# PHY117 HS2026

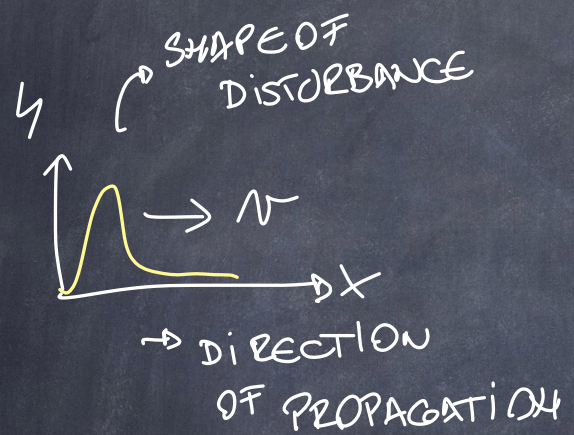
Week 11, Lecture 2
Nov. 25th, 2026
Prof. Ben Kilminster
Prof. Florencia Canelli

Today:
waves
standing waves

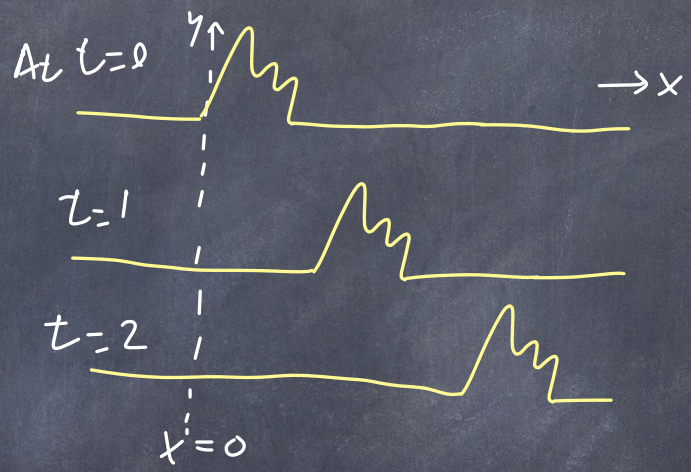
# WAVES

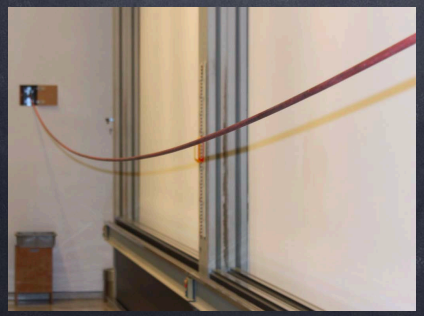
DISTURBANCE THAT PROPAGATE ALONG SOME MEDIUM

↓
WATER
AIR
ETC




TRANSVERSE WAVE: DISTURBANCE ⊥ PROPAGATION



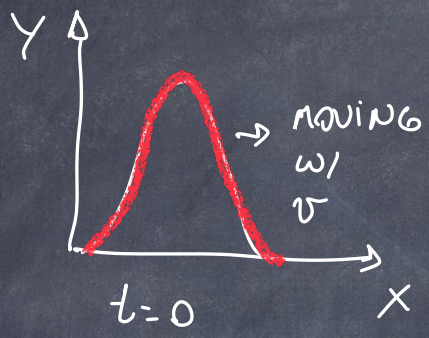
SHAPE OF THE WAVE

$$Y = f(x)$$

WE SEE SAME SHAPE BUT MOVED

WE DEFINE 2 COORDINATE SYSTEM (OR REST FRAMES)

