MMP I
Exercise Sheet 8

HS 21
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https://www.physik.uzh.ch/en/teaching/PHY312

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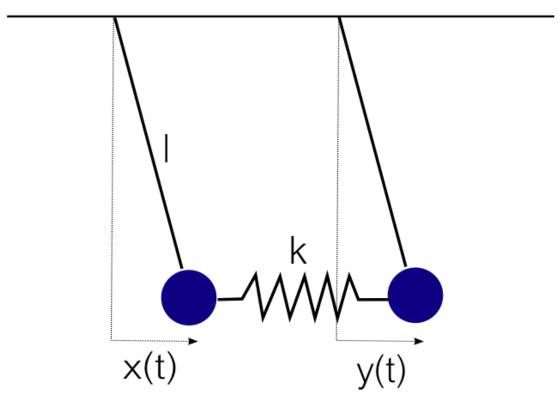
Exercise 1 [Bound oscillators (3 points)]

The motion of two bound pendula of the same mass m and length l is described by the following equations (g is the gravitational acceleration, k is the spring constant):

$$m\ddot{x} = -\alpha x - k(x - y)$$
$$m\ddot{y} = -\alpha y - k(y - x),$$

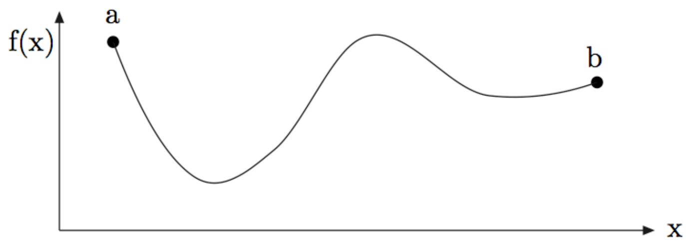
where $\alpha = mg/l$. The origin of the coordinate system is chosen so that $y = x = 0$ corresponds to the rest position. These equations are linearized and only valid for small oscillations.



Find the fundamental system of solutions. Discuss the motion of the oscillators for the following initial conditions: $x(0) = y(0) = \dot{y}(0) = 0$, $\dot{x}(0) = 1$.

Exercise 2 [Calculus of variations (6 points)]

Consider differentiable function $f(x) : [a, b] \to \mathbb{R}_+$ with $f(a) = \alpha$ and $f(b) = \beta$.



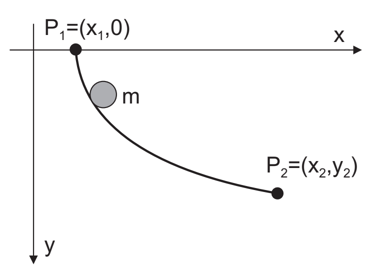
Consider the body, that is created by rotating this curve around the x-axis. Find the function f, for which the surface of the created body is a minimum.
Hint: Consider the square of the differential equation and remember that $\cosh^2(x) - \sinh^2(x) = 1$.

Exercise 3 [Brachistochrone problem (5 points)]

A bead with mass m and zero initial velocity moves on a frictionless wire placed in a (homogeneous) gravitational field from $P_1 = (x_1, 0)$ to $P_2 = (x_2, y_2)$. Find the curve $y(x)$ which minimizes the time T, i.e. for which the integral

$$T[y] = \int_{x_1}^{x_2} F[y(x), y'(x), x]dx$$

is extremal. Which is the shape of the wire? Use your favored plotting program to plot the resulting curve.



Hint: Express T as a path-integral over the inverse velocity which can be found by looking at the total energy of the wire (which is conserved). Use the substitution $y = \kappa \sin^2(\frac{\phi}{2})$ to solve the integral (κ is not a new constant but should be chosen conveniently as a function of the constants already introduced).