions.
  ⇒ Split loop integral numerators into 4-dim part $\mathcal{N}$ and $(D-4)$-dim part $\tilde{\mathcal{N}}$.

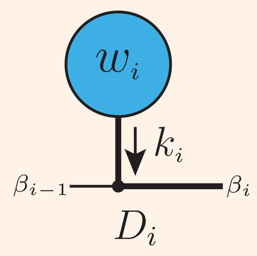
OpenLoops – automated amplitude calculation @ NLO

Exploit **factorisation** of one-loop diagrams into **universal building blocks**



$$\sim \int \mathrm{d}^D q \, \frac{S_1(q)\dots S_N(q)}{D_0(q)\cdots D_{N-1}(q)}$$

Numerator segments $S_i(q) =$



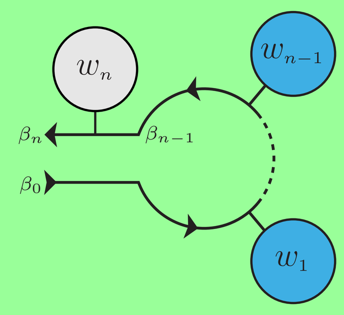
(with external subtrees w_i)

Denominators $D_i(q) = (q+p_i)^2 - m_i^2$

$$\sim \sum_{\mu_1=0}^{3}\cdots\sum_{\mu_N=0}^{3} \underbrace{\mathcal{N}_{\mu_1\cdots\mu_N}}_{\text{4-dim coefficient}} \underbrace{\int \mathrm{d}^D q \, \frac{q^{\mu_1}\cdots q^{\mu_N}}{D_0(q)\cdots D_{N-1}(q)}}_{\text{tensor integral}} + \underbrace{\int \mathrm{d}^D q \, \frac{\tilde{\mathcal{N}}}{D_0(q)\cdots D_{N-1}(q)}}_{(D-4)\text{-dim numerator}}$$

Recursive construction of 4-dim coefficients from the segments of the cut-opened loop [Cascioli, Maierhöfer, Pozzorini 2011; Buccioni, Lang, Lindert, Maierhöfer, Pozzorini, Zhang, M.Z. 2019]

$$\mathcal{N}_n(q) = \mathcal{N}_{n-1}(q) \cdot S_n(q) =$$



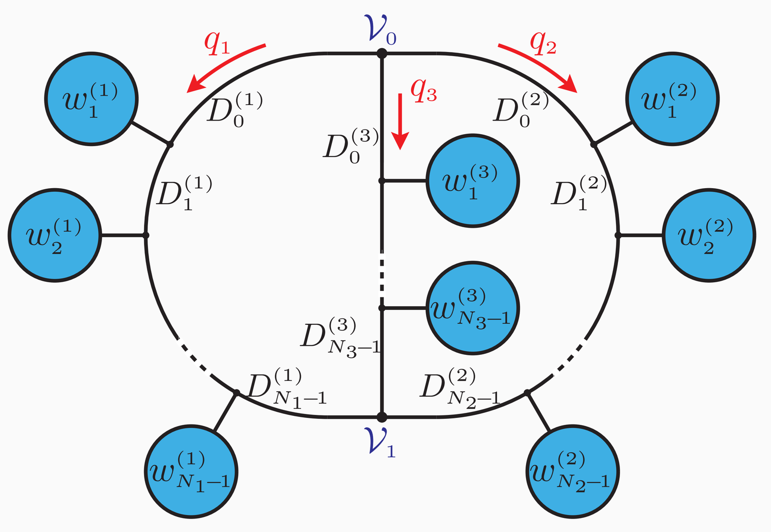
Tensor integrals: On-the-fly reduction [Buccioni, Pozzorini, M.Z. 2017], external tools [Denner, Dittmaier, Hofer; van Hameren]

Restoration of $(D-4)$-dim numerator parts together with renormalization procedure **R** through **universal rational counterterms** [Ossola, Papadopoulos, Pittau 2008]

$$\mathbf{R}\Big[\text{(diagram)}\Big]_{D\text{-dim}} = \Big[\text{(diagram)} + \text{(counterterm)} \times \Big(\underbrace{\delta Z_{1,\Gamma}}_{\text{subtract divergence}} + \underbrace{\delta\mathcal{R}_{1,\Gamma}}_{\text{restore } \tilde{\mathcal{N}}\text{-term}} \Big)\Big]_{4\text{-dim}}$$