S NOT MOVING







ANOTHER REST FRAME

S' MOVING WITH THE SAME VELOCITY ~~AS~~ WAVE, v







TRANSLATE THE SYSTEMS

$$y' = y$$
$$x' = x - vt$$

GALILEAN TRANSFORM

WE CAN DESCRIBE THE WAVE IN THE S' FRAME BY TRANSFORMING ITS POSITION IN THE S FRAME

FOR A GIVEN TYPE OF WAVE, ITS SPEED SAT BY



THE MEDIUM

↓

DISTURBANCE IN WATER

A WAVE MOVING TO THE RIGHT HAS A FORM LIKE

$$y(x,t) = \underbrace{f(x - vt)}_{\text{ANY SHAPE}}$$

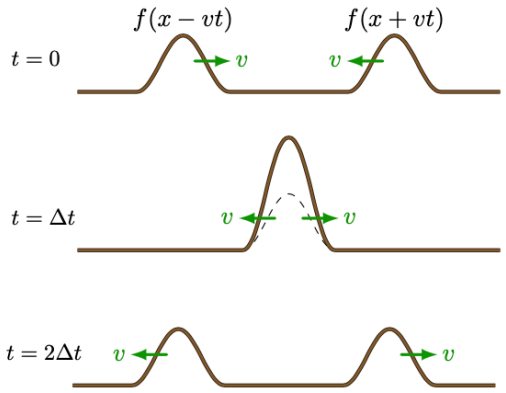
MOVING TO THE LEFT

$$y(x,t) = f(x + vt)$$

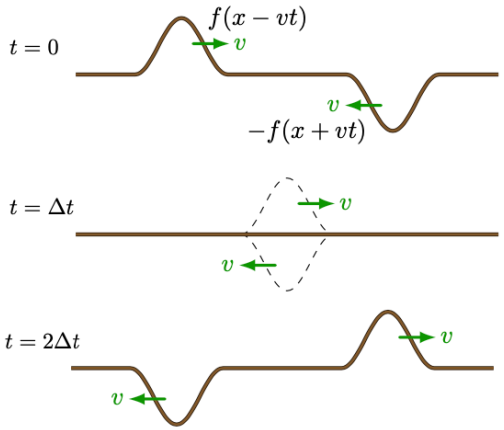
IF WE HAVE 2 WAVES (ONE TO LEFT AND ONE TO RIGHT) WE CAN ADD THEM LINEARLY:

$$y(x,t) = y(x - vt) + y(x + vt)$$

SUPERPOSITION : ADDITION OF WAVES

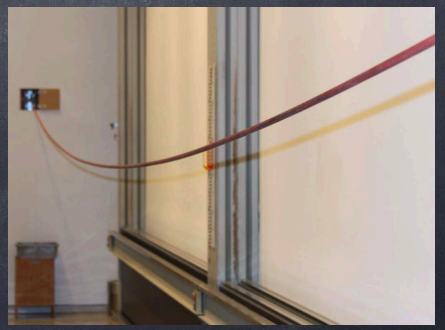



**(a)** Constructive interference happens when two oppositely waves meet on a string.




**(b)** Destructive interference. If the waves are the same but for a sign, they cancel completely.

**Figure 13.5:** Superposition between two oppositely travelling waves in the same medium is a simple linear sum.

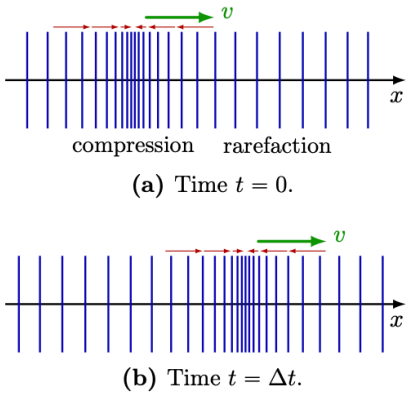


TRANSVERSE WAVES




DISTURBANCE (AMPLITUDE)
⊥ TO DIRECTION OF MOTION

LONGITUDINAL WAVES

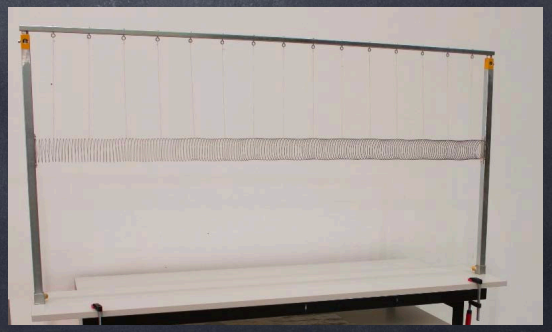



**(a)** Time $t = 0$.

**(b)** Time $t = \Delta t$.

**Figure 13.10:** A traveling longitudinal wave is when the distortion happens along the direction of propagation, here shown as a local displacement.

