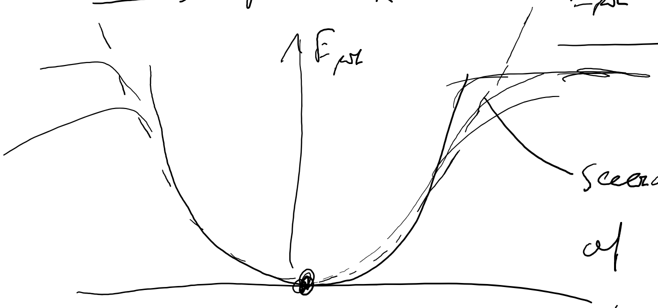
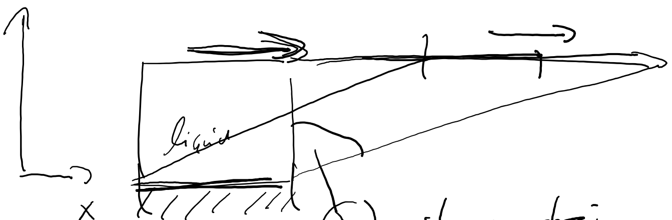
Elastic response

$\sigma = E\varepsilon \longrightarrow F = -kx \longrightarrow E_{pot} = \frac{k}{2}x^2$

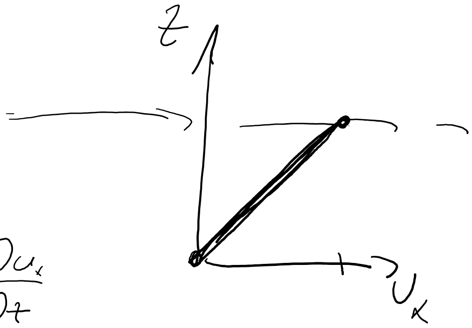


second order Taylor expansion of E_{pot} is the first non-trivial order

liquid behavior



γ = shear strain $= \frac{\partial u_x}{\partial z}$



$$\boxed{\eta \frac{d\gamma}{dt} = \tau}$$

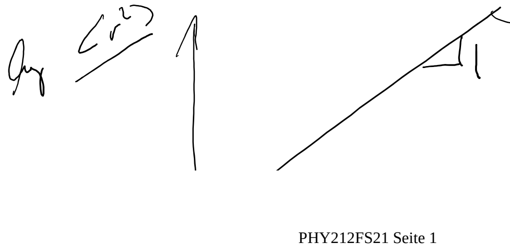
viscosity

compare to

$$\tau = G\gamma \longrightarrow \text{elastic response}$$
$$= \eta\dot{\gamma} \longrightarrow \text{liquid response}$$

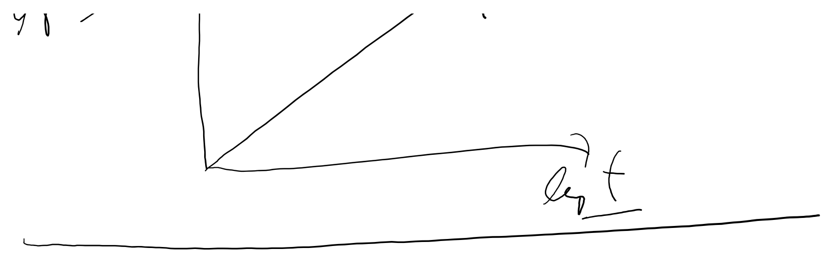
$$\frac{d\gamma}{dt} = \frac{d}{dt}\left(\frac{\partial u_x}{\partial z}\right) = \frac{d^2 u_x}{dt\,dz} = \frac{d^2 u_x}{dz\,dt} = \frac{d}{dz}\left(\frac{du_x}{dt}\right) = \frac{d}{dz} v_x = \frac{dv_x}{dz}$$

$$F = \eta \frac{dv_x}{dz} \cdot A$$



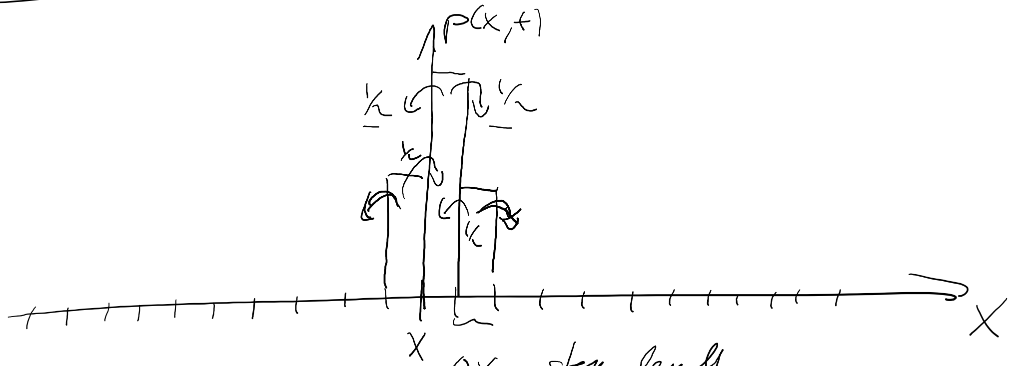
$\rightarrow$ Diffusion / liquid

$\langle r^2 \rangle \sim t$



$\langle r^2 \rangle \sim t$

Diffusion as a random walk



Δx step length

$$p(x, t+\Delta t) = \big(p(x-\Delta x, t) + p(x+\Delta x, t)\big)\frac{1}{2} \qquad \text{random walk}$$

time step

$$\underbrace{p(x, t+\Delta t) - p(x,t)}_{\downarrow} = \frac{1}{2}\big(p(x-\Delta x, t) + p(x+\Delta x, t)\big) - p(x,t)$$

$$\cancel{p(x,t)} + \frac{\partial}{\partial t}p(x,t)\cdot\Delta t - \cancel{p(x,t)} = \frac{1}{2}\Big[\underbrace{-\underbrace{[p(x,t) - p(x-\Delta x,t)]}_{\frac{\partial}{\partial x}p(x,t)\cdot\Delta x} + \underbrace{[p(x+\Delta x,t) - p(x,t)]}_{\frac{\partial}{\partial x}p(x+\Delta x,t)\cdot\Delta x}}_{\frac{\partial^2}{\partial x^2}p(x,t)\cdot\Delta x^2}\Big]$$

$$\boxed{\frac{\partial p(x,t)}{\partial t} = \boxed{\frac{\Delta x^2}{2\Delta t}}\,\frac{\partial^2}{\partial x^2}p(x,t)} \qquad \text{Diffusion equation}$$

$D \qquad \Rightarrow \langle x^2 \rangle = 2D(\Delta t)$

$$D \longrightarrow \langle \Delta x^2 \rangle = 2D \langle \Delta t \rangle$$

$$\langle \Delta t \rangle \downarrow t$$

$$\langle r^2 \rangle = 6Dt$$

3-Dimension: $$\boxed{\frac{\partial}{\partial t} \rho(\vec{r},t) = D \vec{\nabla}^2 \rho(\vec{r},t)}$$