⇒ **Completely general and highly efficient algorithm and tool**

Our approach @ NNLO



$$\sim \sum_{\mu_1=0}^{3}\cdots\sum_{\mu_r=0}^{3}\sum_{\nu_1=0}^{3}\cdots\sum_{\nu_s=0}^{3} \mathcal{N}_{\mu_1\cdots\mu_r\nu_1\cdots\nu_s}$$

$$\times \int \mathrm{d}^D q_1 \int \mathrm{d}^D q_2 \, \frac{q_1^{\mu_1}\cdots q_1^{\mu_r} q_2^{\nu_1}\cdots q_2^{\nu_s}}{\prod_{i,j} D_j^{(i)}(q_i)}$$

$$+ \int \mathrm{d}^D q_1 \int \mathrm{d}^D q_2 \, \frac{\tilde{\mathcal{N}}}{\prod_{i,j} D_j^{(i)}(q_i)} \Bigg|_{q_3=-(q_1+q_2)}$$

Numerical construction of 4-dim tensor coefficients

Exploit factorisation into universal building blocks $\mathcal{K}_n \in \{S_n^{(i)}(q_i), \mathcal{V}_{0,1}(q_1,q_2)\}$ in a new and completely general algorithm [Pozzorini, N.S., M.Z. 2022] with recursion steps $\mathcal{N}_n(q_1,q_2) = \mathcal{N}_{n-1}(q_1,q_2)\cdot\mathcal{K}_n$

Highly efficient and fully implemented for QED and QCD corrections to the Standard Model ✓

Reduction of tensor integrals ⟶ large set of scalar integrals $\mathcal{I}_k$ ⟶ small set of master integrals $\mathcal{M}_l$

$$\int \mathrm{d}^D q_1 \int \mathrm{d}^D q_2 \, \frac{q_1^{\mu_1}\cdots q_1^{\mu_r} q_2^{\nu_1}\cdots q_2^{\nu_s}}{\prod_{i,j} D_j^{(i)}(q_i)} \longrightarrow \sum_k B_k^{\mu_1\cdots\mu_r\nu_1\cdots\nu_s}\mathcal{I}_k \longrightarrow \sum_{k,l} B_k^{\mu_1\cdots\mu_r\nu_1\cdots\nu_s} C_{kl}\mathcal{M}_l$$

- Currently the bottle neck of NNLO automation ⇒ powerful new methods to be developed
- Step $\mathcal{I}_k \to \mathcal{M}_l$ uses integration-by-parts relations [Chetyrkin, Tkachov 1981], e.g. implemented in Kira [Maierhöfer, Usovitsch, Uwer 2017; Klappert, F.L., Maierhöfer, Usovitsch 2020]

Restoration of $(D-4)$-dim numerator parts via small set of **universal two-loop rational terms** [Lang, Pozzorini, Zhang, M.Z. 2020, 2020, 2021] stemming from the interplay of $\tilde{\mathcal{N}}$ with ultraviolet (UV) or infrared (IR) divergences.

$$\mathbf{R}\Big[\text{(diagram)}\Big]_{D\text{-dim}} = \Big[\text{(diagram)} + \text{(diagram)} \times \Big(\underbrace{\delta Z_{1,\gamma} + \delta\tilde{Z}_{1,\gamma}}_{\text{subtract subdivergences}} + \underbrace{\delta\mathcal{R}_{1,\gamma}}_{\text{restore } \tilde{\mathcal{N}}\text{-terms from subdiagrams}} \Big) + \text{(counterterm)} \times \Big(\underbrace{\delta Z_{2,\Gamma}}_{\text{subtract local divergence}} + \underbrace{\delta\mathcal{R}_{2,\Gamma}}_{\text{restore remaining } \tilde{\mathcal{N}}\text{-term}} \Big)\Big]_{4\text{-dim}}$$

Rational terms of UV origin: Fully computed for QED and QCD corrections; IR origin: Currently under investigation (✓)

Conclusions

Automated tools @ NLO have played a key role in the success of the LHC. Similar tools @ NNLO are highly desirable to meet the precision demands of the next years. While there has been huge progress in this field, new methods still need to be developed by our group and others to reach this goal.