DISTORBANCE (AMPLITUDE)
|| TO DIRECTION OF MOTION



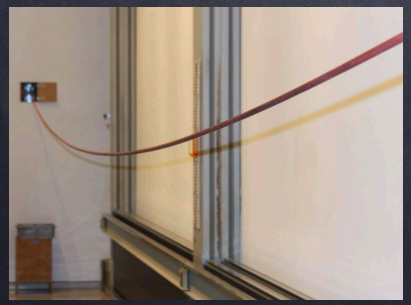
# SPEED OF A WAVE ON A STRING

VELOCITY OF DISTURBANCE $v = \sqrt{\frac{T}{\mu}}$

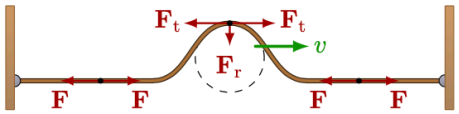
$T$ = TENSION OF STRING

$\mu$ = MASS PER LENGTH

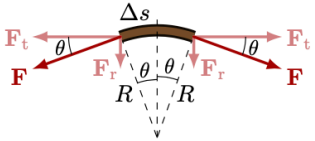
DERIVATION OF THIS FORMULA IN SCRIPT



152 CHAPTER 13. WAVES




**(a)** Forces on a string. All across the string, there is a constant tension **F**.




**(b)** Small segment of length $\Delta s$ experiences a tension **F** on either side.

**Figure 13.3:** The tension in a string is increased due to a disturbance.

GENERAL FORM FOR A WAVE

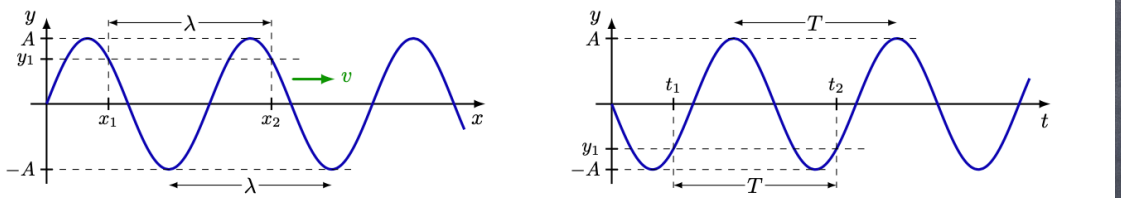
$y(x,t) = f(x - vt)$

A TYPE OF WAVE $y(x,t) = A\sin(Kx - \omega t)$ ①

WAVEFUNCTIONS

IN RADIANS, WE NEED AN ANGLE (NOT x OR t)

