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**Exercise 1** [Gravitational field of a moving particle]

Consider a particle of mass  $M$  moving with constant velocity  $\mathbf{V}$ . Calculate the gravitomagnetic potential and the equation of motion for a test particle to order  $\mathcal{O}(v/c)$ .

*Hint:* Start in the particles rest frame  $\Sigma'$  with the equations (4.9) and (4.10) for the metric perturbation in the script and then perform a coordinate transformation to the global frame  $\Sigma$ . Since we are working at  $\mathcal{O}(h)$  the transformation is nothing than a LORENTZ boost.

**Exercise 2** [Particles in the field of a gravitational wave]

Show that the curves  $r = r(\varphi)$  described by

$$r^2(\varphi) = R^2 \begin{cases} 1 - 2h \cos(2\varphi) \cos(\omega t) \\ 1 - 2h \sin(2\varphi) \cos(\omega t) \end{cases} \quad (1)$$

for  $h \ll 1$  are ellipses. How is the eccentricity  $e$  related to  $h$ ?

*Hint:* Start from the following parametric form of an ellipse,

$$r(\varphi) = \frac{b}{\sqrt{1 - e^2 \cos^2 \varphi}}, \quad (2)$$

and assume  $e^2 \ll 1$ .

**Exercise 3** [Gravitational Bremsstrahlung]

The gravitational wave analogue of Bremsstrahlung can be generated by a small mass  $m$  scattering off a large mass  $M \gg m$  with impact parameter  $b$ . Assume that the large mass sits at  $(0, 0, 0)$ , that  $E = 0$  (parabolic orbit) and that the orbit lies in the x-y-plane. Calculate the gravitational wave amplitude at a position on the z-axis.

*Hint:* For the LORENTZ gauge trace reversed metric  $\gamma_{\mu\nu} = h_{\mu\nu} - 1/2 \eta_{\mu\nu} h$  and slowly moving sources the gravitational wave amplitude can be calculated as<sup>1</sup>

$$\gamma_{ij}(t, \mathbf{x}) = \frac{2G}{r} \frac{d^2 I_{ij}(t_r)}{dt^2}, \quad (3)$$

<sup>1</sup>See Carroll 2004 for derivation and restrictions of this formula.

where  $t_r$  is the retarded time and  $I_{ij}(t) = \int T_{00}(t)y^i y^j dy^3$  is the quadrupole tensor. The hyperbolic orbit of the small mass  $m$  is described by the parametric solution

$$r(\varphi) = \frac{2b}{1 + \cos(\varphi)} \quad \dot{\varphi} = \sqrt{\frac{M}{8b^3}} [1 + \cos(\varphi)]^2. \quad (4)$$