



MMP I

Tutorial 2

HS 2019
Prof. M. Grazzini

S. Devoto, M. Höfer, J. Yook
<https://www.physik.uzh.ch/en/teaching/PHY312/HS2019.html>

Issued: 24.09.2019
Due: 01.10.2019 10:15

Exercise 1: Fourier analysis and minimization (6 Pts.)

f is a 2π periodical function defined as

$$f(x) = \pi - |x|, \quad -\pi \leq x \leq \pi. \quad (1.1)$$

- Find the Fourier coefficients for f .
- Find parameters a and b for which the integral

$$\int_{-\pi}^{\pi} dx (f(x) - a \cos(3x) - b \sin(4x))^2$$

is minimal.

Hint: Use the analogy between the Fourier series expansion of the function $f(x)$ and its expansion over the orthonormal basis given by $\cos(nx)$ and $\sin(nx)$ for $n = 0, 1, \dots$

Exercise 2: Convolution theorem (6 Pts.) Let $f, g : x \rightarrow \mathbb{R}$ be continuous 2π periodical functions which are integrable. The convolution of those two functions is defined as

$$(f * g)(x) = \frac{1}{2\pi} \int_0^{2\pi} f(y)g(x-y)dy. \quad (2.1)$$

Prove the following identities:

- Commutative Property: $f * g = g * f$
- Associativity: $(f * g) * h = f * (g * h)$
- The Fourier coefficients of $f * g$ are given by $c_n(f * g) = c_n(f) \cdot c_n(g)$.

Exercise 3: Fourier Series (4 Pts.) Consider an odd function f , periodic with 2π , with $f(x) > 0$ for $x \in (0, \pi)$. Show that for the Fourier coefficient b_n of f the following is valid:

$$|b_n| \leq nb_1.$$

Hint: Show that

$$|\sin(nx)| \leq n|\sin(x)| \quad \forall x.$$