AT $t=0$ At $x=0$




**(a)** Whole wave in space at time $t = 0$, given by $y(x,0) = A\sin(kx)$.

**(b)** Local disturbance at position $x = 0$, given by $y(0,t) = -A\sin(\omega t)$.

**Figure 13.2:** A space and time slice of a travelling sine wave $y(x,t) = A\sin(kx - \omega t)$.

WHAT IS K? WE SEE THAT THE WAVE REPEATS EVERY WAVELENGTH $\lambda$

FOR $x_2 = x_1 + h$ $y(x_1,0) = y(x_2,0)$

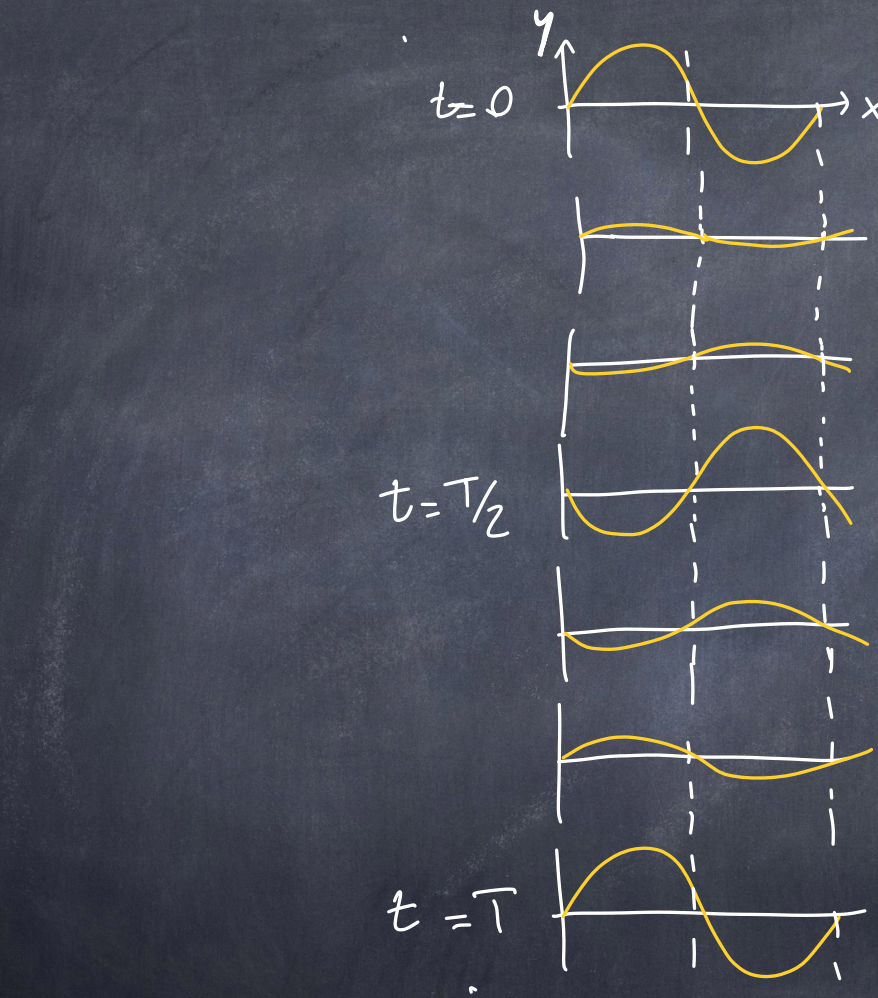
$y(x_1) = A\sin(Kx_1) = y(x_2) = A\sin(Kx_2)$

$A\sin(Kx_1) = A\sin(Kx_1 + Kh)$

ONLY IF $Kh = n(2\pi)$ $n = 0, \pm 1, \pm 2, \ldots$

FOR $n=1 \Rightarrow$ $K = \frac{2\pi}{\lambda}$ WAVE NUMBER

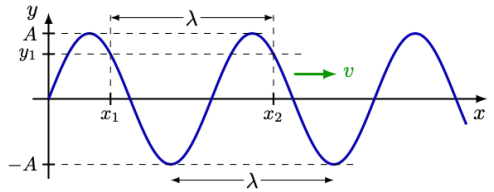
IN TIME, THE SHAPE OF THE WAVE CHANGES IN THE SAME PLACE



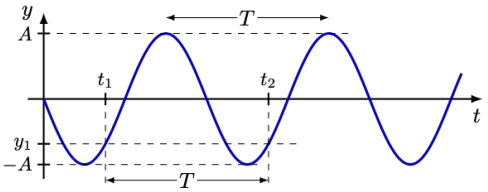

T= PERIOD

THE WAVE REPEATS EVERY TIME T

WHEN $x=0$




**(a)** Whole wave in space at time $t = 0$, given by $y(x, 0) = A\sin(kx)$.




**(b)** Local disturbance at position $x = 0$, given by $y(0, t) = -A\sin(\omega t)$.

**Figure 13.2:** A space and time slice of a travelling sine wave $y(x, t) = A\sin(kx - \omega t)$.

$y(x,t) = A\sin(kx - \omega t)$

$x=0 \quad y(0,t) = A\sin(-\omega t)$

TWO MOMENTS IN TIME $t_1$ AND $t_2 = t_1 + T$

MUST BE SEPARATED BY $2\pi$ RADIANS

$\omega(t_1 + T) = \omega t_1 + 2\pi$

$\omega T = 2\pi$

$$\omega = \frac{2\pi}{T}$$

ANGULAR VELOCITY

SUMMARY

WAVE $\lambda$, WAVELENGTH [m]

$T$, PERIOD [s]

$v$, VELOCITY [m/s]

THE VELOCITY OF THE WAVE IS

$$v = \frac{\lambda}{T} = f\lambda \qquad f = \frac{1}{T} \quad \left[\frac{1}{s}\right] = [Hz] \text{ FREQUEN}$$

THE WAVE NUMBER $K = \frac{2\pi}{\lambda} \left[\frac{1}{m}\right]$

THE ANGULAR VELOCITY $\omega = \frac{2\pi}{T} = 2\pi f \quad \left[\frac{1}{s}\right]$

$$v = \frac{\lambda}{T} = \frac{2\pi/K}{2\pi/\omega} = \frac{\omega}{K} \qquad v = \frac{\omega}{K}$$

EX: $y(x,t) = A \sin(Kx - \omega t) = A \sin\left(2\pi\frac{x}{\lambda} - 2\pi\frac{t}{T}\right)$

# WAVE EQUATION

WAVE FONCTION $y(x,t)$

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

PARTIAL DERIVATIVE $\frac{\partial}{\partial x}$

(SEE FULL DERIVATION IN SCRIPT)

ALL WAVE FONCTIONS SATISFY THE WAVE EQUATION

CHECK $y(x,t) = A\sin(Kx-\omega t)$

$$\frac{\partial y}{\partial x} = AK\cos(Kx-\omega t) \qquad \frac{\partial^2 y}{\partial x^2} = -AK^2\sin(Kx-\omega t)$$

$$\frac{\partial y}{\partial t} = -A\omega\cos(Kx-\omega t) \qquad \frac{\partial^2 y}{\partial t^2} = -A\omega^2\sin(Kx-\omega t)$$

IN WAVE EQUATION

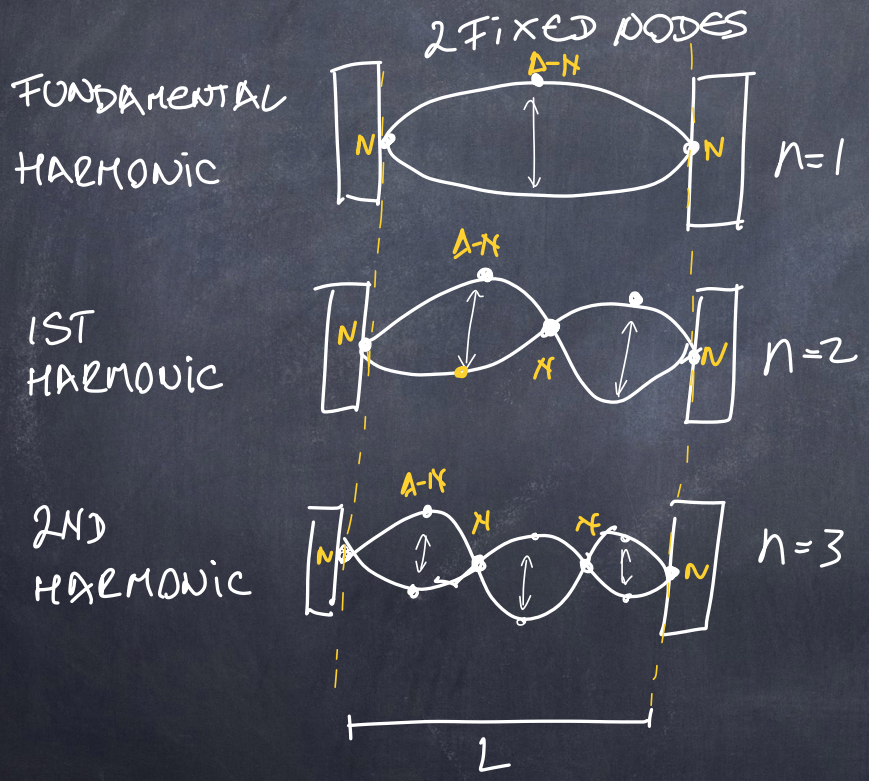
$$-AK^2\sin(Kx-\omega t) = \frac{1}{v^2}\left[-A\omega^2\sin(Kx-\omega t)\right]$$

$$-K^2 = -\frac{1}{v^2}\omega^2 \Rightarrow K^2 = \frac{\omega^2}{v^2} \Rightarrow v = \frac{\omega}{K}$$

TRUE, CHECK WORKS

# STANDING WAVES

WHEN WE CONFINE WAVES, THE WAVES WILL REFLECT AND COMBINE BY THE SUPERPOSITION PRINCIPLE THERE ARE RESONANT FREQUENCIES AND WAVELENGTH THAT CREATE STANDING WAVES



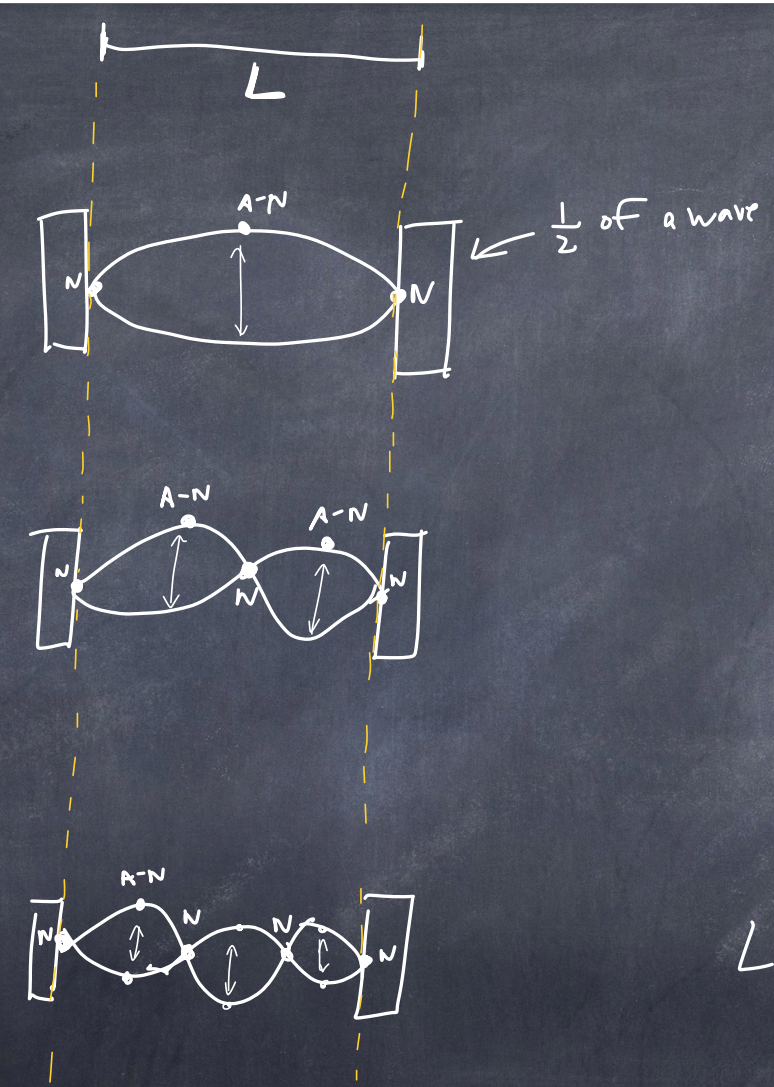

N NODES : POINTS THAT DO NOT MOVE

A-N ANTINODES : MOVE MAXIMALLY



L SIZE OF THE SPACE THAT THE WAVE IS CONFINED

HOW MANY $\lambda$ ARE THERE PER LENGTH




$n = 1$

$n = 2$

$n = 3$

$\frac{1}{2}$ of a wave

ONE $\lambda$ =

$L = \frac{\lambda}{2} \Rightarrow \lambda = 2L$

$L = \lambda$

$L = 1 + \frac{1}{2}\lambda = \frac{3}{2}\lambda$

IN GENERAL, THE WAVELENGTH OF THESE HARMONICS ARE $\lambda = \frac{L}{n} 2$ FOR $n = 1, 2, 3, \ldots$

THE FREQUENCIES OF THE $n^{th}$ HARMONICS

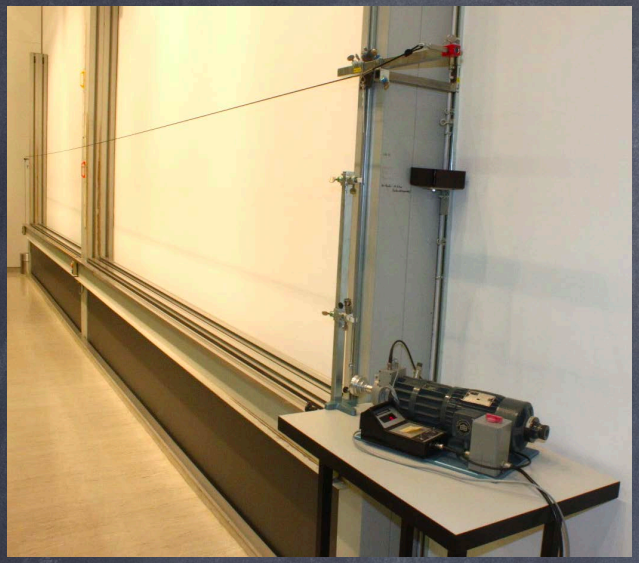
$$f_n = \frac{v}{\lambda_n} = \frac{v}{2L/n} = \frac{nv}{2L} \qquad \text{FOR } n = 1, 2, 3, \ldots$$

FOR $n = 1$ (1ST HARMONIC) $\quad f_1 = \frac{1 \cdot v}{2L} = \frac{v}{2L}$

IF WE KNOW $f_1$, $f_n = n f_1$

FOR A STRING, $v = \sqrt{\frac{T}{\mu}} \Rightarrow f_1 = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$

ALL STANDING WAVES ON A STRING HAVE A FREQUENCY THAT IS A MULTIPLE OF THE FIRST HARMONIC $f_1$, AND DEPEND ON TENSION, LENGTH, MASS



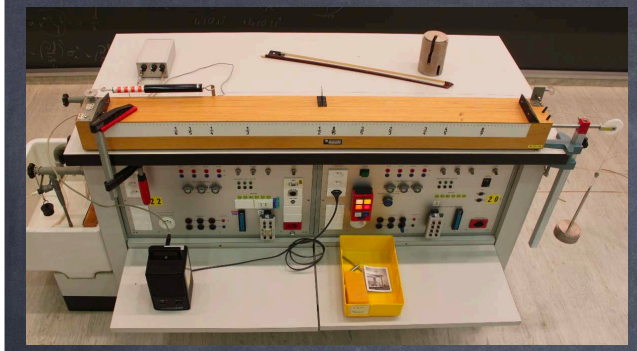
$f_m = m f_1$

FIRST WE FOUND $f_1$

THEN WE CAME UP WITH

$f_2 = 2 f_1$

$f_3 = 3 f_2$



1) $v = f_n \lambda_n \Rightarrow$ IF WE HALF THE WAVELENGTH FREQUENCIES SHOULD DOUBLE (WAVE HAS SAME VELOCITY)

2) $f_1 = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$ T TENSION

$\mu$ MASS PER LENGTH

T INCREASE BY A FACTOR 4 THEN $f_1$ WILL DOUBLE

STANDING WAVE FUNCTIONS

CONSIDER SUPERPOSITION OF WAVE MOVING TO THE RIGHT AND TO THE LEFT

$$y_R = A \sin(Kx - \omega t)$$

$$y_L = A \sin(Kx + \omega t)$$

$$y(x,t) = y_R + y_L = A\left[\sin(Kx - \omega t) + \sin(Kx + \omega t)\right]$$

SUBSTITUTION $\sin\theta_1 + \sin\theta_2 = 2\cos\frac{1}{2}(\theta_1 - \theta_2)\sin\frac{1}{2}(\theta_1 + \theta_2)$

$$\theta_1 = Kx - \omega t$$

$$\theta_2 = Kx - \omega t$$

$$\theta_1 + \theta_2 = 2Kx$$

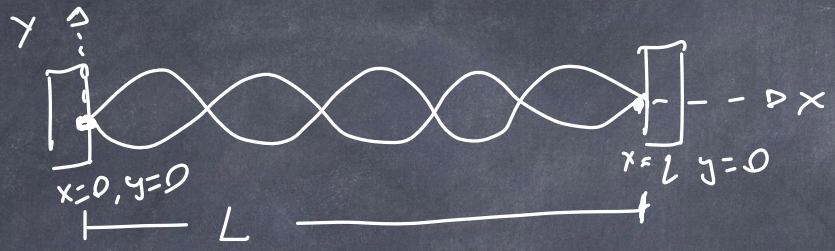
$$\theta_1 - \theta_2 = 2\omega t$$

$$y(x,t) = 2A\cos(\omega t)\sin(Kx)$$

WAVEFUNCTION OF AN STANDING WAVE WITH 2 FIXED NODES

NOT ANY ω, K WORK FOR STANDING WAVES

BOUNDARY CONDITIONS NEED TO BE APPLIED




AT $x=0$ AND $x=L$

$y=0$

WHEN DOES $y=0$? $\Rightarrow$ $\sin(KL)=0$

$K_n L = n\pi \quad n=1,2,3,\ldots$

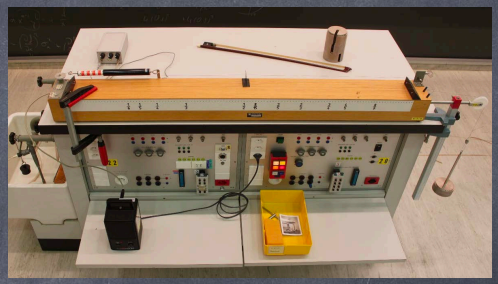
ALLOWED $K_n = \frac{n\pi}{L}$ $\left(\lambda = \frac{2\pi}{K}\right)$

WE KNOW THAT $f_n = \frac{v}{\lambda} = \frac{v n}{2L}$ $n=1,2,3,\ldots$

$f_1 = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$

SINCE $\omega_n = 2\pi f_n$

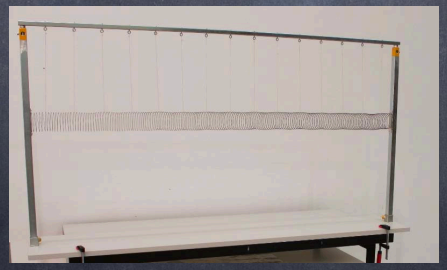
$\omega_n = 2\pi f_n = 2\pi n f_1$ FOR $n=1,2,3,\ldots$



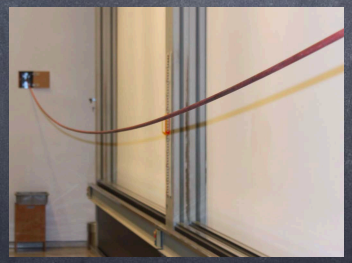
W14



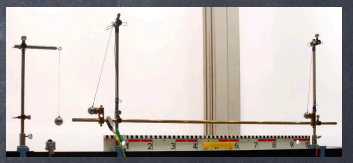
W18



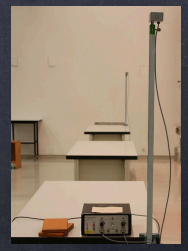
W24



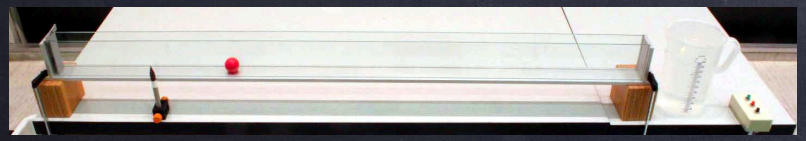
W28



W32



W33



W31